Chapter 2
Hydrodynamics

Nomenclature

\( A \) \hspace{1cm} \text{Cross-sectional area of bed, m}^2
\( a \) \hspace{1cm} \text{Decay constant in axial voidage profile in Eq. (2.17), m}^{-1}
\( C \) \hspace{1cm} \text{Gas concentration}
\( C_1 \) \hspace{1cm} \text{Constant in Eq. (2.3)}
\( C_2 \) \hspace{1cm} \text{Constant in Eq. (2.3)}
\( d_p \) \hspace{1cm} \text{Surface volume mean diameter of bed particles, m}
\( D \) \hspace{1cm} \text{Diameter of bed, m}
\( D_{bmax} \) \hspace{1cm} \text{Maximum stable bubble size, m}
\( D_{eq} \) \hspace{1cm} \text{Equivalent diameter of furnace cross section, m}
\( D_r, D_a \) \hspace{1cm} \text{Radial and axial dispersion coefficients}
\( F \) \hspace{1cm} \text{Mass fraction of particles less than 45 \( \mu \)m}
\( F_D \) \hspace{1cm} \text{Drag force on particles N}
\( G_s \) \hspace{1cm} \text{Net solid circulation rate, kg/m}^2\text{s}
\( G_u \) \hspace{1cm} \text{Upward solid flux, kg/m}^2\text{s}
\( G_d \) \hspace{1cm} \text{Downward solid flux, kg/m}^2\text{s}
\( g \) \hspace{1cm} \text{Acceleration due to gravity, 9.81 m/s}^2
\( L \) \hspace{1cm} \text{Height of a section of bed, m}
\( H \) \hspace{1cm} \text{Total height of the furnace measured above the distributor, m}
\( h \) \hspace{1cm} \text{Height above the grate in the bed, m}
\( h_i \) \hspace{1cm} \text{Height of location of point of inflexion, m}
\( h_o \) \hspace{1cm} \text{Characteristics height in Eq. (2.15), m}
\( r \) \hspace{1cm} \text{Radial distance from the axis of the bed, m}
\( R \) \hspace{1cm} \text{Radius of the bed, m}
\( P \) \hspace{1cm} \text{Pressure, N/m}^2
\( u_c \) \hspace{1cm} \text{Onset velocity for turbulent fluidization, m/s}
\( u_k \) \hspace{1cm} \text{Velocity for completion of transition to turbulent fluidization, m/s}
\( U \) \hspace{1cm} \text{Superficial gas velocity, m/s}
In a circulating fluidized bed (CFB) boiler, hot solids circulate around an endless loop carrying heat from burning fuels to heat-absorbing surfaces and to the flue gas leaving the furnace (see Fig. 1.3, Chap. 1). Here, solids pass through a number of hydrodynamic regimes in different sections of the boiler. These are as follows:
Most of the combustion and sulfur capture reaction takes place in the upper furnace, which is above the secondary air injection level. This zone operates under a special hydrodynamic condition termed “fast fluidization” a member of a family of gas–solid contacting operations broadly classified as fluidization. A CFB boiler has a number of novel features, such as fuel flexibility, low NOx emission, good combustion efficiency, good limestone utilization for sulfur capture, excellent mixing, and fewer feed points. These result directly from this special mode of gas–solid motion in its furnace, a deviation from which results in reduced performance.

The hydrodynamic condition dictates auxiliary power consumption, heat absorption, temperature distribution, combustion condition, bed inventory, erosion to name a few. For optimum design and reliable operation of a CFB boiler, a good comprehension of the hydrodynamics of the fluidized bed boiler is essential. The following section describes some of the above-mentioned hydrodynamic conditions, with special emphasis on the fast fluidization.

### 2.1 Regimes of Fluidization

Fluidization is defined as the operation through which granular solids are transformed into a fluid-like state through contact with either a gas or a liquid. Under the fluidized state, the gravitational pull on granular solid particles is offset by the fluid drag on them. Thus, the particles remain in a semi-suspended condition. A fluidized bed displays characteristics similar to those of a liquid, as explained below with the help of Fig. 2.1.

1. An object denser than the bulk of the bed will sink, while one lighter than the bed will float. Thus, a heavy steel ball sinks in the bed, while a light badminton shuttlecock floats on the surface (Fig. 2.1a).

2. The solids from the bed may be drained like a liquid through an orifice at the bottom or on the side, and its static pressure at any height is approximately equal to the weight of bed solids per unit cross section above that level. For this reason, we note in Fig. 2.1b, solids flow out like a liquid jet, and the lower jet goes further than the upper one as one expects from a tank filled with liquid.
3. The bed surface maintains a horizontal level, irrespective of how the bed is tilted. Also, the bed assumes the shape of the vessel (Fig. 2.1c).
4. Particles are well mixed, and the bed maintains a nearly uniform temperature throughout its body when heated from whatever side.

When the superficial gas velocity through a bed of granular solids increases, one notes the changes in the mode of gas–solid contact in many ways. With changes in gas velocity, the bed moves from one state or regime to another.

Like any reactor, a boiler furnace operates in its distinct gas–solid contacting regime. Figure 2.2 presents a regime diagram illustrating the presence of those regimes in different types of boiler furnaces. It shows that the volume fraction of solids in the combustion zone decreases continuously with increase in superficial gas velocity in the furnace or the combustion zone. Here we note that stoker-fired boilers use the densest combustion zone, while pulverized coal (PC) boilers use the leanest. The furnace of a CFB boiler lies between these two extremes.

Table 2.1 presents a comparison of some characteristic features of different gas–solid processes used in various types of boilers. It should be noted that the term “bed” has been used loosely in Table 2.1 and elsewhere in the text. It refers to a body of gas–solid in one of the above contacting modes.

### 2.1.1 Packed Beds

A bed of particles sitting stationary on a perforated grid through which a gas is passing is generally referred to as a fixed or packed bed (Fig. 2.3a). In moving packed beds, the solids move with respect to the walls of the column, like those in the return leg of a CFB boiler. In either case, the particles do not move relative to each other. As the gas flows through the solids, it exerts a drag force on the
particles, causing a pressure drop across the bed. The pressure drop through unit height of a packed bed of uniformly sized particles, $\Delta P/L$, is correlated as (Ergun 1952)

$$\frac{\Delta P}{L} = 150\frac{(1 - \varepsilon)^2}{\varepsilon^3} \frac{\mu U}{(\phi d_p)^2} + 1.75\frac{(1 - \varepsilon)}{\varepsilon^3} \frac{\rho_g U^2}{\phi d_p}$$  \hspace{1cm} (2.1)

where $U$ is the superficial gas velocity, defined as the gas flow rate per unit cross section of the bed, $\varepsilon$ is the void fraction in the bed, and $d_p$ and $\phi$ are the diameter and sphericity (see Appendix I) of bed solids, respectively. $\mu$ is the dynamic viscosity, and $\rho_g$ is the density of the gas.

