

# Chapter 2

## Design of First Order Controllers for Unstable Infinite Dimensional Plants

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**Abstract** A design method for first order controllers is presented for a class of unstable infinite dimensional plants, including systems with time delays, fractional order systems, and systems represented by PDEs. The design restricts the controllers to be in the form of PI, PD and lead or lag controllers. The approach is based on the small gain theorem and requires minimization of an  $H_\infty$  norm of a transfer function over a low number of parameters. The gain margin optimization problem is solved for PD controllers. For PI controllers, optimization of the integral action gain is also discussed.

### 2.1 Introduction

This work deals with the design of different types of first order controllers for infinite dimensional plants whose transfer functions contain single unstable pole. In this context PI, PD, lead and lag controllers are investigated. The basic idea is to put the characteristic equation of the feedback system into a form where the small gain theorem can be applied. For this purpose, algebraic manipulations similar to those used in [6, 17] play a crucial role. Once the controller structure is fixed, the range of allowable controller gain is estimated by computing the  $H_\infty$  norm of an infinite dimensional transfer function which contains a free parameter. Optimization of this free parameter is helpful for reducing the conservative results obtained in [17].

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It should be noted that when there are only small number of free parameters in the controller, classical stability checks (e.g. Nyquist criterion) can be used to determine the set of all stabilizing controller parameters. However, this brute force method may not be numerically very attractive, especially when the plant considered is unstable and infinite dimensional.

In particular, for time delay systems there are several numerically feasible methods for finding low order controllers, see e.g. [4, 5, 14, 19]. For applications to communication networks see [11] and [20]. The method of [17] has been extended to cover fractional order systems with time delays in [16], see also [3].

This chapter is organized as follows. Section 2.2 contains several examples of engineering applications where plant model falls within the framework of the present study. A sufficient condition for the stability of the feedback system (based on the small gain theorem) is derived in Sect. 2.3. Then in Sect. 2.4 different types of controllers are designed based on this condition. Conclusions and future works are given in Sect. 2.5.

## 2.2 Problem Definition and Examples of Plants Considered

As mentioned above, the plants considered here have transfer functions in the form

$$P(s) = \frac{1}{s - p} G(s) \quad (2.1)$$

where  $p \geq 0$  is the unstable pole and  $G \in H_\infty$  is the stable part of the plant. Note that  $G(s)$  can be irrational (plant is infinite dimensional). The factorization in the form (2.1) also implies that the plant is strictly proper.

The controllers to be designed have the following common structure

$$C(s) = K_p + \frac{K_d s}{\tau s + 1} + \frac{K_i}{s}, \quad K_p, K_d, K_i \in \mathbb{R}, \quad \tau \geq 0. \quad (2.2)$$

Note that PD, PI, lead and lag controllers are special case of (2.2):

$$C_{pd}(s) = K_p (1 + \tilde{K}_d s), \quad \tilde{K}_d = \frac{K_d}{K_p}, \quad (2.3)$$

$$C_{pi}(s) = K_p \left( 1 + \frac{\tilde{K}_i}{s} \right), \quad \tilde{K}_i = \frac{K_i}{K_p}, \quad (2.4)$$

$$C_\ell(s) = K_p \left( \frac{1 + \alpha \tau s}{1 + \tau s} \right), \quad \alpha \tau = \tau + \tilde{K}_d. \quad (2.5)$$

Clearly  $C_\ell$  is a lead controller if  $\tilde{K}_d > 0$  and it is a lag controller if  $0 > \tilde{K}_d > -\tau$ .

**Definition 1** The feedback system formed by the controller  $C$  and the plant  $P$  is stable if  $S := (1 + PC)^{-1}$ ,  $CS$  and  $PS$  are stable, i.e., they are transfer functions in  $H_\infty$ . If this is the case, then the controller  $C$  is said to stabilize the plant  $P$ . The set of all controllers stabilizing a given plant  $P$  is denoted by  $\mathcal{C}(P)$ .

The goal of this chapter is to determine controllers  $C(s)$ , in the form (2.3)–(2.5), stabilizing a given unstable infinite dimensional plant  $P(s)$  whose transfer function is given by (2.1). There are several applications where plant transfer functions have this structure; specific examples are given below.

*Example 1 Integrating systems with transport delay:*

$$P(s) = \frac{K e^{-hs}}{s}, \quad K > 0, \quad h > 0, \quad (2.6)$$

i.e., in this case,  $p = 0$  and  $G(s) = K e^{-hs}$ . There are many application examples and control methods for this plant, [12, 21]. Application examples include oil/gas pipelines, communication networks, manufacturing plants, storage systems, etc., see e.g., [13, 18].

