A few days before leaving Prague, after 15 months spent at the German part of Charles-Ferdinand University, Albert Einstein submitted a paper titled *Relativity and Gravitation. Reply to a Comment by M. Abraham*. It was received by Annalen der Physik on July 4, 1912. Here Einstein summarized the contemporary state of his relativistic theory of gravitation and, remarkably, anticipated what a future theory of gravity should look like.\(^1\)

“Relativity and Gravitation: 100 years after Einstein in Prague,” was the name of the conference held in Prague on June 25–29, 2012, inspired by the title, date, and significance of this last of Einstein’s Prague papers. The aim of the conference was twofold. First, it was to review the present status of the general theory of relativity (both classical and quantum) and its applications in cosmology and astrophysics from a broad perspective. The second aim was to present the newest results in each of these fields. This volume is based on the invited plenary lectures at the conference. In another volume, “Relativity and Gravitation: 100 years after Einstein’s stay in Prague,” appearing in the “Springer Proceedings in Physics,” articles based on contributed talks and posters are included; more on cultural and other events associated with the conference is recalled therein.

The articles included in this volume represent a broad and highly qualified view on the present state of general relativity, quantum gravity, and their cosmological and astrophysical implications. As such, it may serve as a valuable source of knowledge and inspiration for experts in these fields, as well as an advanced source of information for young researchers.

The contents is divided into four broad parts: (i) Gravity and Prague, (ii) Classical General Relativity, (iii) Cosmology and Quantum Gravity, and (iv) Numerical Relativity and Relativistic Astrophysics.

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\(^1\) See the contribution by J. Bičák in this Volume.
Gravity and Prague

In the first contribution, Julian Barbour “honors” Kepler and Mach for their work and fundamental discoveries made in Prague. Barbour starts with Greek planetary astronomy, continues with Copernicus, and comes to a profound description of the work of Kepler, “the true discoverer of heliocentricity.” After a short intermezzo about Doppler, he comes to Ernst Mach. Mach profoundly influenced several generations of experimentalists (not only) in Prague. However, within the context of the conference, the most important issue was Mach’s influence on Einstein in the formulation of general relativity. At the same time, though, Einstein sometimes “distorted” Mach. A novel conception of Mach’s principle within geometrodynamics, called by Barbour the “shape dynamics,” concludes Barbour’s excursion through almost 2,200 years.

Jiří Bičák describes how and why Einstein was invited to Prague, and what his influence was on Czech science and culture. The main themes that occupied him then were the principle of equivalence, the bending of light, gravitational lensing, gravitational redshift, and frame dragging effects. In the article, these topics are discussed from a present-day perspective. Perhaps most importantly, just before leaving Prague, Einstein summarized his views on what basic features a new theory of gravity should possess, including the invariance with respect to a larger group than the Lorentz group, the local significance of the equivalence principle, or the nonlinearity of the field equation for gravity.

Classical General Relativity

The problem of measurement, of precise definition of observers and observables they employ, has been one of the most distinct features of both special and general relativity since their birth. Donato Bini summarizes the results obtained by his colleagues and him during the last two decades on the “measurement process.” The process involves clearly defined geometrical and physical quantities arising from an identification of “space” and “time” relative to a given observer within a congruence of timelike worldlines.

A novel theme, unimaginable to be discussed 100 or even 20 years ago, is reviewed in Gary Gibbons’ article: The Role of General Relativity in Other Parts of Physics. Gibbons concentrates on a number of specific problems in which geometrical ideas are employed: the description of shallow water waves is analogous to the behavior of the rays in the Schwarzschild metric or around straight cosmic strings. In optics, examples of a left-handed light moving in a medium with a negative refractive index can be described with the help of hyperbolic or Lobachevsky space. Zermelo’s problem of minimizing the travel time of a boat moving with a fixed speed in a Riemannian metric under the presence of a “wind” is discussed using Finsler geometry. Other problems include: invisibility cloaks,
hyperbolic metamaterials, gravitational kinks, Bloch walls, liquid crystal droplets, and last but not least the popular subject of graphene.

