Preface

It is often averred that two contrasting cultures coexist in mathematics—the theory-building culture and the problem-solving culture. The present volume was certainly spawned by the latter. This book takes an array of specific problems and solves them, with the needed tools developed along the way in the context of the particular problems.

The book is an unusual hybrid. It treats a mélange of topics from combinatorial probability theory, multiplicative number theory, random graph theory, and combinatorics. Objectively, what the problems in this book have in common is that they involve the asymptotic analysis of a discrete construct, as some natural parameter of the system tends to infinity. Subjectively, what these problems have in common is that both their statements and their solutions resonate aesthetically with me.

The *results* in this book lend themselves to the title "Problems from the Finite to the Infinite"; however, with regard to the *methods of proof*, the chosen appellation is the more apt. In particular, generating functions in their various guises are a fundamental bridge "from the discrete to the continuous," as the book's title would have it; such functions work their magic often in these pages. Besides bridging discrete mathematics and mathematical analysis, the book makes a modest attempt at bridging disciplines—probability, number theory, graph theory, and combinatorics.

In addition to the considerations mentioned above, the problems were selected with an eye toward accessibility to a wide audience, including advanced undergraduate students. The technical prerequisites for the book are a good grounding in basic undergraduate analysis, a touch of familiarity with combinatorics, and a little basic probability theory. One appendix provides the necessary probabilistic background, and another appendix provides a warm-up for dealing with generating functions. That said, a moderate dose of the elusive quality known as mathematical maturity will certainly be helpful throughout the text and will be necessary on occasion.

The primary intent of the book is to introduce a number of beautiful problems in a variety of subjects quickly, pithily, and completely rigorously to graduate students and advanced undergraduates. The book could be used for a seminar/capstone course in which students present the lectures. It is hoped that the book might also be of interest to mathematicians whose fields of expertise are away from the subjects treated herein. In light of the primary intended audience, the level of detail in proofs is a bit greater than what one sometimes finds in graduate mathematics texts.

I conclude with some brief comments on the novelty or lack thereof in the various chapters. A bit more information in this vein may be found in the chapter notes at the end of each chapter. Chapter 1 follows a standard approach to the problem it solves. The same is true for Chap. 2 except for the probabilistic proof of Theorem 2.1, which I haven't seen in the literature. The packing problem result in Chap. 3 seems to be new, and the proof almost certainly is. My approach to the arcsine laws in Chap. 4 is somewhat different than the standard one; it exploits generating functions to the hilt and is almost completely combinatorial. The traditional method of proof is considerably more probabilistic. The proofs of the results in Chap. 5 on the distribution of cycles in random permutations are almost exclusively combinatorial, through the method of generating functions. In particular, the proof of Theorem 5.2 makes quite sophisticated use of this technique. In the setting of weighted permutations, it seems that the method of proof offered here cannot be found elsewhere. The number theoretic topics in Chaps. 6–8 are developed in a standard fashion, although the route has been streamlined a bit to provide a rapid approach to the primary goal, namely, the proof of the Hardy–Ramanujan theorem. In Chap. 9, the proof concerning the number of cliques in a random graph is more or less standard. The result on tampering detection constitutes material with a new twist and the methods are rather probabilistic; a little additional probabilistic background and sophistication on the part of the reader would be useful here. The results from Ramsey theory are presented in a standard way. Chapter 10, which deals with the phase transition concerning the giant component in a sparse random graph, is the most demanding technically. The reader with a modicum of probabilistic sophistication will be at quite an advantage here. It appears to me that a complete proof of the main results in this chapter, with all the details, is not to be found in the literature.

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