**Fig. 2.2** Different commercial combustion systems operate under different gas–solid flow regimes. A stoker-fired boiler burns coal in a fixed bed (E) having the highest density and lowest gas velocity (relative to the terminal velocity of average particles in the bed). It is followed by a bubbling fluidized bed (A) boiler; an Ignifluid boiler, where the bed is in turbulent fluidization (B); a CFB boiler, where upper parts of the combustor, in fast fluidization (C); and pulverized coal firing where the coal burns under entrained or pneumatic transport condition (D).
2.1.2 Bubbling Fluidized Beds

As the gas flow rate through the fixed bed is increased, the pressure drop continues to rise according to Eq. (2.1), until the superficial gas velocity reaches a critical value known as the minimum fluidization velocity, \( U_{mf} \). This is defined as the velocity at which the fluid drag is equal to the weight of particles less its buoyancy. At this stage, the particles feel “weightless,” and the fixed bed transforms into an incipiently fluidized bed. In this regime, the body of solids behaves like a liquid, as illustrated in Fig. 2.1. Since the pressure drop across the bed equals the weight of the bed, the fluid drag \( F_D \) is written as

\[
F_D = \Delta P A = AL(1 - \varepsilon)(\rho_p - \rho_g)g
\]

(2.2)

where \( A \) and \( L \) are the cross section and height of the bed, respectively.

The superficial gas velocity at which the bed turns just fluidized \( (U_{mf}) \) is called minimum fluidization velocity. It may be obtained by solving Eqs. (2.1) and (2.2) simultaneously to obtain

Table 2.1 Comparisons of principal gas–solid contacting combustion processes

<table>
<thead>
<tr>
<th>Property</th>
<th>Packed bed</th>
<th>Fluidized bed</th>
<th>Fast bed</th>
<th>Pneumatic transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application in boilers</td>
<td>Stoker fired</td>
<td>Bubbling fluidized</td>
<td>Circulating fluidized</td>
<td>Pulverized coal fired</td>
</tr>
<tr>
<td>Mean particle diameter (mm)</td>
<td>&lt;300</td>
<td>0.03–3</td>
<td>0.05–0.5</td>
<td>0.02–0.08</td>
</tr>
<tr>
<td>Gas velocity through combustor zone</td>
<td>1–3</td>
<td>0.5–2.5</td>
<td>3.5–6</td>
<td>15–30</td>
</tr>
<tr>
<td>Typical ( U/U_t )</td>
<td>0.01</td>
<td>0.3</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>Gas motion</td>
<td>Up</td>
<td>Up</td>
<td>Up</td>
<td>Up</td>
</tr>
<tr>
<td>Gas mixing</td>
<td>Near plug flow</td>
<td>Complex two phases</td>
<td>Dispersed plug flow</td>
<td>Near plug flow</td>
</tr>
<tr>
<td>Solids motion</td>
<td>Static</td>
<td>Up and down</td>
<td>Mostly up, some down</td>
<td>Up</td>
</tr>
<tr>
<td>Solid–solid mixing</td>
<td>Negligible</td>
<td>Usually near perfect</td>
<td>Near perfect</td>
<td>Near plug flow</td>
</tr>
<tr>
<td>Overall voidage</td>
<td>0.4–0.5</td>
<td>0.5–0.85</td>
<td>0.85–0.99</td>
<td>0.98–0.998</td>
</tr>
<tr>
<td>Temperature gradient</td>
<td>Large</td>
<td>Very small</td>
<td>Small</td>
<td>May be significant</td>
</tr>
<tr>
<td>Typical bed-to-surface. Heat transfer coefficient (W/m² K)</td>
<td>50–150</td>
<td>200–550</td>
<td>100–200</td>
<td>50–100</td>
</tr>
<tr>
<td>Attrition</td>
<td>Little</td>
<td>Some</td>
<td>Some</td>
<td>Considerable</td>
</tr>
<tr>
<td>Agglomeration</td>
<td>Considerable</td>
<td>Some</td>
<td>Less</td>
<td>No problem</td>
</tr>
</tbody>
</table>

2.1.2 Bubbling Fluidized Beds

As the gas flow rate through the fixed bed is increased, the pressure drop continues to rise according to Eq. (2.1), until the superficial gas velocity reaches a critical value known as the minimum fluidization velocity, \( U_{mf} \). This is defined as the velocity at which the fluid drag is equal to the weight of particles less its buoyancy. At this stage, the particles feel “weightless,” and the fixed bed transforms into an incipiently fluidized bed. In this regime, the body of solids behaves like a liquid, as illustrated in Fig. 2.1. Since the pressure drop across the bed equals the weight of the bed, the fluid drag \( F_D \) is written as

\[
F_D = \Delta P A = AL(1 - \varepsilon)(\rho_p - \rho_g)g
\]

(2.2)

where \( A \) and \( L \) are the cross section and height of the bed, respectively.

The superficial gas velocity at which the bed turns just fluidized \( (U_{mf}) \) is called minimum fluidization velocity. It may be obtained by solving Eqs. (2.1) and (2.2) simultaneously to obtain
Fig. 2.3 As the gas velocity through a perforated grate supporting a mass of granular solids is increased, the gas–solid contacting mode moves from a packed or fixed bed (a) to bubbling bed (b) and then, under certain conditions, to a slugging bed (c). If the gas velocity is further increased, the bed transforms into turbulent bed. Solids in the above four regimes generally remain within a certain height above the grate and hence are said to be in the captive stage. a Fixed. b Bubbling. c Slugging. d Turbulent
\[ Re_{mf} = \frac{d_p U_{mf} \rho_g}{\mu} = [C_1^2 + C_2 Ar]^{0.5} - C_1 \]  

(2.3)

where

\[ Ar = \text{Archimedes number} = \frac{\rho_g (\rho_p - \rho_g) g d_p^3}{\mu^2} \]

and \( d_p \) = surface volume mean diameter of particles (see Appendix I).

The values of the empirical constants \( C_1 \) and \( C_2 \) may be taken from experiments as 27.2 and 0.0408, respectively (Grace 1982).

At minimum fluidization, the bed behaves as a pseudo-liquid. For Group B and D particles (Appendix I), a further increase in gas flow can cause the extra gas to flow in the form of bubbles. The section of the bed outside the bubbles is called the emulsion phase, in which the superficial gas velocity is of the order of \( U_{mf} \) and it has a characteristic voidage \( \varepsilon_{mf} \). However, for Group A particles, the bed does not form bubbles until the superficial gas velocity reaches a velocity \( U_{mb} \), referred to as minimum bubbling velocity. Above \( U_{mf} \), the bed expands until the superficial velocity reaches \( U_{mb} \). A typical bubbling bed is shown in Fig. 2.3b. The minimum bubbling velocity for Group A particles is given by Abrahamsen and Geldart (1980):

\[ U_{mb} = U_{mf} \left( \frac{2300 \mu^{0.523} \rho_g^{0.126} \exp(0.716F)}{d_p^{0.8} g^{0.934} (\rho_p - \rho_g)^{0.934}} \right) \]  

(2.4)

where \( F \) is the mass fraction of particles less than 45 \( \mu \)m, \( d_p \) is the mean diameter of particles in m, \( \rho_g \) is the density of gas in kg/m\(^3\), and \( \mu \) is the viscosity of gas in kg/m s.