A very old theme in general relativity is the N-body problem. For over 30 years a leading figure in this field has been Thibault Damour. He first reviews ongoing post-Newtonian calculations, continuing to still higher orders in v/c, and combined with formalisms yielding gravitational waveforms. Next he describes the effective field theory approach employing diagrammatic methods of quantum field theory, numerical relativity simulations, and gravitational self-force theory. The main attention is then paid to the “Effective One Body” (EOB), formalism that Damour and his collaborators started to develop at the end of the 1990s. The goal of the EOB formalism is to obtain an analytical description of the motion and radiation of coalescing binaries during the whole process, from inspiral to the final black-hole ring-down. The results obtained by employing the EOB formalism are compared with those coming from numerical relativity and self-force computations. The ways in which EOB theory may progress are indicated and the conclusion is taken from Henri Poincaré: “There are no solved problems, there are only more-or- less solved problems... .”

The article by Leor Barack on the gravitational self-force follows. Originally, the self-force theory was developed for so-called “Extreme Mass Ratio Inspirals” (EMRIs), in which a test particle (say a 10M\(_{\odot}\) black hole) moves along a geodesic of a stationary background geometry of a large mass (say a 10\(^6\)M\(_{\odot}\) black hole). If the mass of the particle is taken into account, the effect of the perturbation of the background geometry on the particle gives rise to a gravitational self-force. To calculate the resulting deviation from the geodesic motion is a formidable task. Barack, a principal author of a powerful, practical method for doing this for EMRI orbits gives a brief but comprehensive review of both successes and difficulties. As an illustration, important cases of the self-force effects are discussed: the induced shift in the frequency of the innermost stable circular orbit or the correction to the periastron shift.

For a number of years, the problem of motion has been studied by Gerhard Schäfer and his collaborators concentrating on the use of the Hamiltonian treatments. The Hamiltonian formalisms of Arnowitt-Deser and Misner (ADM), Dirac and Schwinger are compared. The results based on the ADM approach are combined with the post-Newtonian/post-Minkowskian approximations. Lastly, Schäfer tells how the complicated problem of incorporating the spin of the particles into the formalism has been recently analyzed.

The article by Marc Mars focuses on geometric inequalities involving physical quantities like mass, charge, area, or angular momentum. This work is interwoven by definitions, propositions, lemmas, and theorems, though it also contains helpful intuitive remarks. The best known inequalities are the famous positive mass theorem and the Penrose inequality (the theorem under the presence of trapped surfaces). More recently, with new concepts of Marginally Outer Trapped Surfaces (MOTS) and dynamical horizons, a great interest arose in inequalities considering area and angular momentum. Other inequalities involving charge, the cosmological constant, and the topology of MOTS’s, are concisely reviewed.
It has been a long-standing problem whether the spin–spin interaction between two black holes can balance their gravitational attraction. The article by Gernot Neugebauer and Jörg Hennig, based on Neugebauer’s talk in Prague, provides a survey of the numerous papers addressing this problem in the past. It also describes the rigorous non-existence proof based on the formulation of a boundary value problem for the nonlinear Ernst equation under the assumption of the existence of two disconnected Killing horizons.

Robert Wald’s article discusses a recently obtained dynamical stability criterion for black holes in $D \geq 4$ spacetime dimensions. This relates stability with respect to axisymmetric perturbations to the positivity of a certain canonical energy. This energy is determined by second variations in the mass, angular momentum, and horizon area like those appearing in the first law of black hole thermodynamics. One consequence is that black branes, corresponding to thermodynamically unstable black holes, are dynamically also unstable. Wald concludes that “The remarkable relationship between the laws of black hole physics and the laws of thermodynamics [...] extends to dynamical stability.”

A brief contribution by Piotr Bizoń and Andrzej Rostworowski, based on the talk given by Rostworowski, summarizes their numerical and perturbative work on the instability of anti-de Sitter spacetime based on numerical and perturbative calculations. Results for spherically symmetric massless scalar fields strongly indicate that anti-de Sitter space is unstable to arbitrarily small perturbations—eventually forming black holes.

Higher dimensional black holes within classical general relativity in higher dimensions are discussed in the last two articles of this part. Harvey Reall analyzes stationary vacuum solutions within (i) Kaluza–Klein theory and (ii) general relativity. In addition to Myers-Perry black holes, solutions include the famous black rings and black Saturns found by Reall and Roberto Emparan. Generalizations of these are mentioned, as well as the problem of instabilities and the search for perturbative solutions.

The next article is based on the talk by Valeri Frolov. It concentrates on black holes with topologically spherical horizons but non-vanishing cosmological constant and NUT parameter. Crucial is the existence of the principal conformal Killing-Yano tensor which is admitted by Kerr-NUT-AdS metrics in four and higher dimensions. This object enabled Carter in 1968 to separate geodesic and wave equations in the four-dimensional Kerr geometry. The principal conformal Killing-Yano tensor allows similar results to be derived in higher dimensions as well.

