GBI Method: A Powerful Technique to Study Drying of Complex Shape Solids

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Abstract This chapter briefly focuses on the theory and applications of drying process (heat and mass transfer) with particular reference to arbitrarily-shaped wet capillary-porous bodies. Here, a modeling based on the liquid diffusion theory and the mathematical formalism to obtain the exact solution of the governing equation via Galerkin-based integral method are presented. The model considers constant thermo-physical properties and convective boundary conditions at the surface of the solid. Applications have been done to different solids of revolution and wheat grain. Predicted results of the average moisture content, average temperature, and moisture content and temperature distributions within the porous solids are presented and discussed, and for particular situations they are compared with experimental drying data.

Keywords Drying · Analytical · Wheat · Prolate spheroid

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1 Introduction

Drying is a processing operation that play important role in many industries such as food, pharmaceutical, ceramics, chemical and waste treatment. Drying is the removal of unbounded water molecules in the liquid and/or vapor phases caused by a partial pressure difference of water vapor between the surface of the product to be dried and the air surrounding. It is the process most often used for quality assurance and stability during storage of biological products such as fruits, vegetables and grains, considering that reduced amount of water in the material reduce both biological activity and deterioration caused by microorganisms, and chemical and physical changes that occur during storage [1]. In the manufacturing process of ceramics products, clay minerals, when mixed with water, become highly plastic (hydroplasticity) and it can be molded easily without cracking. These physical and chemical characteristics maintain the body shape during handling and drying. The water added to ceramic clay cover the surface of the clay particle, fills small capillaries and porous, and causes a separation of particles [2]. After plastic forming and casting, products must be dried, in order, to removal of the moisture prior for further processing and/or firing. If this moisture content is not removed, the extreme temperature in the kiln will force out this water during firing, causing cracking and until explosion of the product.

Drying is a phenomenon that occurs with simultaneous heat and mass transport and volume variation (shrinkage by moisture removal and expansion by heating). During the drying process appear strong thermo-hydro-mechanical stresses which are caused by the emergence of moisture and temperature gradients within the body, mainly when the solid has a geometry that disfavor its mechanical strength as for example tips. Then, to study drying of solids it is important to control and optimize the process, avoiding damage to the material.

The heat and mass transport within the porous solids had been subject of the investigation for many years, because of their wide variety of industrial applications involving drying and wetting, and scientific interest. Because of the complexities of the mechanism involved in the process through the irregular void configuration within the porous bodies many theories has been proposed to predict moisture movements into the porous solids. For instance, liquid diffusion theory, capillary theory, evaporation-condensation theory, Luikov's theory, Philip and De Vries'theory, Krischer's theory, Berger and Pei's theory, and Fortes and Okos theory. In the hygroscopic porous bodies the pores are partially filled with liquid water and partially filled with non condensable gas and water vapor mixture. Thus, moisture can be transferred both in liquid and in gaseous phases, and the following possible mechanisms for moisture migration in a porous solid are cited in the literature: liquid diffusion, vapor diffusion, surface diffusion, effusion flow, thermodiffusion, transport by capillary forces, transport by gravity forces, transport by osmotic pressure, transport due to pressure gradients, and transport due to shrinkage [1, 3–5]. The liquid diffusion theory has gained the preference of the researchers. It considers that the water inside the solid migrates to surface only in the liquid phase. This assumption facilitates the analysis of the physical problem.

The theoretical treatment to predict moisture removal by drying formally is based upon the traditional method and start with a potential balance in a differential control volume with arbitrary shape, in macroscopic scale. The geometry of the body is one of the factors that are used to establish certain hypotheses in the description of a physical process. When liquid diffusion theory is used to predict drying process, the Fick's second law and the Fourier's law has been utilized to predict mass diffusion and heat conduction inside the solid, respectively.

On the basis of many researches in early works, the heat and mass diffusion within a capillary-porous body can be predicted solving the governing equations by mean of different techniques such as: numerical (finite-difference, finite-element, boundary-element and finite-volume), analytical (separation of variables, Laplace transform and Galerkin-based integral method) and semi-analytical (Generalized integral transform). For several cases, appropriate considerations and boundary conditions, and often constant thermo-physical properties for the medium are assumed.