*Example 2 Abstract model of an aircraft:*

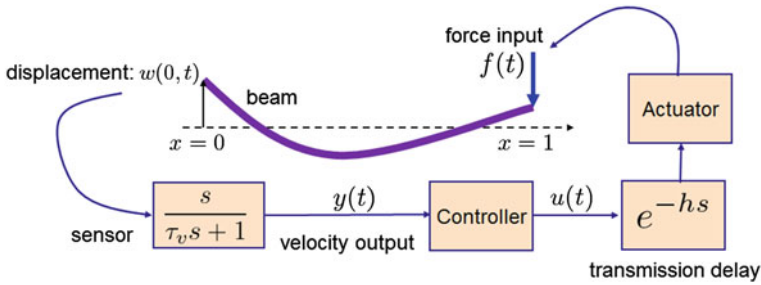
$$P(s) = \frac{e^{-hs}}{s - p}, \quad h > 0, \quad p > 0, \quad G(s) = e^{-hs}. \quad (2.7)$$

This model is used for the purpose of controlling the high frequency longitudinal dynamics (short period) of an aircraft. Dynamics due to elasticity, sensor, actuator, sampling, contribute to the time delay. The product  $h \cdot p$  represents how difficult it is to control this open loop unstable system. Depending on the operating regime, it is observed that  $0.06 < h \cdot p < 0.37$  for an X-29 aircraft [2].

*Example 3 Flexible beam with non-collocated actuator and sensor:* Typically, mathematical models of flexible beams are given by partial differential equations, [1], and their transfer functions are irrational. For the free-free beam model (with normalized material parameters) shown in Fig. 2.1, the following infinite product expansion of  $G(s)$  converges in  $H_\infty$  (see [9, 10]):

$$P(s) = \frac{1}{s} G(s), \quad G(s) = \frac{2e^{-hs}}{(\tau_v s + 1)} \prod_{n=1}^{\infty} \left( \frac{1 + \varepsilon s - s^2/\omega_n^2}{1 + \varepsilon s + s^2/\tilde{\omega}_n^2} \right), \quad (2.8)$$

where  $\tau_v > 0$  is the sensor parameter,  $h > 0$  is the input delay,  $\varepsilon > 0$  is the damping parameter of the beam and  $\omega_n, \tilde{\omega}_n > 0$  with  $\omega_n \rightarrow 2\left(\frac{\pi}{4} + n\pi\right)^2$  and  $\tilde{\omega}_n \rightarrow \left(\frac{\pi}{2} + n\pi\right)^2$  as  $n \rightarrow \infty$ .



**Fig. 2.1** Flexible beam control loop under delayed point force input and velocity feedback

*Example 4 Interconnected systems with time delays:*

$$P(s) = \frac{e^{-hs}}{s+2} \left( \frac{s+1+2(s-1)e^{-2s}}{s+1-2e^{-0.4s}} \right) = \frac{1}{s-p} G(s), \quad (2.9)$$

where  $h > 0$  and  $p \approx 0.5838$  is the unique root of  $(s+1-2e^{-0.4s}) = 0$  in  $\overline{\mathbb{C}}_+$ . So,

$$G(s) = e^{-hs} \left( \frac{(s+1)+2(s-1)e^{-2s}}{s+2} \right) \left( \frac{s-p}{s+1-2e^{-0.4s}} \right).$$

*Example 5 A non-laminated magnetic suspension system:* The following fractional order plant model is taken from [8]:

$$P(s) = \left( (s^\alpha)^5 + (s^\alpha)^4 - c \right)^{-1}, \quad \alpha = 0.5, \quad c > 0. \quad (2.10)$$

It has been shown that  $P$  can be factored as in the standard form (2.1), see [7]:

$$P(s) = \frac{1}{s-p} G(s) \quad \text{with} \quad p = r^2, \quad G(s) = \frac{(s^\alpha + r)(s^\alpha - r)}{(s^\alpha)^5 + (s^\alpha)^4 - c}$$

where  $r > 0$ , is the unique root of  $(z^5 + z^4 - c) = 0$  on  $\mathbb{R}_+$ .

