Preface

In many disciplines, problems appear which can be formulated with the aid of differential or integral equations. In simpler cases, such equations can be solved analytically, but for more complicated cases, numerical procedures are needed. At present we have, for this purpose, several methods and programs. In recent times, the wavelet-based methods have gained great popularity, where different wavelet families such as Daubechies Coiflet, Symlet, etc., wavelets are applied. A shortcoming of these wavelets is that they do not have an analytic expression. For this reason, differentiation and integration of these wavelets are very complicated and doubts about the expediency of these wavelets in Calculus arise.

From wavelets which have an analytic expression mathematically the simplest are the Haar wavelets, which consist of pairs of piecewise constant functions. Such functions were introduced by Alfred Haar in 1910 and they have been used for solving problems of Calculus only from 1997.

When compared with other methods of solution, the Haar wavelet approach has some preferences, as mathematical simplicity, possibility to implement standard algorithms, and high accuracy for a small number of grid points. The solutions based on the Haar wavelets are usually simpler and faster than in the case of other methods. For these reasons, the Haar wavelets have obtained a great popularity and the number of papers about Haar Wavelets is rapidly increasing. According to the Science Direct in 12.04.13, there were 3,295 publications about Haar wavelets, among these 1,266 items are on differential and integral equations. To the reader it is difficult to find his way among the great number of these publications; therefore, a text-book about the applications of the Haar wavelets in Calculus is extremely necessary. Unfortunately such a book has been missing up to now.

The aim of the present book is to fulfill this gap, even if partially. At present, time different variants of the Haar method exist. It is not reasonable to handle and analyze all of them in detail; it would make the book less understandable and could confuse the reader. Therefore, we have decided to choose a method of solution, which is sufficiently universal and is applicable to solve all the problems by a unit approach. Other treatments will be referred and discussed in the section related papers, which is added to each chapter.

The book is put together on the basis of 19 papers, which we have published in prereviewed international journals. A unit method of solution is applied for solution of a wide range of problems (different types of differential and integral
equations, fractional integral equations, optimal control theory, buckling and vibrations of elastic beams). To demonstrate efficiency and accuracy of the proposed method, a number of examples is solved. Mostly test problems, for which the exact solution or solution obtained by other methods is known, are considered.

The book is meant for researchers in applied mathematics, physics, engineering, and related disciplines, also for teachers of higher schools, graduate and postgraduate students. To make the book accessible for a wider circle of readers, some mathematical finesses are left out.

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