In the literature are cited several works which use common geometry shaped bodies, such as plates, cylinders and spheres [6–9]. Thus, different numerical studies in oblate or prolate spheroidal solids are reported in the literature [10–24]. These formal solutions of the diffusion equation has been obtained for various boundary conditions with constant or variable diffusion coefficient, in homogeneous or heterogeneous and isotropic or anisotropic bodies, and in steady or unsteady cases. However, when we consider one- or two-dimensional geometries for describe drying of solids with arbitrary shape discrepancies in the drying and heating kinetics and distribution of moisture and temperature within the solid are found when compared with experimental results.

For obtaining the analytical solution of the governing transport equation is a difficult task especially when the solid has an irregular shape. Despite of this disadvantage, the exact solution of the diffusion equation applied to bodies with complex geometry and constant or convective boundary conditions are reported in the literature [25–40]. These mentioned works are directed to study uncoupled heat and/or mass transfer.

Since, rigorous solutions to the heat and mass diffusion in different geometrical shapes are essential, for the predictions of both the performance and control of the processes like heating, cooling, wetting and drying, for examples, so, we notice that there is a lack of studies that must take into account the heat and mass transfer in bodies with complex geometric shape, which is necessary in order to describe more adequately the drying process.

In this sense, this chapter aims to develop a mathematical model and its analytical solution to predict drying of solids of revolution via Galerkin-based integral method. Applications have been done to different solids of revolution and wheat grain (prolate spheroid).

2 Heat and Mass Transfer Modeling Applied to Irregularly-Shaped Solids

The complexity of real processes taking place in a capillary-porous materials not only leads to uncertainties and difficulties with their mathematical formulation, but also cause considerable problem with the numerical and analytical treatment of resulting equation. According to Farias et al. [41], in the macroscopic scale, diffusion modeling approach presents like advantages: less mathematics than the microscopic transport analysis, external effects can be incorporated in boundary conditions, easy to obtain analytical solution for some particular cases. However, like disadvantages we can cite: many questionable theoretical assumptions are required, extrapolation to areas outside region of experimental data are questionable [42].

A realistic approach of the physical-mathematical model for the drying process of a solid, it can be influenced by internal and external conditions and mechanism of moisture migration and heat flux inside the material.

From the transport general equation applied to a infinitesimal control volume, when source and convective terms are nulls, we obtain the diffusion equation in the short form expressed as:

$$\frac{\partial(\lambda\Phi)}{\partial t} = \nabla \cdot \left(\Gamma^{\Phi}\nabla\Phi\right) \tag{1}$$

The solution of the diffusion problems for various physical situations of interest often requires the need to establish certain assumptions in describing the physical process. One of them is related to the geometry of the body in which occurs the transport of matter or energy. Here, in order to enable the solution of the physical issue, the following considerations were taken:

(a) The solid is homogeneous and isotropic;
(b) The distribution of the potential Φ inside the solid is uniform at the beginning of the process;
(c) Thermophysical properties are constant throughout the process;
(d) The solid consists of dry substance and water in the liquid phase;
(e) The phenomenon of drying occurs by diffusion of liquid water inside the solid and by evaporation of water at the surface.

Considering the transport coefficients Γ^{Φ} and λ constant, the formal solution of Eq. (1) can be written as follows [27]:

$$\Phi = \sum_{n=1}^{N} C_n \psi_n e^{-\gamma_n t} + \Phi_e \tag{2}$$

whereas the $C_{n,}$ γ_n and Φ_e are constants, and Ψ_n is a function independent of time. Putting Eq. (2) in (1) we obtain:

$$\frac{\partial}{\partial t}\left[\lambda\sum_{n=1}^{N}C_n\psi_n e^{-\gamma_n t}+\Phi_e\right]=\nabla\cdot\left\{\nabla\left[\Gamma^{\Phi}\left(\sum_{n=1}^{N}C_n\psi_n e^{-\gamma_n t}+\Phi_e\right)\right]\right\} \tag{3}$$