Bubbles are gas voids with very little or no solids within. Due to the buoyancy force, bubbles rise through the emulsion phase, bypassing the particles. The bubble size increases with particle diameter, \( d_p \); excess gas velocity, \( U - U_{mf} \); and its position above the distributor or grid of the bed.

The bubble size can increase only to a maximum stable size, \( D_{bmax} \), given by

\[ D_{bmax} = \frac{2(U_t^*)^2}{g} \]  

(2.5)

where \( U_t^* \) is the terminal velocity (see Appendix I) of particles having a diameter 2.7 times the average size of bed solids (Grace 1982).

A bubble carries some solids upward in its wake. Bubbles erupt at the surface of the bed throwing or entraining particles into the space above the bubbling bed, known as the freeboard. The entrained particles travel upward due to their momentum and local gas-particle drag. Some of the particles may disengage from the gas and return to the dense bed due to gravitational force. This process of disengagement reduces the
upward flux of particles exponentially along the height in the freeboard. Beyond a certain height, termed “transport disengaging height (TDH),” the particles that disengage from the gas to return to the dense bed are negligible. The flux rate of particles carried away beyond the TDH is known as the elutriation rate.

2.1.3 Slugging

For a given bed, the size of the bubble increases as the fluidizing velocity or the bed height is increased. If the bed is small in cross section and deep, the bubble may increase to a size comparable to the diameter or width of the bed. In this case, the bubble passes through the bed as a slug (Fig. 2.3c); this phenomenon is known as slugging. A necessary condition for the formation of slugs is that the maximum stable bubble size, \( D_{b\text{max}} \), must be greater than 0.6 times the diameter of the bed, \( D \) (Geldart 1986). Slugging does not generally occur in commercial fluidized bed boilers or reactors, which are fairly large in diameter. The criterion for slug formation at choking is given by Yang (1976)

\[
\frac{U_t^2}{gD} \geq 0.123
\]  

where \( U_t \) is the terminal velocity of the average-sized bed solid. The minimum slugging velocity, \( U_{sl} \), is given by Stewart and Davidson (1967):

\[
U_{sl} = U_{mf} + 0.07(gD)^{0.5}
\]  

where \( D \) is the diameter of the bed. For a rectangular bed or a bed of any other cross section, \( D \) may be taken as four times the bed area over bed perimeter (4A/P).

2.1.4 Turbulent Beds

When the velocity of gas through a bubbling fluidized bed is increased above its minimum bubbling velocity, the bed expands. A continued increase in the velocity may eventually show a change in the pattern of the bed expansion. This may be due to an increase in the bubble fraction, an expansion of the emulsion phase (Nakajima et al. 1991), as well as thinning of the emulsion walls separating the bubbles. As the velocity continues to increase, bubble fraction increases till it reaches a stage when the bubble phase loses its identity due to rapid coalescence and breakup. This results in a violently active and highly expanded bed and a change in the pattern of bed expansion. Particles are thrown into the freeboard above the bed. The bed will have an upper surface, but it is considerably diffused. Such beds are referred to as turbulent beds (Fig. 2.3d).
At this stage, the pressure drop across the bed fluctuates randomly. The amplitude of fluctuation reaches a peak at the velocity $u_c$ and then reduces to a steady value as the fluidizing velocity is increased further to the velocity $u_k$ (Fig. 2.4). The transition from the bubbling to turbulent bed does not take place suddenly. The onset of this transition starts at the velocity $u_c$ and is completed at the velocity $u_k$. The transition appears to start at the upper surface of the bed and then moves downward. Presently, no general correlation for the calculation of the velocity of transition from the bubbling to the turbulent bed is available. However, some restricted correlations (Grace 1982) based on small-diameter beds are given below.

\[
uc = 3.0 \sqrt{\rho_p d_p} - 0.17 \quad \text{(SI unit)} \tag{2.8}
\]

\[
uk = 7.0 \sqrt{\rho_p d_p} - 0.77 \quad \text{(SI unit)} \tag{2.9}
\]

where $(\rho_p d_p)$ is in the range 0.05–0.7 kg/m².

Horio and Morishita (1988) presented an alternative set of equations for this transition:

\[
Re_c = \frac{u_c d_p \rho_g}{\mu} = 0.936 Ar^{0.472} \tag{2.10}
\]

\[
Re_k = \frac{u_k d_p \rho_g}{\mu} = \begin{cases} 
1.46 Ar^{0.472} & (Ar < 10^4) \\
1.41 Ar^{0.56} & (Ar > 10^4)
\end{cases} \tag{2.11}
\]

Table 2.2 presents experimental values of velocities of transition to turbulent fluidization for some solids.

**Fig. 2.4** Amplitude of pressure fluctuation across the bed increases as the bed approaches turbulent fluidization. It reaches a peak and then drops to a steady value.
The transition from bubbling to turbulent fluidization occurs at a lower velocity in larger diameter vessels (Sun and Chen 1989). An absence of data in large beds prevents defining the actual extent of the effect of bed diameter in large commercial size units.

Fine particles enter turbulent fluidization at a velocity sufficiently above the terminal velocity of the solids, whereas coarser particles may enter turbulent fluidization at a velocity less than their terminal velocity. The gas–solid contact in this regime is good, and the reactor performance approaches an ideal back-mix reactor.

In the regimes discussed so far, solids are generally retained within a certain height above the grid. Except for some entrainment, there is no large-scale migration of particles with the gas; thus, these regimes are called the captive stage. Next we discuss regimes with large scale migration of solids from the reactor.

**Example 2.1** Find the minimum fluidization, minimum bubbling, terminal velocity, and velocity for the onset of transition to turbulent fluidization for 300 μm sand ($\rho_p = 2500 \text{ kg/m}^3$) in a 0.203 m × 0.203 m bed operating under the following conditions:

<table>
<thead>
<tr>
<th>Bed temperature</th>
<th>Gas density (kg/m³)</th>
<th>Gas viscosity ($\text{N s/m}^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>825 °C</td>
<td>0.316</td>
<td>$4.49 \times 10^{-5}$</td>
</tr>
<tr>
<td>27 °C</td>
<td>1.16</td>
<td>$1.84 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

**Solution**

1. First check which group the particle belongs to:

$$\rho_p = 2500 \text{ kg/m}^3 = 2.5 \text{ gm/cm}^3$$
\[ \rho_p - \rho_g = 2.5 - 0.00116 = 2.49 \text{ gm/cm}^3 \text{ (27 °C)} \]
\[ = 2.5 - 0.000316 = 2.49 \text{ gm/cm}^3 \text{ (825 °C)}. \]

Figure I.2 in Appendix I suggests that 300-μm particle will belong to Group B.