Cosmology and Quantum Gravity

Lars Andersson’s article deals with cosmological models and their stability. The principle of equivalence and Mach’s principle are mentioned and shown to have roles different from the “hierarchy of cosmological principles.” A brief discussion
follows of cosmological models and issues such as the coincidence problem and the role of inhomogeneities. The second part has a more technical character: The asymptotics and nonlinear stability of various models is surveyed, including de Sitter space, the deformed Milne model, and generalized anisotropic Kasner models. The present status of the Belinskii, Khalatnikov, Lifshitz proposal is also discussed.

The contribution by Misao Sasaki discusses inflation and the birth of cosmological perturbations. Inflationary universe, despite various objections, is being accepted by more and more active cosmologists not only because it helps to explain various problems of the standard big-bang theory, but also appears to be in agreement with new observational data on the anisotropy of the microwave background. Sasaki describes slow-roll inflation, the curvature perturbations arising from vacuum fluctuations of the inflation field, and tensor perturbations of the metric. Possible primordial non-Gaussianities, if observed, could significantly constrain cosmological models. The review concludes with a brief description of a powerful formalism which allows curvature perturbations to be calculated on superhorizon scales.

What preceded inflation? This question is asked and an answer is suggested in the article on Loop Quantum Cosmology (LQC) by Abhay Ashtekar. His discovery of new variables in 1986 initiated the birth of the Loop Quantum Gravity (LQG). Ashtekar suggests that “even though we are far from a complete theory [of quantum gravity], advances can occur by focusing on specific physical problems.” One “grand” problem is the LQC based on a truncated LQG. It concentrates on (i) the resolution of the “initial” singularity, (ii) the formulation of effective LQC dynamics which leads to inflation, and (iii) the extension of cosmological perturbation theory to the Planck regime. Ashtekar reviews progress in these directions. He also indicates how LQC relates initial conditions (at a bounce) with observations.

One open aspect of inflation is the issue of how quantum fluctuations in the inflaton field transmute into observed classical inhomogeneities. This problem is addressed by Daniel Sudarsky. In the standard “philosophy” of quantum theory there exists a collapse of the wave function during a measuring process. How can this happen in the Universe? Some believe in Everett’s many world interpretation. More radical ideas require “novel physics,” such as gravity-induced collapse of the wave function. Such an approach is adopted by Sudarsky. He investigates it within a semiclassical treatment of gravity interacting with quantum fields during inflation.

A broad view on the state of quantum gravity (QG) from a particle physics perspective is given by Hermann Nicolai. It covers a large territory in a fairly nontechnical style. There are difficulties in both general relativity and quantum field theory associated with the use of a continuum at and below Planck-scale distances. The principle differences in the approaches to QG between two main candidates, string theory and LQG, are elucidated and considered to be a sign that “we are probably still very far from the correct answer!” A number of other issues that should be answered by a future theory of QG include the divergence problem.
of perturbative general relativity, the hierarchy problem (the smallness of $G$), and the question of whether the Standard Model of particle physics remains true up to the Planck scale. The incompleteness of the Standard Model is considered as “one of the strongest arguments in favor of quantizing gravity and searching for new concepts replacing classical notions of space and time.”

**Numerical Relativity and Relativistic Astrophysics**

Relativistic astrophysics celebrated its 50th birthday in December 2013. Seven articles included in this part demonstrate remarkably well the enormous progress this field experienced in the last half century.

Luciano Rezzolla illustrates the progress in numerical relativity in three examples: (1) numerical calculations of a binary merger of two neutron stars producing an extremely strong magnetic field, (2) numerical calculations leading to a black hole due to the collision of two self-gravitating fluids moving toward each other with ultra-relativistic velocities, (3) numerical study of the recoil dynamics of a black hole formed from head-on collision of two black holes with different masses. The presence of an “anti-kick” is studied from various points of view, including instructive figures, observational data, and issues of interest in mathematical relativity.

A comprehensive review on instabilities of relativistic stars is given by John Friedman and Nikolaos Stergioulas. It represents an extension of the talk given by John Friedman in Prague and is partially based on the book *Rotating Relativistic Stars* published by the authors in 2013. An action for perturbations leads to the canonical energy and momentum and to the criterion for stability. Various types of instabilities have been analyzed: convective instability, axisymmetric instability, nonaxisymmetric instabilities leading to the formation of bar modes, and instabilities driven by gravitational waves like the Chandrasekhar-Friedman-Schutz (CFS) instability. The CFS instability is primarily analyzed and questions of how it can be influenced by the complex physics in neutron stars are discussed.