Developing the derivative of the Eq. (3) we obtain:

$$\lambda\sum_{n=1}^{N}C_n\psi_n(-\gamma_n)e^{-\gamma_n t}=\sum_{n=1}^{N}C_n e^{-\gamma_n t}\nabla\cdot\nabla(\Gamma^{\Phi}\psi_n) \tag{4}$$

By organizing Eq. (4) it can be obtained:

$$\sum_{n=1}^{N}\left[\nabla\cdot\left[\Gamma^{\Phi}\nabla\psi_n\right]+\lambda\gamma_n\psi_n\right]=0 \tag{5}$$

The specific function Ψ_n is obtained as a linear combination of a set of basis function, f_j, [27] as follows:

$$\psi_n=\sum_{j=1}^{N}d_{nj}f_j \tag{6}$$

where d_{nj} are constants which must be determined.

Substituting Eq. (6) into (5) and applying the Galerkin method which consists in multiplying both members of Eq. (5) by $f_i dV$ and integrate over the volume of the solid we obtain [43]:

$$\sum_{j=1}^{N}d_{nj}\left[\frac{1}{V}\int_V f_i\nabla\cdot\left(\Gamma^{\Phi}\nabla f_j\right)dV+\lambda\gamma_n\frac{1}{V}\int_V f_i f_j dV\right]=0 \tag{7}$$

In matrix form Eq. (7) can be rewrite as follows:

$$(\overline{A}+\gamma_n\overline{B})\overline{d}_n=0 \tag{8}$$

where $\bar{A}$ and $\bar{B}$ are square matrices of N×N elements, whose elements are calculated by the following equations:

$$a_{ij}=\left[\frac{1}{V}\int_V f_i\nabla\cdot\left(\Gamma^{\Phi}\nabla f_j\right)dV\right] \tag{9}$$

$$b_{ij}=\frac{1}{V}\int_V \lambda f_i f_j dV \tag{10}$$

Coefficients $d_{n1}, d_{n2}, \ldots, d_{nN}$ in Eq. (6) are elements of vector $\bar{d}_n$ in Eq. (8). It can be observed also that matrix $\overline{B}$ is symmetrical, so $b_{ij} = b_{ji}$. Matrix $\overline{A}$ is symmetrical as well.

Since the linear equations originating in Eq. (8) are homogeneous, $\gamma_1, \gamma_2, \ldots, \gamma_N$ can be obtained to make the determinant of the matrix $\left(\overline{A} + \gamma\overline{B}\right)$ equal to zero.

Once the eigenvalues, γ_n, have been determined the values of coefficients d_{nj} corresponding to each γ_n can be obtained. Further, since the simultaneous equations resulting from Eq. (8) are homogeneous, one of the coefficients d_{nj} can be selected arbitrarily to be equal to 1 without any loss of generality. Therefore, for a specified d_{nj}, a system of (N−1) equations should be solved to obtain $d_{n2}, d_{n3}, \ldots, d_{nN}$.

The diffusion equation relates time and space variations of the potential Φ; it governs the transfer process within a body. For determine the distribution of Φ inside a porous body at any instant, i.e., to solve the diffusion equation, we necessitate to know the distribution of Φ at the initial instant (initial conditions), the geometry of the body, and the law of interaction between the surrounding medium and the body surface (boundary condition). Thus, we have a well-posed mathematical model.