2. Minimum fluidization velocity

From Eq. (2.3) \( Re_{mf} = [27.2^2 + 0.0408Ar]^{0.5} - 27.2 \)

For 27 °C,

\[ Ar = \frac{1.16 \times (2500 - 1.16) \times 9.81 \times 0.0003^3}{(1.84 \times 10^{-5})^2} = 2267 \]

\[ Re_{mf} = [27.2^2 + 0.0407 \times 2267]^{0.5} - 27.2 = 1.65 \]

\[ U_{mf} = \frac{1.84 \times 10^{-5}}{0.0003 \times 1.16} \times 1.65 = 0.086 \text{ m/s} \]

Similarly for 825 °C,

\[ Ar = 103.7, \]

\[ Re_{mf} = 0.0777, \]

\[ U_{mf} = 0.0368 \text{ m/s}. \]

3. Minimum bubbling velocity

One would normally use Eq. (2.4) to find \( U_{mb} \). Since it is a Group B particle, the bed will bubble immediately after \( U_{mf} \). So,

\[ U_{mb} = U_{mf} = 0.086 \text{ m/s at 27 °C}. \]

4. Minimum slugging velocity

At an equivalent bed diameter, \( D = \frac{4 \times 0.203 \times 0.203}{4 \times 0.203} = 0.203 \)

Using Eq. (2.7) for 27 °C,

\[ U_{sl} = 0.087 + 0.07 \times (9.81 \times 0.203)^{0.05} = 0.1858 \]

Similarly for 825 °C, \( U_{sl} = 0.1356 \text{ m/s} \)

5. Transition to turbulent fluidization

Using Eqs. (2.8) and (2.9),

\[ u_c = 3 \times (2500 \times 0.0003)^{0.5} - 0.1 = \begin{array}{ll} 27 \text{ °C} & 825 \text{ °C} \\ 2.498 & 2.498 \text{ m/s} \end{array} \]

\[ u_h = 7 \times (2500 \times 0.0003)^{0.5} - 0.77 = \begin{array}{ll} 27 \text{ °C} & 825 \text{ °C} \\ 5.292 & 5.292 \text{ m/s} \end{array} \]
Equations (2.8) and (2.9) are valid only for \(0.05 < \rho_p d_p < 0.7\). Thus, to find transition velocities we use Eqs. (2.10) and (2.11) with Archimedes number 2267 at 25 °C and 103.7 at 825 °C.

\[
Re_c = 0.936 \times (2267)^{0.472} = 35.9 \quad 825 \quad 8.36
\]

\[
u_c = \frac{35.9 \times 1.84 \times 10^{-5}}{0.0003 \times 1.16} = 1.90 \text{ m/s} \quad 3.96 \text{ m/s}
\]

\[
Re_k = 1.46 \times (2267)^{0.472} = 56.0 \quad 3.05
\]

\[
u_k = \frac{56.0 \times 1.84 \times 10^{-5}}{0.0003 \times 1.16} = 2.96 \text{ m/s} \quad 6.18 \text{ m/s}
\]

6. Terminal velocity
At 27 °C, \(Ar = 2267\). So, using Eq. (I.16),

\[
U_t = \frac{1.84 \times 10^{-5}}{0.0003 \times 1.16} \times \left(\frac{2267}{7.5}\right)^{0.666} = 2.37 \text{ m/s}
\]

Similarly, at 825 °C, \(Ar = 103.7\). So, we use Eq. (I.18) to get

\[
U_t = \frac{4.49 \times 10^{-5}}{0.0003 \times 0.316} \times \left(\frac{103.7}{7.5}\right)^{0.666} = 2.72 \text{ m/s}
\]

2.2 Fast Fluidized Bed

In the context of its use in CFB boilers, fast fluidized bed may be defined as: a high velocity gas–solid suspension where particles, elutriated by the fluidizing gas above the terminal velocity of particles, are recovered and returned to the base of the furnace at a rate sufficiently high as to cause a degree of solid refluxing that will ensure a minimum level of temperature uniformity in the furnace.

2.2.1 Characteristics of Fast Beds

Yershalmi and his co-workers (1976) coined the term “fast bed” for a regime lying between the turbulent fluidized bed and the pneumatic transport. In a typical fast fluidized bed, one observes a non-uniform suspension of slender particle agglomerates moving up and down in a very dilute up-flowing gas–solid continuum (Fig. 2.5).
High slip velocity between gas and solid, formation and disintegration of particle agglomerates, and excellent mixing are major characteristics of this regime. Some axial as well as radial variation in suspension density is other physical characteristics of the fast bed.

The formation of solid agglomerates or clusters is not a sufficient condition for the fast fluidized bed, but is an important necessary feature of this regime.

A qualitative description of the phenomenon leading to the formation of clusters in a pneumatic transport column is presented with the help of Fig. 2.6. A solid is continuously fed to the rising gas stream in the column. At a very low feed rate, the particles will be spread uniformly in the gas stream. Each particle will travel in isolation. The relative velocity between the gas and solids may form a small wake behind each particle (Fig. 2.6a).

For a given gas velocity, the feed rate may be increased to a level where the solid concentration will be so high that one particle will enter into or interfere in the wake of the other. When that happens, the fluid drag on the leading particle will decrease, and it will fall under gravity to drop on the trailing particle (Fig. 2.6b). The effective surface area of the pair just formed is reduced. So the fluid drag will be lower than their combined weight, making the pair fall further to collide with other particles. Thus, an increasing number of particles combine together to form particle agglomerates known as clusters. These clusters are, however, not permanent. They are continuously torn apart by the up-flowing gas. Thus, the formation of clusters and their disintegration continue.
2.2.2 Transition to Fast Fluidization

There are divergent views on the transition to or from fast fluidization. So, the description of the process of transition to fast fluidization as presented below is only tentative.

Imagine that gas is flowing upward through a vertical column to which solid is fed at a given rate, \( W_1 \), and the velocity is sufficiently high for the suspension to be in pneumatic transport. If the superficial gas velocity through the column is decreased (Fig. 2.7) without changing the solid feed rate, the pressure drop per unit height of the column will decrease due to the reduced fluid friction on the wall (C–D). However, the suspension will become increasingly denser with decreasing gas velocities. Thus, pressure drop may not be as much as one would expect if there were no solids in the column. As the gas velocities continue to decrease pressure drop due to solids, drag dominates that due to wall drag. Soon the pressure drop begins to increase (D–E) with a continued decrease in the superficial velocity (Fig. 2.7). This point of reversal (D) marks the onset of fast bed (Reddy-Karri and Knowlton 1991) from pneumatic transport.