In “Gravity talks: observing the universe with gravitational waves” (GW), Bernard Schutz indicates that, indeed, the detectors are like microphones, not pointed in some direction; and phases of the waves are more important than the amplitudes. The network of six GW detectors is portrayed eloquently. In this field, data analysis is most important. Its use at present is demonstrated by Einstein@Home platform which led to the discovery of new pulsars. The information we can obtain from detection (expected to occur by 2017) not only yields masses and spins of colliding black holes or neutron stars but also, for example, the distance to the sources. The properties of likely sources (neutron star or black hole binary coalescence, neutron stars interiors and pulsars) are discussed. The article concludes with information about LISA, substituted by somewhat restricted
eLISA, which should be launched if the LISA Pathfinder is successful. The enormous advantages of detectors in space are highlighted.

**Gerhard Heinzel** (talking in Prague), and **Karsten Danzmann**, write about the LISA and eLISA missions in greater detail, focusing also on financial problems. It became clear in March 2011 that the originally designed joint ESA/NASA Mission cannot rely on support from NASA. The redesigned eLISA (“evolved” LISA) could be supported by ESA alone. Despite the cost reduction, thousands of compact white dwarfs binaries and hundreds of black hole binaries inspirals could still be observed. The authors conclude that “the technology is well developed, the team is strong and convinced that LISA must fly in the early 2020s…”

A wide range of relativistic effects, including the strong-field regime, can be investigated by observing pulsars. **Michael Kramer** wrote a readable and comprehensive article on the current state-of-the art experiments involving these “cosmic lighthouses.” Going from individual pulsars, he shows how about 10% of about 2,000 known pulsars are in binary systems. These enable tests of general relativity and alternative theories with extraordinary precision. The effects include the precession of periastron, gravitational redshift, Shapiro delay, GW, and spin-orbit coupling; at the same time, the basic principles of the theory are tested. The detailed properties of the famous Hulse-Taylor pulsar are summarized and, in particular, of the double pulsar. It provides tests of the GW quadrupole formula far below the 0.1% level. The masses and the orbital and relativistic spin-precession are measured with extremely high accuracy. There are good prospects of detecting GW by the “Pulsar Timing Array” method and a great challenge exists to test fundamental predictions like the “no-hair theorem” if a pulsar orbiting the black hole in the center of our Galaxy is discovered with the future Square Kilometer Array.

In the last two articles, based on talks by **Marek Abramowicz** and by **Ramesh Narayan** (with co-authors **Jeffrey McClintock** and **Alexander Tchekhovskoy**), regions of extreme gravity effects are discussed. Three instruments planned for missions in the near future (e.g. “The event horizon telescope”) will provide angular and time resolution that will enable us to investigate the immediate neighborhoods of event horizons, ergospheres, innermost stable circular orbits (ISCO), and circular photon orbits around black holes. All these features of black holes are discussed in Abramowicz’s review, the main attention being paid to ISCO because here the standard paradigm might have to be modified by the magneto-rotational instability leading to turbulence around ISCO.

In the last contribution, the authors deal with a long-standing, complex, and fascinating astrophysical problem: how jets from both stellar-mass and supermassive black holes form, and how are they powered. Results of recent computer simulations of black hole accretion and jets, in which magnetic fields twisted by the rotating black hole play an essential role, combined with recent observations, imply that (i) jets are powered from the black hole energy (rather than from a surrounding disk), (ii) “the first observational evidence for a correlation between jet power and black hole spin has finally been obtained.”
The conference was organized\(^2\) under the auspices of the Rector of Charles University, the oldest university north of the Alps (founded in 1348). It was sponsored by the Faculty of Mathematics and Physics of Charles University, and, in particular, by the Karel Janeček Foundation.\(^3\) We also acknowledge a continuous support of the Czech Science Foundation, now under the grant No. 14-37086G (Albert Einstein Centre).

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The local organizing committee included: J. Bičák, M. Bursa, P. Hadrava, D. Heyrovský, V. Karas, D. Kofroň, P. Krtouš, J. Langer, T. Ledvinka, J. Podolský, V. Pravda, O. Semeráč, Z. Stuchlík, O. Svítek, V. Špička, and M. Žofka, all of them associated (now or in the past) with the Institute of Theoretical Physics of the Faculty of Mathematics and Physics of the Charles University in Prague.

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