In this research we used the identity, Eq. (11) to apply the boundary conditions,

$$\int_V f_i \nabla \cdot \left(\Gamma^\Phi \nabla f_j\right) dV = \int_V \nabla \cdot \left(\Gamma^\Phi f_i \nabla f_j\right) dV - \int_V \Gamma^\Phi \nabla f_i \cdot \nabla f_j dV \tag{11}$$

Since Γ^Φ is constant, Eq. (11) can be written as follows:

$$\int_V f_i \nabla \cdot \left(\Gamma^\Phi \nabla f_j\right) dV = \int_S \Gamma^\Phi f_i \nabla f_j \vec{n} \cdot d\vec{S} - \int_V \Gamma^\Phi \nabla f_i \cdot \nabla f_j dV \tag{12}$$

or yet,

$$\int_V f_i \nabla \cdot \left(\Gamma^\Phi \nabla f_j\right) dV = \int_S \Gamma^\Phi f_i \left(\frac{\partial f_j}{\partial n}\right) dS - \int_V \Gamma^\Phi \nabla f_i \cdot \nabla f_j dV \tag{13}$$

For analysis consider the following boundary condition, which is a specification of a linear combination of the values of a function Φ and the values of its derivative on the boundary of the domain:

$$-\Gamma^\Phi \frac{\partial \Phi}{\partial \vec{n}} = h(\Phi - \Phi_e) \tag{14}$$

where h represent the convective transfer coefficient and $\vec{n}$ is the normal vector to surface. If h tends to an infinity value we have a so called Dirichlet condition

(Φ prescribed). When h tend to zero we have a so called Newmann condition (flux prescribed), and for another situation we have a so called Robin condition (convective boundary condition).

Appling Eq. (2) in the (14) it is possible to prove that:

$$-\Gamma^{\Phi}\frac{\partial f_j}{\partial \vec{n}} = hf_j \tag{15}$$

For boundary conditions of the 1st kind (Φ prescribed) $f_j = 0$. For boundary conditions of the 2nd kind (flux prescribed) we have that $\partial f_j/\partial n = 0$ already for the boundary conditions of the 3rd kind (convective boundary condition), we have $-\Gamma^{\Phi}\partial f_j/\partial n = hf_j$.

For obtain the constants C_n of Eq. (2) we use the condition at t = 0, i.e., $\Phi = \Phi_0$. Then, applying this value in Eq. (2) we obtain the following equation:

$$\Phi_0 = \sum_{n=1}^{N} C_n \Psi_n + \Phi_e \tag{16}$$

Multiplying Eq. (16) by f_i dV and integrating over the volume we obtain [43]:

$$\int_v f_i(\Phi_0 - \Phi_e)dV = \int_v f_i \sum_{n=1}^{N} C_n \Psi_n dV \tag{17}$$

The solution of Eq. (17) will be a set of N linear algebraic equations, which allow determine the parameters C_n, completing the solution of the physical-mathematical problem.

Function f_j is called the Galerkin function and it is obtained by the multiplication of function of position φ by an element of a complete set of functions. Function φ is selected satisfying the homogeneous boundary condition. Function f_j with j varying from 1 to N constitutes a set of base functions.

The method to select basis functions for boundary conditions of the first kind, $f_j^{(1)}$, which correspond to the equilibrium boundary condition at the surface of the body, is given in the literature [36, 42].

The basis functions of first kind are determined from the first basis function, which is given by:

$$f_1 = \varphi_1 \varphi_2 \varphi_{3\ldots} \varphi_m \tag{18}$$

where $\varphi_m = 0$ is an equation that describes one of the surfaces of the body, and m is the number of surfaces that define the body to be studied.

Each base function should tend towards zero at the boundary of the solid. Some, but not all, of the basis functions can be zero at some point in the solid.

The basis functions of the second and third kind are defined by [36]:

$$f_j^{(2)} = f_j^{(1)}(\varphi_m H - 1) \tag{19}$$

where H is given by

$$H = \left. \frac{\nabla f_j^{(1)} \cdot \nabla \varphi_m}{f_j^{(1)} \cdot \nabla \varphi_m \cdot \nabla \varphi_m} \right|_{\varphi_m = 0} \tag{20}$$

and

$$f_j^{(3)} = f_j^{(2)} \left(\varphi_m H' - \frac{\Gamma^\Phi}{h} \right) \tag{21}$$

where H' can be determined by:

$$H' = \left. \frac{1}{\frac{\partial \varphi_m}{\partial \vec{n}}} \right|_{\varphi_m = 0} = \left. \frac{1}{\nabla \varphi_m \cdot \nabla \varphi_m} \right|_{\varphi_m = 0} \tag{22}$$

The average value of the potential Φ within the solid can be calculated as follows:

$$\overline{\Phi} = \frac{1}{V} \int_V \Phi dV \tag{23}$$

where V is the volume of the solid.