If the gas velocity is decreased further, the solid concentration in the column increases up to a point when the column is saturated with solids, i.e., the gas can no longer carry the solids in the column. The solids start accumulating, filling up the column. This is marked by a steep rise in the pressure drop. This condition (E) is termed “choking.” In smaller diameter columns, the bed starts slugging, while in larger ones, it undergoes transformation into a non-slugging dense-phase fluidized
bed, such as a turbulent bed or bubbling bed. The gas–solid regime below this velocity has the generic name captive state. The captive state may include turbulent, slugging, bubbling, and fixed beds.

### 2.2.3 Transition from Bubbling to Fast Bed

The discussion above suggests that the choking velocity, $U_{ch}$, may be used to mark the transition from captive (turbulent/bubbling/slugging) to fast fluidization. Yang’s (1983) correlation developed for Group A particles at room temperatures using small-diameter (<0.3 m) pneumatic transport columns may be used for a first approximation of the choking velocity in fluidized bed boilers. However, one must recognize its limitations.

$$
\frac{U_{ch}}{\varepsilon_c} = U_t + \left[ \frac{2gD(\varepsilon_c^{-4.7} - 1)\rho_p^{2.2}}{6.81 \times 10^5 \rho_g^{2.2}} \right]^{0.5}
$$

$$
G_s = (U_{ch} - U_t)(1 - \varepsilon_c)\rho_p, \quad D < 0.3 \text{ m}
$$

where $G_s$ is the solid circulation rate, $\varepsilon_c$ is the voidage at choking, and $U_t$ is the terminal velocity of single particles. For a given $G_s$, $U_t$, $\rho_p$, and $\rho_g$, one can calculate $U_{ch}$ through the simultaneous solution of the above two equations.

This correlation suggests a square root dependence on bed diameter, while experimental data (Knowlton 1990b) show the dependence to be 0.2 power of the diameter, and for beds larger than 0.5 diameter, the bed diameter may not have any effect on the transition velocity (Knowlton 1990b). Furthermore, its validity in
high-temperature CFB boilers is yet to be verified. So Eqs. (2.12) and (2.13) should be used with caution.

A qualitative flow regime diagram is developed in Fig. 2.8 following the description of Reddy-Karri and Knowlton (1991). The line A–B (locus of choking velocity) marks the boundary between the captive and fast beds. Figure 2.8 further shows that at higher circulation rates, the transition to fast fluidization occurs at higher velocities.

There is, however, a minimum velocity below which fast fluidization cannot occur, irrespective of the circulation rate. It is known as the transport velocity. It may be described as follows. If a bed is fluidized above the terminal velocity (described in Appendix I.1) of individual bed particles, all solids are entrained out of the column in a finite period of time, unless they are replaced simultaneously. As the velocity is decreased from a level much in excess of the terminal velocity, the time taken to empty the vessel increases gradually until it reaches a critical velocity below which there is a sudden increase in the time for emptying the vessel (Fig. 2.9). This velocity is called the transport velocity, $U_{tr}$. Perales et al. (1991) gave an empirical relation for the transport velocity based on their experiments on a 92-mm-diameter bed.

$$U_{tr} = 1.45 \times \frac{\mu}{\rho_g d_p} Ar^{0.484}, \quad 20 < Ar < 50,000 \quad (2.14)$$

Thus, a fast bed is more rigorously defined by as a regime of gas–solid fluidization above the transport velocity and above a minimum rate of recycling of entrained solids. For a given solid circulation rate, it is bounded by the choking velocity on the lower side and the velocity corresponding to the minimum pressure drop on the gas–solid contacting phase diagram on the higher side.

---

**Fig. 2.8** The fast fluidization is bounded by two velocities, which depend on the circulation rate.
For the design of CFB boiler, Yue et al. (2005) suggested an alternative expression of minimum value of fast fluidization velocity, $U_{tr}$, and circulation rate, $G_{s,min}$.

\[
U_{tr} = (3.5 \text{ to } 4.0) \cdot U_t; \quad G_{s,min} = \frac{U_{tr}^{2.25} \rho_g^{1.627}}{0.164 [gd_p(\rho_p - \rho_g)]^{0.627}}
\]

### 2.2.4 Transition from Pneumatic Transport to Fast Bed

It is apparent from Fig. 2.8 that for a given solid circulation rate, two velocities bound the fast bed, the choking velocity on the lower side and another velocity on the higher side. The higher limiting velocity marks the transition from fast fluidization to pneumatic transport. As indicated earlier, it is the locus of the velocity at which, for a given circulation rate, the average pressure drop across the height of the column reaches a minimum value (point D, Fig. 2.7). It is difficult to find this point experimentally. A simpler experimental method can be used (Biswas and Leung 1987) to find this velocity by exploiting the axial non-uniformity feature of the fast fluidized bed.

In pneumatic transport, there is very little difference in pressure drops across unit height of the upper and lower sections of a vertical column. Now, if the gas velocity is decreased at a given circulation rate, the pressure drop (hence, suspension densities) at those two points will increase to identical degrees until the velocity for transition to fast fluidization is reached, at which point the bed becomes axially non-uniform. Thus, the suspension density near the lower section of the column will start exceeding that at the top. No comprehensive correlation is available at the moment to estimate this velocity. Only experimental data can provide the necessary guide to this transition.
2.2.5 The Flow Regime Diagram

The transition to fast fluidization is influenced by a number of parameters, such as particle size, particle density, gas viscosity, gas density, and tube size. Figure 2.10 is a qualitative representation of transition values of those parameters as a function of superficial gas velocity. Shingles and Dry (1986) used this to describe the regime transition. A simpler form of the regime diagram is shown in Fig. 2.8.

These diagrams may be used for qualitative assessment only. An important point, shown in Fig. 2.10, is that the operating range of fast beds reduced for coarser particles. This is why it is more difficult to maintain fast fluidization for Group B particles than for Group A particles.

**Example 2.2** Find the minimum velocity for fast fluidization for 300-μm sand particles at 27 °C and 825 °C for the following conditions. The desired solid circulation rate in the fast regime is 30 kg/m² s. The cross section of the bed is 0.203 m × 0.203 m. The density of the particles is 2500 kg/m³.

<table>
<thead>
<tr>
<th></th>
<th>825 °C</th>
<th>27 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature</strong></td>
<td>825 °C</td>
<td>27 °C</td>
</tr>
<tr>
<td><strong>Gas density</strong></td>
<td>0.316 kg/m³</td>
<td>1.16 kg/m³</td>
</tr>
<tr>
<td><strong>Gas viscosity</strong></td>
<td>$4.49 \times 10^{-5}$ N s/m²</td>
<td>$1.84 \times 10^{-5}$ N s/m²</td>
</tr>
<tr>
<td><strong>Terminal velocity</strong></td>
<td>2.72 m/s</td>
<td>2.27 m/s</td>
</tr>
</tbody>
</table>
### Solution

1. We will first use Eq. (2.14) for the transport velocity. For 27 °C,

\[
Ar = \frac{1.16 \times (2500 - 1.16) \times 9.81 \times 0.0003^3}{(1.84 \times 10^{-5})^2} = 2267
\]

\[
U_tr = \frac{1.45 \times 1.84 \times 10^{-5} \times 2267^{0.484}}{1.16 \times 0.0003} = 3.226 \text{ m/s}
\]

Similarly at 825 °C, \( U_tr = 6.49 \text{ m/s} \).