### 2.3 A Sufficient Condition for Feedback System Stability

In this section the controller is taken to be in the form  $C_{pd}$  or  $C_\ell$ . Such a controller is stabilizing a plant in the form (2.1) if and only if there exists a constant  $a > 0$  such that  $U_a$  is unimodular (i.e.  $U_a, U_a^{-1} \in H_\infty$ ):

$$U_a(s) := \frac{s-p}{s+a} + \frac{K_p}{s+a} G(s)C_0(s),$$

where  $C_0(s) := \frac{1+\alpha\tau s}{1+\tau s}$  when  $C = C_{pd}$  or  $C_0(s) := (1 + \tilde{K}_d s)$  when  $C = C_\ell$ . Define

$$K_p := (p+a)G(0)^{-1} \quad \text{and} \quad G_0(s) := G(s)G(0)^{-1} \quad (2.11)$$

then

$$U_a(s) = 1 + (p+a) \frac{s}{s+a} \Psi_0(s) \quad \text{where} \quad \Psi_0(s) = \frac{1}{s} (G_0(s)C_0(s) - 1).$$

Thus, using the fact that  $\left\| \frac{s}{s+a} \right\|_\infty \leq 1$ ,  $U_a$  is unimodular if

$$(p+a) < \|\Psi_0\|_\infty^{-1}. \quad (2.12)$$

The condition (2.12) was derived earlier in [6, 17]. Clearly, a less conservative condition for  $U_a$  to be unimodular is

$$(p+a) < \|\Psi_a\|_\infty^{-1}, \quad (2.13)$$

where

$$\Psi_a(s) := \frac{1}{s+a} (G_0(s)C_0(s) - 1).$$

Note that

$$\|\Psi_a\|_\infty \leq \|\Psi_0\|_\infty \quad \forall a > 0.$$

Therefore, the controller defined as above is a stabilizing controller for the plant if

$$pG(0)^{-1} < K_p < (p+a_o)G(0)^{-1},$$

where  $a_o > 0$  is the largest  $a > 0$  satisfying (2.13).

In order to illustrate the computations involved in the above discussion, let us consider the plant defined by (2.6) with  $K = 1$  and  $h > 0$ . Let  $C_0(s) = 1$  (i.e., consider proportional control only). The exact value of the upper bound of the controller gain can be easily computed as

$$K_p = a < \frac{\pi}{2h} \approx 1.57h^{-1}.$$

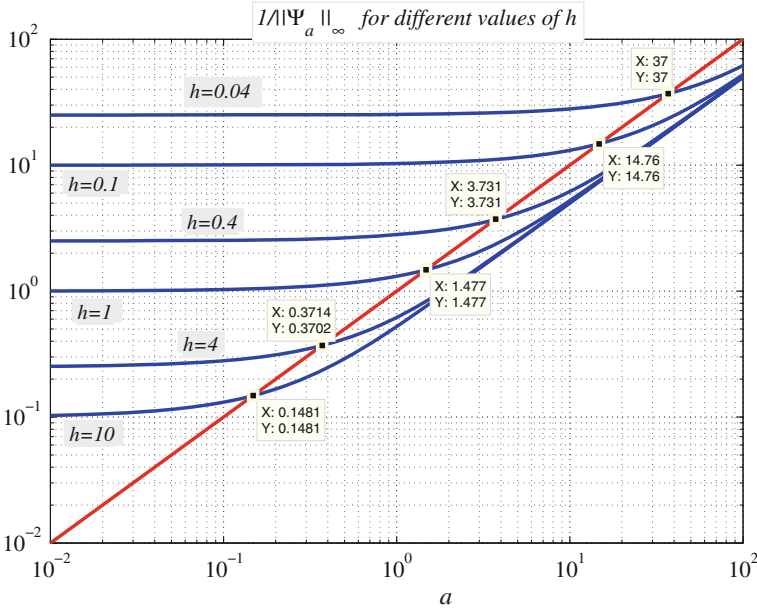


Fig. 2.2 The graph of  $1/\|\Psi_a\|_\infty$  for different values of  $h$ : the largest  $a$  satisfying (2.13) is  $1.48/h$

If one uses the condition (2.12), the conservative upper bound of the controller gain is

$$K_p = a < \frac{1}{\|\Psi_0\|_\infty} = h^{-1}.$$

On the other hand, if (2.13) is used, then

$$K_p = a < \tau_o h^{-1} \quad \text{with} \quad \tau_o \approx 1.48,$$

where  $\tau_o$  is computed as shown in Fig. 2.2.