To describe mass transport within the solid with arbitrary geometry, it is considered in the Eq. (1), $\lambda = 1$, $\Gamma^\Phi = D$ (mass diffusion coefficient), and $\Phi = M$ (moisture content in dry basis), and $h = h_m$ (convective mass transfer coefficient), thus the mass diffusion equation in transient state without mass generation (Fick's law) will be written as follows:

$$\frac{\partial M}{\partial t} = \nabla \cdot (D \nabla M) \tag{24}$$

For heat transfer we consider in Eq. (1) $\Gamma^\Phi = k$ (thermal conductivity), $\lambda = \rho c_p$ (density and specific heat) and $\Phi = \theta$ (temperature), and $h = h_c$ (convective heat transfer coefficient), thus the heat conduction equation in transient state without energy generation (Fourier's law) will be written as follows:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (\alpha \nabla M) \tag{25}$$

where $\alpha = k/(\rho c_p)$ represents the thermal diffusivity.

3 Applications

3.1 Drying of Solids of Revolution

A solid of revolution is a solid figure obtained by rotating a plane curve in space around some straight line (the axis) that lies on the same plane (axis coplanar). It can be defined yet like a three-dimensional figure formed by revolving a plane area about a given axis. However the curve does not intercept the axis. Thus, considers a solid of revolution pictured in Fig. 1.

The contour of the solid of revolution represented in Fig. 1 is defined by:

$$\frac{r^m}{a^m} + \frac{z^2}{b^2} = 1 \tag{26}$$

where m is a number that defines the geometric shape of the solid being studied.

Thus, a set of basis functions of the 1st kind is defined for instance as follows:

$$f_j^{(1)} = \varphi_1 r^{m_j} z^{n_j} \tag{27}$$

where

$$\varphi_1 = 1 - \frac{r^m}{a^m} - \frac{z^2}{b^2} \tag{28}$$

being, j = 0, 1, 2, 3,…, $m_j = 0, 2, 4, 6\ldots$ and $n_j = 0, 2, 4, 6\ldots$

The volume of the solid shown in Fig. 1 can be calculated by:

$$V = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^m/a^m}} rdzdrd\theta \tag{29}$$

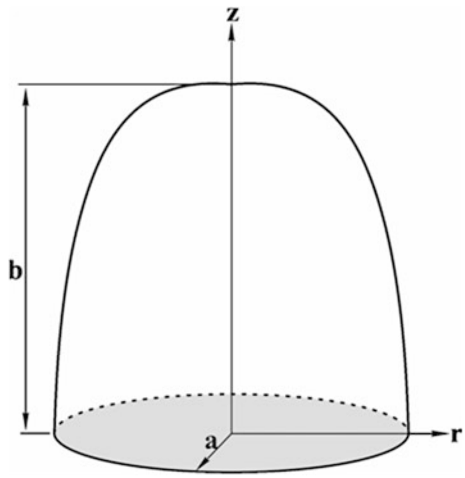
Then, the coefficients presented in Eqs. (9), (10) and (17) can be given as follows:

$$a_{ij} = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^m/a^m}} f_i \nabla \cdot (\Gamma^{\Phi} \nabla f_j) rdzdrd\theta \tag{30}$$

$$b_{ij} = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^m/a^m}} f_i f_j rdzdrd\theta \tag{31}$$

$$\int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^m/a^m}} f_i (\Phi_0 \text{-} \; \Phi_e) rdzdrd\theta = \int_0^{2\pi}\int_0^{a}\int_0^{b\sqrt{1-r^m/a^m}} f_i \sum_{n=1}^{N} C_n \Psi_n rdzdrd\theta \tag{32}$$

