Contents

1 Basic Concepts ........................................... 1
  1.1 Introduction: Whys and Hows of Quantum Many-Body Theory .................................................. 1
    1.1.1 Screening of Coulomb Potential in Metal .............. 3
    1.1.2 Time-Dependent Effects: Plasmons .................... 6
  1.2 Propagation Function in a One-Body Quantum Theory ...... 8
    1.2.1 Propagator: Definition and Properties .............. 8
    1.2.2 Feynman’s Formulation of Quantum Mechanics: Path (Functional) Integrals ..................... 13
    1.2.3 Quantum Transport in Mesoscopic Rings: Path Integral Description ......................... 20
  1.3 Perturbation Theory for the Propagator .................. 24
    1.3.1 General Formalism .................................. 24
    1.3.2 An Example: Potential Scattering ................... 30
  1.4 Second Quantization ................................... 33
    1.4.1 Description of Large Collections of Identical Particles: Fock’s Space ....................... 33
    1.4.2 Bosons ............................................. 36
    1.4.3 Number and Phase Operators and Their Uncertainty Relation .................................. 43
    1.4.4 Fermions .......................................... 46
  1.5 Problems ............................................. 49
  References .............................................. 50

2 Green’s Functions at Zero Temperature ......................... 53
  2.1 Green’s Function of The Many-Body System: Definition and Properties ................................. 53
    2.1.1 Definition of Green’s Functions of the Many-Body System ...................................... 53
    2.1.2 Analytic Properties of Green’s Functions .............. 62
    2.1.3 Retarded and Advanced Green’s Functions ................ 67
    2.1.4 Green’s Function and Observables ..................... 70
2.2 Perturbation Theory: Feynman Diagrams
   2.2.1 Derivation of Feynman Rules. Wick’s and Cancellation Theorems
   2.2.2 Operations with Diagrams. Self Energy. Dyson’s Equation
   2.2.3 Renormalization of the Interaction. Polarization Operator
   2.2.4 Many-Particle Green’s Functions. Bethe–Salpeter Equations. Vertex Function

2.3 Problems

References

3 More Green’s Functions, Equilibrium and Otherwise, and Their Applications
   3.1 Analytic Properties of Equilibrium Green’s Functions
      3.1.1 Statistical Operator (Density Matrix): The Liouville Equation
      3.1.2 Definition and Analytic Properties of Equilibrium Green’s Functions
   3.2 Matsubara Formalism
      3.2.1 Bloch’s Equation
      3.2.2 Temperature (Matsubara) Green’s Function
      3.2.3 Perturbation Series and Diagram Techniques for the Temperature Green’s Function
   3.3 Linear Response Theory
      3.3.1 Linear Response Theory: Kubo Formulas
      3.3.2 Fluctuation-Dissipation Theorem
   3.4 Nonequilibrium Green’s Functions
      3.4.1 Nonequilibrium Causal Green’s Function: Definition
      3.4.2 Contour Ordering and Three More Nonequilibrium Green’s Functions
      3.4.3 The Keldysh Formalism
   3.5 Quantum Kinetic Equation
      3.5.1 Dyson’s Equations for Nonequilibrium Green’s Functions
      3.5.2 The Quantum Kinetic Equation
   3.6 Application: Electrical Conductivity of Quantum Point Contacts
      3.6.1 Quantum Electrical Conductivity in the Elastic Limit
      3.6.2 Elastic Resistance of a Point Contact: Sharvin Resistance, the Landauer Formula, and Conductance Quantization
3.6.3 The Electron–Phonon Collision Integral in 3D Quantum Point Contact .................. 145
3.6.4 Calculation of the Inelastic Component of the Point Contact Current .................. 147

3.7 Method of Tunneling Hamiltonian .................................................. 149
3.8 Problems .................................................................................... 154
References ...................................................................................... 155

4 Methods of the Many-Body Theory in Superconductivity ............. 157
4.1 Introduction: General Picture of the Superconducting State ...... 157
4.2 Instability of the Normal State ..................................................... 168
4.3 Pairing (BCS) Hamiltonian .......................................................... 172
   4.3.1 Derivation of the BCS Hamiltonian ....................................... 172
   4.3.2 Diagonalization of the BCS Hamiltonian: The Bogoliubov Transformation—Bogoliubov-de Gennes Equations ....................... 175
   4.3.3 Bogolons ........................................................................... 177
   4.3.4 Thermodynamic Potential of a Superconductor ................. 179
4.4 Green’s Functions of a Superconductor: The Nambu-Gor’kov Formalism .................................................. 181
   4.4.1 Matrix Structure of the Theory .......................................... 181
   4.4.2 Elements of the Strong Coupling Theory ......................... 182
   4.4.3 Gorkov’s Equations for the Green’s Functions ............... 185
   4.4.4 Current-Carrying State of the Superconductor ............ 189
   4.4.5 Destruction of Superconductivity by Current .............. 194
4.5 Andreev Reflection ................................................................. 197
   4.5.1 The Proximity Effect in a Normal Metal in Contact with a Superconductor .......... 203
   4.5.2 Andreev Levels and Josephson Effect in a Clean SNS Junction ............................... 204
   4.5.3 Josephson Current in a Short Ballistic Junction: Quantization of Critical Current in Quantum Point Contact ........................................ 206
   4.5.4 Josephson Current in a Long SNS Junction ..................... 209
   4.5.5 Transport in Superconducting Quantum Point Contact: The Keldysh Formalism Approach ........................................... 215
4.6 Tunneling of Single Electrons and Cooper Pairs Through a Small Metallic Dot: Charge Quantization Effects .................................................. 217
   4.6.1 Coulomb Blockade of Single-Electron Tunneling ............ 218
   4.6.2 Superconducting Grain: When One Electron Is Too Many ............... 220
4.7 Problems .................................................................................... 223
References ...................................................................................... 224
Quantum Theory of Many-Body Systems
Techniques and Applications
Zagoskin, A.
2014, XVI, 280 p. 154 illus., Hardcover
ISBN: 978-3-319-07048-3