2. We will now use Eq. (2.12) to find the choking velocity:

\[
\frac{U_{ch}}{\varepsilon_c} = U_t + \left[ \frac{2gD(\varepsilon_c^{-4.7} - 1)\rho_p^{2.2}}{6.81 \times 10^5 \rho_g^{2.2}} \right]^{0.5}
\]

\[
G_s = (U_{ch} - U_t)(1 - \varepsilon_c)\rho_p
\]

For an equivalent bed diameter,

\[
D = \frac{4 \times 0.203 \times 0.203}{0.203 + 0.203 + 0.203 + 0.203} = 0.203 \text{ m}
\]

The value of \( U_t \) is taken from the example on p. 29. \( U_{t, 27 \ ^\circ C} = 2.37 \text{ m/s} \) and \( U_{t, 825 \ ^\circ C} = 2.72 \text{ m/s} \). By substituting values at 27 °C, we get

\[
\frac{U_{ch}}{\varepsilon_c} = 2.37 + 11.229(\varepsilon_c^{-4.7} - 1)^{0.5}
\]

\[
30 = (U_{ch} - 2.37)(1 - \varepsilon_c) \times 2500
\]

For 825 °C, the above equation takes the form

\[
\frac{U_{ch}}{\varepsilon_c} = 2.722 + 46.9(\varepsilon_c^{-4.7} - 1)^{0.5}
\]

\[
30 = (U_{ch} - 2.722)(1 - \varepsilon_c) \times 2500
\]

Iterative solutions of above equations give values of \( U_{ch} \) and \( \varepsilon_c \). These values can be compared with the transport velocities, which are also believed to mark the transition between turbulent and fast beds.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>( U_{ch} ) (m/s)</th>
<th>( \varepsilon_c )</th>
<th>( U_tr ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 °C</td>
<td>4.19</td>
<td>0.9937</td>
<td>3.22</td>
</tr>
<tr>
<td>825 °C</td>
<td>7.70</td>
<td>0.9976</td>
<td>6.49</td>
</tr>
</tbody>
</table>
The restricted validity of the correlation for choking velocity makes it imperative that the transition velocity computed from Eqs. (2.12) and (2.13) be used with caution. Experiments should be the best guide. Further research is needed in this area for more reliable prediction of the transition velocity.

2.3 Structure of Fast Beds

The fast fluidized bed is non-uniform in both axial and lateral directions. The hydrodynamic structure of the fast bed is described below in light of several models (Bolton and Davidson 1988; Kunii and Levenspiel 1990; Yang 1988). Also, empirical equations are presented to predict the distribution of local suspension densities in the radial and axial directions.

2.3.1 Axial Voidage Profile

A typical axial distribution of the cross-sectional average voidage of a fast bed is shown in Fig. 2.11. It shows that the lower section of the fast bed, especially below the secondary air injection level, is dense, and the upper section is relatively dilute.

There is a gradual transition of density between two sections. Kwauk et al. (1986) explained this on the basis of cluster diffusion in the axial direction and concluded that the axial distribution of the voidage is “S” shaped, as shown in Fig. 2.11. This profile is represented by the following correlation (Kwauk et al. 1986) for Group A and weakly Group B particles.

\[
\frac{\varepsilon - \varepsilon_a}{\varepsilon_d - \varepsilon} = \exp \left( \frac{h - h_i}{h_o} \right)
\]  

(2.15)

where \(\varepsilon\) is the voidage at a height, \(h\) measured from the bottom of the bed, \(\varepsilon_a\) is the asymptotic voidage in the dense section \( (h = -\infty)\), and \(\varepsilon_d\) is the asymptotic voidage in the dilute sections \( (h = +\infty)\) of the S-shaped voidage profile. The characteristic height \(h_o\) can be found from empirical equations given by Kwauk et al. (1986). The height \(h_i\) is the point of inflexion in the profile. The asymptotic voidage, \(\varepsilon_d\), in the dilute phase is described as the voidage at which clusters start forming. This may be taken to be equal to the choking voidage, \(\varepsilon_c\), of Eq. (2.12).

The location of the point of inflexion (B in Fig. 2.12) separating the lean and dense regions of a fast bed is a function of the solid circulation rate and the solids inventory in the system. This effect arises out of the pressure balance around the CFB loop. In a CFB boiler, the location of B is influenced by the height at which secondary air is injected, but the suspension density at this point depends on circulation rate and fluidizing velocity. Figure 8.5 in Chap. 8 shows how the suspension density changes at B while other parameters are changed.
**Fig. 2.11** Axial profile of cross-sectional average bed voidage. Measured in a 0.24-m-diameter and 54-m-tall CFB riser in support of 600 MWe supercritical CFB boiler at Baima, China [with permission from Yue et al. 2014]

**Fig. 2.12** Voidage profile is governed by the pressure balance around the CFB loop
The lower section of the fast bed is denser, and therefore, it results in a pressure drop per unit height of the bed higher than that in the upper section, which is leaner. The total solid inventory is distributed between the riser bed and the return leg in such a way that pressure drops through two legs of the loop (Fig. 2.12) balance each other. The pressure drop across the L-valve or loop seal controlling the solid flow (Fig. 2.12) is proportional to its flow rate. One can increase solid flow through the L-valve by increasing the aeration air, which will in turn increase the pressure drop per unit length of the moving packed bed in the return leg (EF). The pressure drop across the cyclone is proportional to the square of the inlet gas velocity. The pressure drop across the standpipe depends on the level of solids in it. For stable operations, the pressure balance around the loop may be written as

\[ \Delta P_{F-G} + \Delta P_{G-A} + \Delta P_{A-B} + \Delta P_{B-C} + \Delta P_{C-D} = \Delta P_{E-D} + \Delta P_{F-E} \quad (2.16) \]

This pressure balance depends on different operating parameters. The response of the bed to the variation of an operating parameter can be predicted from the above pressure balance, as shown in Table 2.3.