Fig. 1 Geometrical representation of a solid of revolution



By varying the parameter m, in Eq. (26), and fixed the aspect ratio (b/a = 2.0), we generated the following solids of revolution presented in Fig. 2. Further, for fixed value of the parameters "m" and "a", for example, a = 1.0 and m = 2.0 and varying the value of the parameter "b" we obtain different ellipsoids of revolution. When the solid in which the axis of revolution is greater than the other axis (b > a), the solid is said to be prolate spheroid (Fig. 2c). For the other hand, if the axis of revolution is smaller than the other (b < a), it is called oblate spheroid. When the axis of revolution is equal to the other axis (b = a), so, the ellipsoid is known like sphere.

Figure 3 shows the dimensionless average moisture content as a function of mass transfer Fourier number (Dt/a^2) for different values of the parameter m and fixed aspect ratio b/a considering equilibrium boundary condition at the surface of the solid. It is clear from Fig. 3 that for the same Fourier number there are different moisture levels depending of the "m" parameter value, so factor m in Eq. (22) defines how the body shape influenced the drying kinetics. By examining this figure, one can observe that for m = 0.5, the dimensionless average moisture content within the solid decreases faster than for m = 4.0. In other words, for the same aspect ratio, the higher the value of m in Eq. (24) the lower the area/volume ratio will be formed. Since the drying velocity (drying rate) is directly proportional to the area/volume relationships, it is noticed that for m = 4.0, the drying process of this solid will be slower when compared to other solids of revolution studied. This observation is consistent with existing works in literature [14, 15, 35, 38, 44]. Higher rates of drying (high concentration gradients) and heating (high temperature gradients) induce elevated thermal, hydric and mechanical stresses within the solid, which could result in the manifestation of cracks or distortions in the product, which may come to compromise its final quality post-drying.

Figures 4 and 5 show the distribution of the dimensionless moisture content inside the solids of revolution with aspect ratio b/a = 2.0, in two dimensionless