Clearly, when  $C_0(s) \neq 1$ , for example,  $C_0(s) = \frac{1+\alpha\tau s}{1+\tau s}$  or  $C_0(s) = (1 + \tilde{K}_d s)$ , the free parameters  $(\alpha, \tau)$  or  $\tilde{K}_d$  can be used to further maximize  $a_o$ , the largest  $a > 0$  satisfying (2.13).

*Remark 1* There are some plants which do not admit a feasible stabilizing controller in the form  $C_{pd}$  or  $C_\ell$ . For example, if the plant does not satisfy the parity interlacing property (PIP), then there does not exist a *stable* stabilizing controller. In order to illustrate this point, consider the plant

$$P(s) = \frac{1}{s-p} \left( \frac{1-s/z}{1+\tau s} \right) \quad p > 0, z > 0, \tau > 0,$$

for which there exist a stable stabilizing controller if and only if  $p < z$ . If the proportional controller is defined as  $C(s) = K_p = (p + a)$  then, using the notation set above,

$$\Psi_a(s) = \frac{-s(\tau + z^{-1})}{(s + a)(1 + \tau s)} \Rightarrow 1/\|\Psi_a\|_\infty = z \left( \frac{1 + \tau a}{1 + \tau z} \right).$$

So, the condition (2.13) becomes

$$p + a < z \left( \frac{1 + \tau a}{1 + \tau z} \right) = \frac{z}{1 + \tau z} + \frac{\tau z}{1 + \tau z} a,$$

which is stronger than the PIP, i.e.,  $p < z$ .

## 2.4 PD and PI Controller Designs

### 2.4.1 PD Controller Design

Recall that for the plant (2.1), a PD controller is in the form  $C_{pd}(s) = K_p C_0(s)$  where  $K_p = (p + a)G(0)^{-1}$  and  $C_0(s) = (1 + \tilde{K}_d s)$ . Based on the results of Sect. 2.3 the largest  $a > 0$  satisfying (2.13) should be computed depending on  $\tilde{K}_d \in \mathbb{R}$ . For this purpose define  $G_0(s) = G(s)G(0)^{-1}$ ,  $Q := \tilde{K}_d \in \mathbb{R}$  and

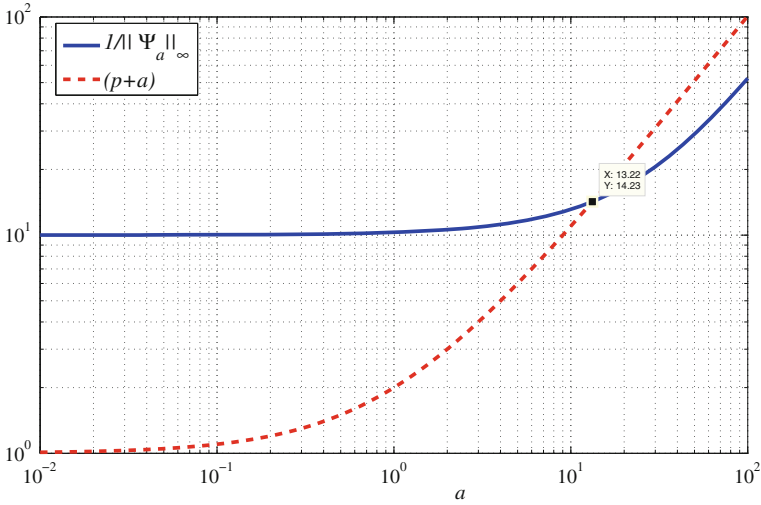
$$\gamma(Q, a) := \left\| \frac{G_0(s) - 1}{s + a} + Q \frac{s}{s + a} G_0(s) \right\|_\infty. \quad (2.14)$$

In order to maximize the gain margin (GM) of the system one should try to minimize  $\gamma(Q, 0)$  (the conservative approach) or try to find the largest  $a$  satisfying (2.13). See [17] for a detailed discussion on the computation of the optimal  $Q$  minimizing  $\gamma(Q, 0)$  for the conservative approach. The main idea can be extended to the case  $a > 0$  easily; see the algorithm given below.