An alternative approach to the prediction of axial distribution of voidage in a fast bed is based on the entrainment model (Rhodes and Geldart 1986; Bolton and Davidson 1988; Kunii and Levenspiel 1990). The equation of axial voidage distribution developed by Kunii and Levenspiel (1991) is similar to Eq. (2.15).

\[ \frac{\varepsilon_d - \varepsilon}{\varepsilon_d - \varepsilon_a} = \exp[-a(h - h_i)], \quad h > h_i \quad (2.17) \]

For fast beds, the density decay constant, \( a \), can be correlated with superficial velocities using experimental data, as shown in Fig. 2.13. Typical values of asymptotic voidage in the denser section, \( \varepsilon_a \), for Group A particles are given below as:

<table>
<thead>
<tr>
<th>Situations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Operating parameters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas velocity</td>
<td>–</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Valve opening</td>
<td>0</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>System inventory</td>
<td>0</td>
<td>0</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>0</td>
</tr>
<tr>
<td><strong>Response to</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid circulation rate</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Valve pressure drop</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>Dilute-phase density</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Dense-phase density</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Return leg inventory</td>
<td>–</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
</tbody>
</table>

Adapted from Matsen (1988)

Symbols used: – decrease; + increase; 0 unchanged
0.88–0.78 for fast beds
0.60–0.78 for turbulent beds
0.45–0.60 for bubbling beds (Kunii and Levenspiel 1990).

For bed materials typically used in commercial boilers (coal ash, sorbent, sand), $\varepsilon_a$ would be even higher. In fast beds with secondary air injections, as in CFB boilers, the point of inflexion ($h_i$) generally occurs at or below the secondary air injection level. The asymptotic voidage in the dilute phase (upper section), $\varepsilon_d$, may be associated with the voidage beyond which the cluster formation starts. So one can approximate $\varepsilon_d$ as $\varepsilon_c$, which can be found from Eq. (2.12).

The voidage at the furnace exit, $\varepsilon_e$, at height $H$ can be found as

$$\varepsilon_e = \varepsilon_d - (\varepsilon_d - \varepsilon_a) \exp\left[-a(H - h_i)\right]$$

(2.18)

The mean bed voidage, $\varepsilon_s$, above the point of inflexion (or above the secondary air level in CFB boilers) can be found integrating Eq. (2.18) between $H$ and $h_i$ or the height of the secondary air level.

$$\varepsilon_s = \varepsilon_d - \frac{\varepsilon_c - \varepsilon_a}{\ln(1 + (H - h_i)/\varepsilon_a)}$$

(2.19)

So for an uniform cross-sectional bed, the amount of solids in the furnace, $W$, can be found as

$$W = A\rho_p[h_i(1 - \varepsilon_a) + (H - h_i)(1 - \varepsilon_s)]$$

(2.20)

As mentioned above, for a first approximation, one can take $h_i$ to be equal to the height of the secondary air level or the lower combustion section.
If one assumes that the voidage at the top of the furnace is so high that all particles are completely dispersed and there is no refluxing, then the voidage of the gas–solid suspension leaving the bed may be approximated as \([G_c/\rho_p U]\). If there is appreciable refluxing at that height, then it could be estimated from the following equation:

\[
(1 - \varepsilon_c)\rho_p = \left[ \frac{G_u}{U_s} + \frac{G_d}{U_d} \right]
\]

where \(G_u\) is the solid flux moving upward, \(G_d\) is the downward solid flux near the exit, \(U_s\) is the linear velocity of upward-moving solids, and \(U_d\) is the velocity of downward-moving solids.

In addition to Eqs. (2.15) and (2.17), the voidage profile above the secondary air level is sometimes also fitted by a simpler equation of the form \([c(h-h_0)^n]\), where \(c\) and \(n\) are fitted constants and \(h\) is the height measured above the grate.

If a boiler designer decides to use a constricted furnace exit or place some tube panels restricting the gas–solid flow at the furnace exit, the solids will decelerate, leading to an increased suspension density. However, most boiler designers would normally avoid such a situation.

### 2.3.2 Lateral Distribution of Voidage

Macroscopically, the fluidizing gas (with thinly dispersed solids) moves upward in a plug flow, but detailed measurements show that the velocity near the wall is considerably lower than that in the core of the bed. As a result of this, solids near the wall move downward while they move up in the core. Thus, the riser of a CFB may be divided into two vertical regions: the low gas velocity region near the wall where solids generally move down is called annular zone. The central region where gas velocity is high and solids generally move is called core region. Such up-and-down motion of solids keeps the solids in a fast fluidized bed largely back-mixed.

Bulk of the solids in a fast bed move up and down about the bed in the form of particle agglomerates of fine bed solids that move together in the fast bed as a single body for a brief period of time and then dissolve (break apart). Then another cluster is formed. The formation of the clusters is explained in Fig. 2.6. The clusters tend to assume shapes of least drag. This results in their slender shapes. The concentration of clusters is higher near the wall than at the axis of the furnace. Clusters are, however, less likely to appear in very dilute beds of coarse particles.

Solids move upward in the core through a dilute suspension, with the occasional presence of clusters. These solids drift sideways due to hydrodynamic interactions. Upon reaching the wall, they experience low gas velocity and hence lower fluid drag. So, the solids start falling. Occasionally, one would observe on the wall an upward-moving cluster from the core, suddenly swept to the wall, traveling a short
distance with its upward momentum. The solids falling near the wall are occasionally picked up in the up-flowing gas of the core, initiating their upward journey. This results in two lateral fluxes: one toward the wall and another from the wall.

The up-and-down movements of solids in the core and annulus set an internal circulation in the bed, in addition to the external circulation, where solids captured by the cyclone are returned to the bed. Experimental measurements (Horio and Morishita 1988) have shown the internal circulation rate to be 2.3 times the external circulation rate. The temperature uniformity of the bed is a direct result of this internal solid circulation.

(a) **Thickness of wall layer**

Solids in the annular zone descend downward along the wall. The thickness of the annular zone decreases from the bottom to the top of the bed. In commercial CFB boilers, the wall layers measured in some commercial CFB boilers were in the range of 70–350 mm (Johansson et al. 2007), while it was of the order of few millimeters in laboratory-scale units (Horio and Morishita 1988; Knowlton 1990a). The axial variation of thickness, $\delta$, of the annulus layer for large CFB boiler may be estimated by Johansson et al. (2007).

$$
\delta = D_{eq}[0.008 + 4.52(1 - \varepsilon_{av}(z))]
$$

(2.22)

where $D_{eq}$ is equivalent diameter of furnace and $\varepsilon(z)$ is cross-sectional average at a given height. If the average suspension density along the furnace height is unknown, one can use the following expression (Johansson et al. 2005).

or

$$
\delta = 0.00385H \exp\left(1 - \frac{z}{H}\right)
$$

(2.23)

where $H$ is the furnace height and $z$ is the height where the thickness is being calculated.

It may be noted that the thickness of solid layer sliding down the wall as calculated from Eq. (2.23) is for the part of the wall away from the corners of the furnace or bed. The layer thickness is higher on the corners (Zhou et al. 1995).