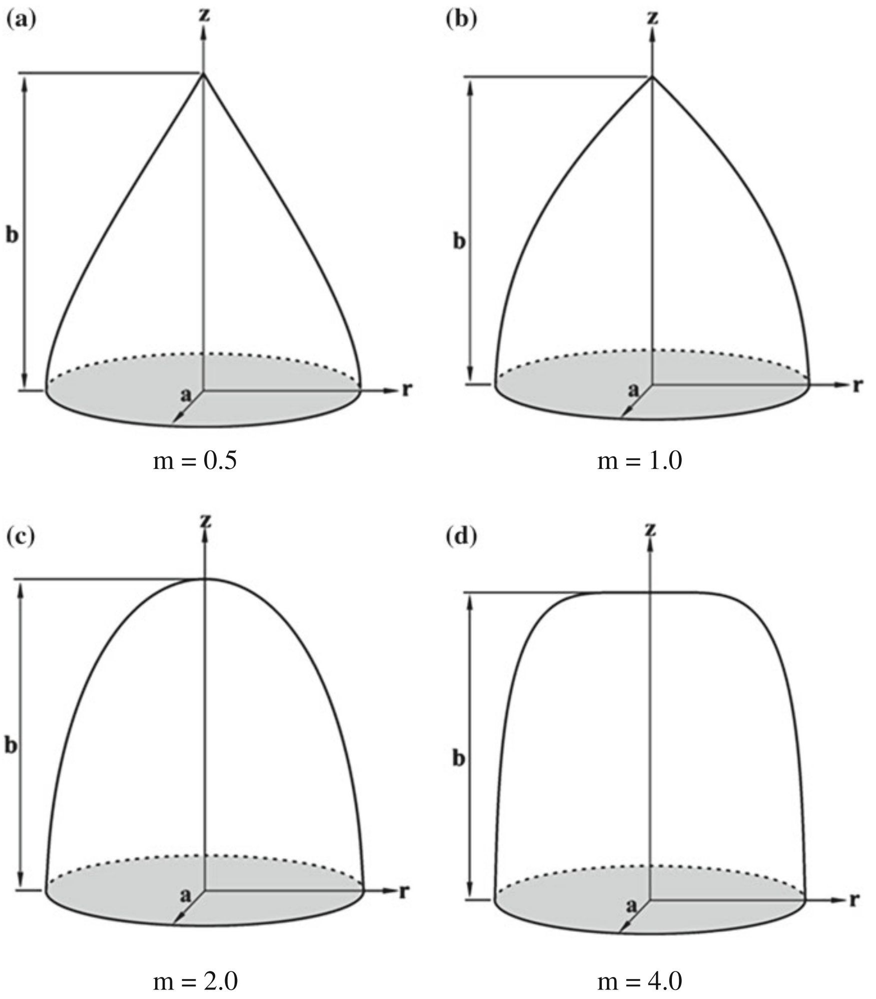


Fig. 2 Solids of revolution with different shapes, **a** m = 0.5, **b** m = 1.0, **c** m = 2.0, **d** m = 4.0

time (Fourier number, Fo = Dt/a^2): 0.0708617 and 0.141723. From these figures, one can verify the existence of concentration gradients during the drying process and that they are higher in early times for each of the solids. However, it is observed that these moisture gradients are higher in solids that have a lower "m" value, which represent the shape factor of the body, being consistent with Fig. 3. Since higher moisture gradients will result in large movement of fluid within the solid, it dries more quickly.

It is also worth pointing out that the region near the point z = 2.0 upholds the lowest moisture concentrations for all times in anyone of the solids. This phenomenon is related with the highest concentration gradients that has occurred at

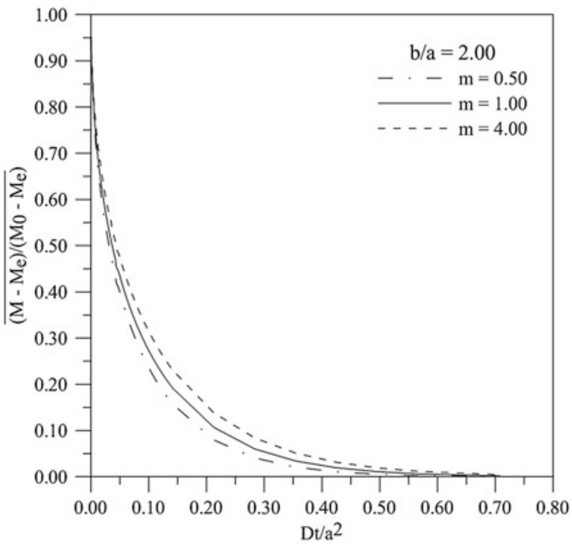


Fig. 3 Dimensionless average moisture content within three solids of revolution with the same aspect ratio as a function of Fourier number

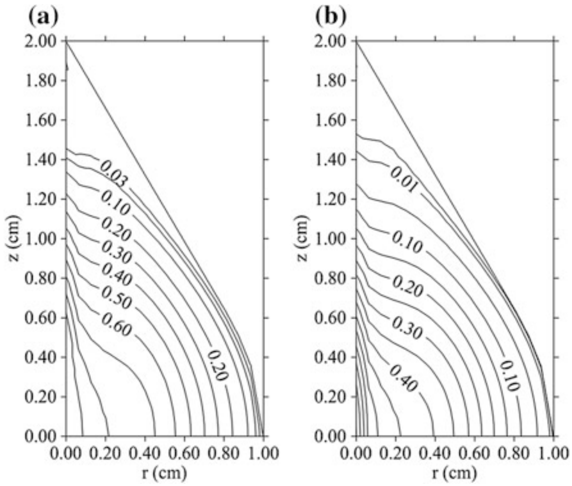


Fig. 4 Dimensionless moisture content distribution $[(M - M_e)/(M_0 - M_e)]$ within a solid of revolution formed with m = 0.5 and b/a = 2.0 for dimensionless times: **a** Fo = 0.0708617 and **b** Fo = 0.141723

the edge of the solid close to z = 2.0, being lower when the solid tend to a sphere (z = 1.0). Therefore, if the drying rate is such that it will cause a fracture or a deformation in the solid, it is very likely that such effects occur in the edge region. For this reason, several authors mention this edge effect [14, 15, 23, 24, 35, 44]. Further, one can observe the presence of iso-concentration lines, which have the same form of the solid. The moisture concentrations are higher in the center of the solid thus showing that the movement of liquid occurs from the interior to the surface of the solid of revolution, as expected.