**Initialize:** Determine a range of  $Q \in [Q_{\min}, Q_{\max}] \subset \mathbb{R}$

**Step 1.** For each fixed  $Q$  in this interval  
if it exists find the largest  $a_{\max}(Q)$  such that

$$(p + a) < 1/\gamma(Q, a) \quad \forall a < a_{\max}(Q).$$



**Fig. 2.3**  $(p + a)$  and  $1/\|\Psi_a\|_\infty$  versus  $a$  for the plant (2.7) with  $h = 0.1$  and  $p = 1$

**Step 2.** Plot  $Q$  versus  $a_{\max}(Q)$  find the maximum of  $a_{\max}(Q)$  and define

$$Q_{opt} := \arg \max\{a_{\max}(Q)\}.$$

**End:** An allowable range of the controller gain  $K_p$  is

$$pG(0)^{-1} < K_p < (p + a_o)G(0)^{-1} \quad \text{with } a_o := a_{\max}(Q_{opt}).$$

For  $p > 0$ , gain margin optimizing (see [15]) PD controller parameters are

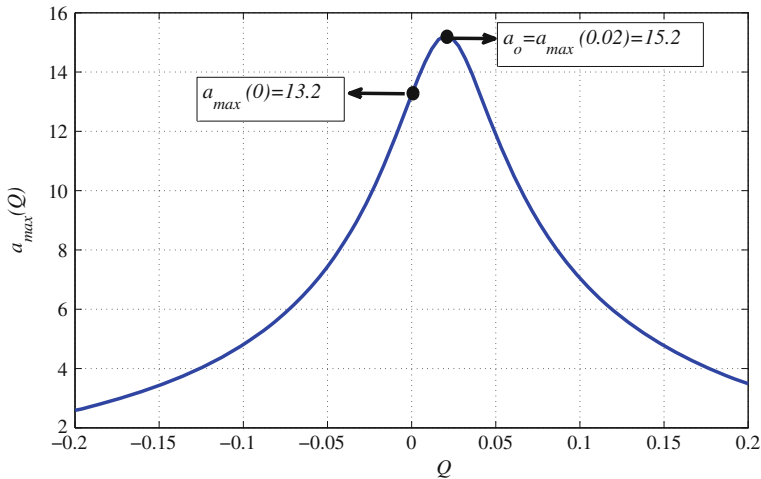
$$\tilde{K}_{d,opt} = Q_{opt}, \quad K_{p,GMopt} = \sqrt{p(p + a_o)}G(0)^{-1}.$$

Alternatively, one can choose the *least fragile* proportional gain

$$K_{p,LF} = \left(p + \frac{a_o}{2}\right)G(0)^{-1}.$$

Step 1 of the algorithm involves drawing a graph like the one shown in Fig. 2.2. To illustrate the numerical computations, consider the plant (2.7) with  $h = 0.1$  and  $p = 1$ . If proportional controller is used, then  $Q = 0$  and  $a_{\max}(0) = 13.2$  as seen in Fig. 2.3; that means the allowable range of the gain is  $1 < K_p < 14.2$ .





**Fig. 2.4**  $a_{\max}$  versus  $Q$  for the plant (2.7) with  $h = 0.1$  and  $p = 1$

On the other hand, it is possible to enlarge this interval by adding a derivative action. Figure 2.4 shows how  $a_{\max}$  change as a function of  $Q$ . Clearly, the optimal choice is  $\tilde{K}_{d,opt} = Q_{opt} = 0.02$  and that leads to  $a_o = \max a_{\max}(Q) = 15.2$  which means that the allowable gain is in the interval  $1 < K_p < 16.2$  and

$$C_{pd,GMopt}(s) = 4.025 (1 + 0.02 s) \quad \text{and} \quad C_{pd,LF}(s) = 8.6 (1 + 0.02 s).$$

*Remark 2 On Lead-Lag Controller Design.*

Recall that for the lead or lag controller design  $C_0(s)$  is in the form

$$C_0(s) = 1 + \frac{Q_1 s}{1 + Q_2 s} \quad \text{with} \quad Q_1 := \tilde{K}_d > -\tau, \quad Q_2 := \tau > 0.$$

Then, similar to the PD controller design, the parameter  $a$  which determines the controller gain should be such that  $(p + a) < 1/\gamma_a$ , where

$$\gamma_a(Q_1, Q_2) = \left\| \frac{G_0(s) - 1}{s + a} + \left( \frac{Q_1}{1 + Q_2 s} \right) \frac{s}{s + a} G_0(s) \right\|_{\infty}.$$

So, to find  $a_{\max}(Q_1, Q_2)$ , in Step 1 of the corresponding gain margin optimization algorithm, the computations are done for two parameters in nested loops. Then in Step 2, a surface plot of  $a_{\max}(Q_1, Q_2)$  is obtained and its maximum is determined.