The local voidage and the gas and solid velocities change continuously from the axis toward the wall (Horio and Morishita 1988; Hartge et al. 1988). The voidage is highest on the axis of the riser, and it is lowest on the wall (Tung et al. 1988; Arena et al. 1988). The radial voidage distribution is much flatter in the upper section of the riser-bed, as well as at lower circulation rates. A typical radial voidage profile in a fast bed is shown in Fig. 2.14. Measurements in laboratory-scale units suggest that the local voidage, $\varepsilon(r)$, is a function of only the cross-sectional average voidage, $\varepsilon_{av}$, and the non-dimensional radial distance, $(r/R)$, from the axis of the bed. One such correlation prepared for CFB riser of radius $R$ (Issangya et al. 2001) is given in Fig. 2.14.
2.3 Structure of Fast Beds

Large commercial units also show similar distributions of voidage (Schaub et al. 1989), but a lack of adequate data in commercial units does not allow verification of the above empirical equation. In commercial CBF boilers, the suspension density determined from static pressures measured on the wall often shows values as low as 1–5 kg/m³, while indirect assessments suggest a much higher inventory of solids in the furnace. This suggests a very large volume of solids in downward motion near the wall. Since they are not fully supported by the drag of upward-moving gas, the static pressure drop may not necessarily detect these solids. This effect is, however, less pronounced in small-diameter bench-scale CFB columns.

The actual mechanism of solid movement and mixing in the bed is more complex than portrayed above. In view of the rapid changes in our understanding of the system, no deeper treatment of this topic is presented here.

Example 2.3 Estimate the bed inventory in a CFB furnace operating at 825 °C, and the bed voidage at 4 m above a fast bed that is 20 m tall. Also, find the voidage at the wall at this height using the empirical equation of Eq. (2.22), given \( \rho_p = 2500 \) kg/m³, \( U = 8 \) m/s, and \( d_p = 300 \) μm, \( \epsilon_{mf} = 0.48 \).

The secondary air is injected at the level of 3 m. The average bed cross section is 2.5 m \( \times \) 10 m below and 5 m \( \times \) 10 m above this level.

\[
\epsilon(r, z) = \epsilon_{mf} + [\epsilon_{av}(z) - \epsilon_{mf}] \epsilon_{av}(z) \left[ -1.5 + 2.1 \left( \frac{z}{R} \right)^3 + 5.0 \left( \frac{z}{R} \right)^8 \right]
\] (2.24)
Solution
We assume the height of the secondary air level, \( h_i \), as equal to 3 m.
The asymptotic voidage is taken from Fig. 2.13b as \( \varepsilon_a = 0.85 \). For details on this choice, refer to Kunii and Levenspiel (1991).
The choking voidage in the dilute phase was calculated in Example 2.2 as 0.9976. We assume the asymptotic voidage in the dilute phase to be equal to this. So, \( \varepsilon_c = 0.9976 \).
The decay constant, \( a \), for \( U = 8 \) m/s is taken from Fig. 2.12 as \( a = 1.0 \) m\(^{-1} \). In the absence of adequate data, the use of Fig. 2.12 for \( a \) is somewhat arbitrary. Further research is needed for more precise value.
The voidage at the furnace exit, \( \varepsilon_e \), can be found from Eq. (2.18):
\[
\varepsilon_e = \varepsilon_d - (\varepsilon_d - \varepsilon_a) \exp[-a(H - h_i)]
\]
\[
\varepsilon_e = 0.9976 - (0.9976 - 0.85) \exp[-1(20 - 3)] \approx 0.9976
\]
The axial mean bed voidage, \( \varepsilon_s \), above the secondary air level is found from Eq. (2.19)
\[
\varepsilon_s = \varepsilon_d - \frac{\varepsilon_e - \varepsilon_a}{a(H - h_i)} = 0.9976 - \frac{0.9976 - 0.85}{1.0(20 - 3)} = 0.9889
\]
The solid inventory, \( W \), is found from Eq. (2.20) for average lower cross-sectional area \( A_i \) and upper cross-sectional area \( A_s \):
\[
W = \rho_p \left[ A_i h_i (1 - \varepsilon_a) + (H - h_i)(1 - \varepsilon_s) A_s \right]
\]
\[
W = 2500 \left[ 3 \times 2.5 \times 10 \times (1 - 0.85) + (20 - 3) \times 5 \times 10 \times (1 - 0.9889) \right]
\]
\[
W = 51,712 \text{ kg}
\]
The voidage at a height 4 m is found from Eq. (2.17):
\[
\varepsilon_e = 0.9976 - (0.9976 - 0.85) \exp[-1(4 - 3)] \approx 0.9433
\]
The voidage on the wall at this level is calculated from Eq. (2.22):
\[
\varepsilon(r) = \varepsilon_{av} \left[ \frac{3.62(r)^{0.47} + 0.191}{\pi} \right]
\]
\[
\varepsilon(r) = 0.9433 \left[ \frac{3.62(0.191)}{\pi} \right] = 0.8
\]
2.4 Gas–Solid Mixing

The mixing of gas and solids is an important aspect of the design of any CBF system involving reactions. This section presents a brief account of some of the important information related to this.

2.4.1 Gas–Solid Slip Velocity

The difference in bulk velocity of gas and solids in a fast bed could give some indication of the gas solid slip velocity, $U_{\text{slip}}$, in it.

$$U_{\text{slip}} = \frac{U}{\varepsilon} - \frac{G_s}{(1 - \varepsilon)\rho_p}$$  \hspace{1cm} (2.25)

However, we know that solids generally travel upward in a dispersed phase through the core of the bed, where the slip velocity will be of the order of the terminal velocity of individual particles. Near the wall, the gas velocity is much lower, and it may even be downward in some cases. Thus, when the solids descend along the wall, they do not necessarily come across a very high slip velocity. This picture, however, ignores the presence of clusters, which travel at a much different velocity, which may account for the high mass transfer rate (an indication of slip velocity) observed in many processes.

Direct measurement of local velocity of gas and particles shows the slip velocity to be higher toward the wall. The magnitude of the local slip velocity is higher than that of the terminal velocity, but lower than that predicted by Eq. (2.25).

2.4.2 Dispersion

The gas in a fast fluidized bed is generally assumed to be in a plug flow. However, there is some dispersion of gas in both the radial and axial directions. The gas dispersion from a point can be written as

$$D_r \frac{1}{r} \frac{d}{dr} \left[ r \frac{dC}{dr} \right] + D_a \frac{d^2C}{dz^2} = U \frac{dC}{dz}$$  \hspace{1cm} (2.26)

when $D_r$ and $D_a$ are dispersion coefficients in radial ($r$) and axial ($z$) directions.

The radial or lateral dispersion is much lower than the axial dispersion. The axial dispersion coefficient, $D_a$, decreases with velocity and voidage. Based on their experiments with dispersion in a bed of 58 μm FCC (1575 kg/m³), Li and Wu (1991) found that a simple empirical relation can express the axial dispersion coefficients for turbulent, fast, and entrained beds as a function of voidage:
\[ D_a = 0.1953 \ e^{-4.197} \ \text{m}^2/\text{s} \]  

The solid mixing in the radial mixing can be described using the two-channel model of Dingrong et al. (1988), but a reliable mixing coefficient is not available yet.

**References**


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