## 2.4.2 PI Controller Design

Consider the design of a PI controller in the form

$$C_{pi}(s) = C_1(s) + \frac{K_i}{s}, \quad (2.15)$$

where  $C_1(s) = K_p$  is such that  $C_1 \in \mathcal{C}(P)$ . In other words, a controller  $C_1$  is already designed to stabilize  $P$  and now the integral action is to be added to the controller. The following discussion is valid for  $C_1 = C_{pd}$  as well, in that case the addition of integral term will give a PID controller  $C_2 = C_{pid}$ .

Since  $C_1 \in \mathcal{C}(P)$  the following statement holds:

$$H_1(s) := \frac{P(s)}{1 + C_1(s)P(s)} \quad \text{is in } H_\infty.$$

The characteristic equation of the feedback system formed by  $C_2$  and  $P$  is

$$1 + C_1(s)P(s) + \frac{K_i}{s}P(s) = (1 + C_1(s)P(s)) \left( 1 + \frac{K_i}{s}H_1(s) \right) = 0.$$

Using the fact that  $C_1 \in \mathcal{C}(P)$  it can be concluded that

$$C_2 \in \mathcal{C}(P) \iff V_1^{-1} \in \mathcal{H}_\infty \quad \text{with} \quad V_1(s) = \left( 1 + \frac{K_i}{s}H_1(s) \right).$$

Now define

$$b := K_i H_1(0),$$

then  $V_1$  can be re-written as

$$V_1(s) = \left( 1 + \frac{b}{s} \right) \left( 1 + \left( 1 + \frac{b}{s} \right)^{-1} b \left( \frac{H_1(s)H_1(0)^{-1} - 1}{s} \right) \right). \quad (2.16)$$

Let us now assume that  $b > 0$  (this is without loss of generality, since the sign of  $K_i$  can be adjusted according to the sign of  $H_1(0)$ ). Then, note that

$$\left( 1 + \frac{b}{s} \right)^{-1} = \frac{s}{s+b} \in H_\infty \quad \text{with} \quad \left\| \frac{s}{s+b} \right\|_\infty = 1.$$

The following result can be derived from the small gain theorem:  $V_1^{-1} \in H_\infty$ , i.e.,  $C_2 \in \mathcal{C}(P)$ , if  $b$  satisfies

$$0 < b < 1/\|\Phi_0\|_\infty \quad \text{where} \quad \Phi_0(s) = \left( \frac{H_1(s)H_1(0)^{-1} - 1}{s} \right). \quad (2.17)$$

In fact, a careful examination of (2.16) shows that, rather than (2.17), the following less conservative sufficient condition on  $b$  can be used for  $C_2$  to be in  $\mathcal{C}(P)$ :

$$0 < b < 1/\|\Phi_b\|_\infty \quad \text{where} \quad \Phi_b(s) = \left( \frac{H_1(s)H_1(0)^{-1} - 1}{s + b} \right). \quad (2.18)$$

Clearly, there is an analogy between  $\Psi_a$  and  $\Phi_b$ , and the conditions (2.13) and (2.18). Note that  $\Phi_b$  depends on  $K_p$  which is assumed to be in  $\mathcal{C}(P)$ . So, the optimal PI controller  $C_{pi,opt}(s) = K_{p,opt} + (K_{i,opt}/s)$  can be designed as follows.

For each fixed  $K_p \in \mathcal{C}(P)$ , find the largest allowable  $b > 0$  satisfying (2.18) and let it be denoted as  $b_{\max}(K_p)$ . Accordingly, define

$$K_{p,opt} := \arg \max \{ b_{\max}(K_p) : K_p \in \mathcal{C}(P) \}.$$

Then, the least fragile integral action gain is

$$K_{i,opt} = \frac{b_{\max}(K_{p,opt})}{2} H_1(0)^{-1}.$$

In order to illustrate the computations involved in the design method described above, let us consider once more the plant (2.7) with  $h = 0.1$  and  $p = 1$ . Recall from Fig. 2.3 that  $C_1(s) = K_p$  is a stabilizing controller if  $K_p \in (1, 14.2)$ . For each fixed  $K_p$  in this interval, define

$$H_1(s) = \frac{e^{-0.1s}}{s - 1 + K_p e^{-0.1s}}, \quad \text{clearly} \quad H_1(0) = \frac{1}{K_p - 1}.$$

Simple computations give  $\Phi_b$  as:

$$\Phi_b(s) = \frac{H_1(s)H_1(0)^{-1} - 1}{s + b} = \left( \frac{1}{s + b} \right) \left( \frac{1 - s - e^{-0.1s}}{s - 1 + K_p e^{-0.1s}} \right).$$

Following the above procedure, for each  $K_p \in (1, 14.2)$  and  $b > 0$ , the  $H_\infty$ -norm  $\|\Phi_b\|_\infty$  is computed. Then, from the graph of  $1/\|\Phi_b\|_\infty$  versus  $b$ , the largest  $b$ , denoted by  $b_{\max}(K_p)$ , satisfying (2.18) is determined. Figure 2.5 shows  $b_{\max}(K_p)$  versus  $K_p$ . Clearly, the largest  $b_{\max}(K_p) = 5.7$  is achieved at  $K_p = K_{p,opt} = 4.8$ . For the least fragile integral gain, let  $K_{i,opt} = H_1(0)^{-1} b_{\max}(K_{p,opt})/2 = 10.8$ . The resulting controller is

$$C_{pi,LF}(s) = 4.8 + \frac{10.8}{s}.$$

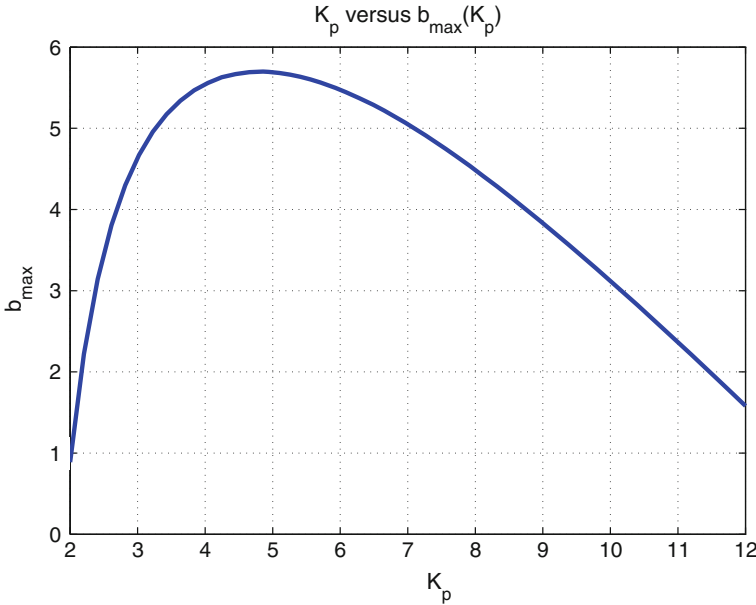


Fig. 2.5  $b_{\max}(K_p)$  versus  $K_p$  for the plant (2.7) with  $h = 0.1$  and  $p = 1$

### 2.5 Conclusions and Future Extensions

In this chapter of the book, a method is proposed for the design of stabilizing first order controllers (PD, PI and lead or lag controllers) for a class of infinite dimensional plants. The main assumption is that the plant has a single unstable pole (at the origin, or on the positive real axis). Examples from several applications are given to justify the plant model considered. These examples include systems with time delays, fractional order systems, and systems represented by PDEs.

The approach is based on the small gain theorem and requires minimization of the  $H_\infty$  norm of an infinite dimensional stable transfer function over a low number of parameters.

Another way to obtain  $C_\ell(s) = K_p C_0(s)$  with a large gain margin would be to find a first order approximation of an infinite dimensional stable controller determined from the following  $H_\infty$  control problem. For a fixed  $a > 0$ , first, solve the one block problem

$$\gamma_o(a) = \inf_{Q \in \mathcal{H}_\infty} \left\| \frac{1}{s+a} (1 - G_0(s)Q(s)) \right\|_\infty.$$

If

$$(p + a) < 1/\gamma_o(a), \tag{2.19}$$

then define

$$Q_a(s) := \arg \gamma_o(a).$$

Now, all controllers in the form  $K_0 G(0)^{-1} Q_a(s)$  stabilize the plant, which is given by  $P(s) = \frac{1}{s-p} G(s)$ , provided that the gain is in the interval

$$p < K_0 < (p + a).$$

Thus, to maximize the allowable controller gain the maximum  $a$  defined below should be determined:

$$a_{\max} = \arg \max \{ a : a \in \mathbb{R}_+ \text{ and (2.19) holds } \}.$$

The least fragile *stable* controller, in this framework, is

$$C_{s,LF}(s) = (p + \hat{a}) G(0)^{-1} Q_{\hat{a}}(s) \quad \text{where} \quad \hat{a} := \frac{a_{\max}}{2}.$$

Approximation of  $Q_{\hat{a}}(s)$  by a first order controller, then, gives a lead or lag controller in the form  $C_{\ell}(s)$ . The above approach (and other alternative methods of approximating the plant first and then designing a low order controller) must be further compared with the proposed design of Sect. 2.4.1 on practical application examples. Currently, this is left open for a future study.

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## References

1. Curtain, R., Morris, K.: Transfer functions of distributed parameter systems: a tutorial. *Automatica* **45**, 1101–1116 (2009)
2. Enns, D., Özbay, H., Tannenbaum, A.: Abstract model and controller design for an unstable aircraft. *AIAA J. Guidance Control Dyn.* **15**, 498–508 (1992)
3. Fioravanti, A.R., Bonnet, C., Özbay, H., Niculescu, S.-I.: A numerical method for stability windows and unstable root-locus calculation for linear fractional time-delay systems. *Automatica* **48**, 2824–2830 (2012)
4. Gumussoy, S., Michiels, W.: Fixed-order  $H_{\infty}$  control for interconnected systems using delay differential algebraic equations. *SIAM J. Control Opt.* **49**, 2212–2238 (2011)
5. Gündeş, A.N., Özbay, H.: Low order controller design for systems with time delays. In: *Proceedings of the 50th IEEE Conference on Decision and Control*, Orlando, pp. 5633–5638 (2011)
6. Gündeş, A.N., Özbay, H., Özgüler, B.: PID controller synthesis for a class of unstable MIMO plants with I/O delays. *Automatica* **43**, 135–142 (2007)
7. Karagül, A.E., Özbay, H.: On the  $H_{\infty}$  controller design for a magnetic suspension system model. In: *Preprints of the IFAC Joint Conference 5th SSSC, 11th TDS, 6th FDA*, Grenoble, FR (2013)

8. Knospe, C., Zhu, L.: Performance limitations of non-laminated magnetic suspension systems. *IEEE Trans. Control Syst. Technol.* **19**, 327–336 (2011)
9. Lenz, K., Özbay, H.: Analysis and robust control techniques for an ideal flexible beam. *Multidisciplinary Engineering Systems: Design and Optimization Techniques and their Applications*. In: Leondes, C.T. (ed.) *Control and Dynamic Systems*, vol. 57, pp. 369–421. Academic Press, New York (1993)
10. Lenz, K., Özbay, H., Tannenbaum, A., Turi, J., Morton, B.: Frequency domain analysis and robust control design for an ideal flexible beam. *Automatica* **27**, 947–961 (1991)
11. Melchor-Aguilar, D., Niculescu, S.-I.: Computing nonfragile PI controllers for delay models of TCP/AQM networks. *Int. J. Control* **82**, 2249–2259 (2009)
12. Michiels, W., Niculescu, S.-I.: *Stability and Stabilization of Time-Delay Systems: An Eigenvalue-Based Approach*. SIAM, Philadelphia (2007)
13. Niculescu, S.-I.: *Delay Effects on Stability: A Robust Control Approach*. Springer, London (2001)
14. Ou, L.-L., Zhang, W.-D., Yu, L.: Low-order stabilization of LTI systems with time delay. *IEEE Trans. Autom. Control* **54**, 774–787 (2009)
15. Özbay, H.: *Introduction to Feedback Control Theory*. CRC Press LLC, Boca Raton (2000)
16. Özbay, H., Bonnet, C., Fioravanti, A.R.: PID controller design for fractional-order systems with time delays. *Syst. Control Lett.* **61**, 18–23 (2012)
17. Özbay, H., Gündeş, A.N.: Resilient PI and PD controller designs for a class of unstable plants with I/O delays. *Appl. Comp. Math.* **6**, 18–26 (2007)
18. Quet, P.-F., Ataslar, B., İftar, A., Özbay, H., Kalyanaraman, S., Kang, T.: Rate-based flow controllers for communication networks in the presence of uncertain time-varying multiple time-delays. *Automatica* **38**, 917–928 (2002)
19. Silva, G.J., Datta, A., Bhattacharyya, S.: *PID Controllers for Time-Delay Systems*. Birkhauser, Boston (2005)
20. Üstebay, D., Özbay, H., Gündeş, A.N.: A new PI and PID control design method for integrating systems with time delays: applications to AQM of TCP flows. *WSEAS Trans. Syst. Control* **2**, 117–124 (2007)
21. Visioli, A., Zhong, Q.: *Control of Integral Processes with Dead Time*. Springer, New York (2011)



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