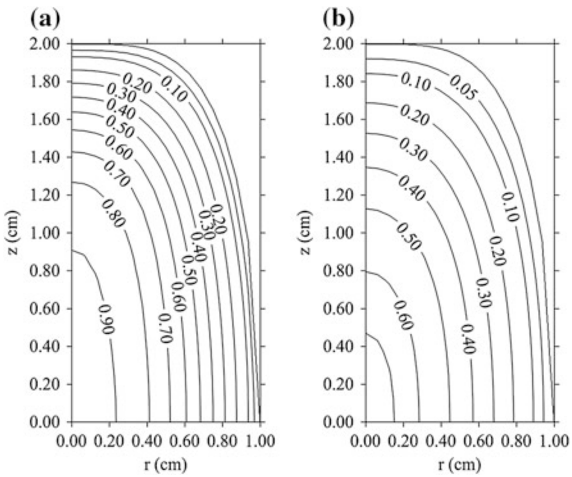


Fig. 5 Dimensionless moisture content distribution $[(M - M_e)/(M_0 - M_e)]$ within a solid of revolution formed with m = 4.0 and b/a = 2.0 for dimensionless times: **a** Fo = 0.0708617 and **b** Fo = 0.141723

3.2 Drying of Wheat Grain

For the theoretical study of drying considered in this topic we use a prolate spheroid (ellipsoid of revolution formed when m = 2.0) whose geometry is resembled to wheat grain, as illustrated in Fig. 6.

To determine the mass diffusion coefficient it was used the equation proposed by [45], as follows:

$$D = 0.543\bar{M}^{\left(-2.8554\times10^{-5}T+1.6432\right)}\mathrm{Exp}[(0.4113T - 30.2634)\bar{M} + (0.022776T - 9.7271)] \tag{33}$$

From the Eq. (33), the mass diffusion coefficient was obtained using the average value of moisture content as follows:

$$\overline{D} = \frac{1}{(M_0 - M_e)}\int_{M_e}^{M_0} D(\overline{M})d\overline{M} \tag{34}$$

For determination of the convective mass transfer coefficient was used the following relations reported by [45]:

$$h_m = \frac{D_{atm}}{RT_{abs}d_p}\left[2.0 + 0.6Re^{1/2}Sc^{1/3}\right] \tag{35}$$

where D_{atm} represents the molecular diffusion coefficient of water in air, Sc and Re represent the Reynolds and Schmidt numbers, R is the Universal gas constant (as applied to water vapour) and dp is the equivalent diameter.

The convective heat transfer coefficient was calculated as follows [45]:

Fig. 6 Wheat grain kernel

$$h_c = \frac{k_a}{d_p}\left[2.0 + 0.6Re^{1/2}Pr^{1/3}\right] \quad (36)$$

where Pr represents the Prandtl number.

The Reynolds, Schmidt and Prandtl numbers are given by:

$$Re = \frac{\rho v d_p}{\mu} \quad (37)$$

$$Sc = \frac{\mu}{\rho D_{atm}} \quad (38)$$

$$Pr = \frac{C_p \mu}{k} \quad (39)$$

In the Eqs. (37)–(39), ρ, v, μ, c_p and k represent density, velocity, viscosity, specific heat and thermal conductivity of the air, respectively. Equivalent diameter (d_p) was obtained considering the wheat grain as a prolate spheroid. Initially, the volume of the grain was calculated using Eq. (29). Following, this volume was assumed to be equal to the volume of a sphere of diameter d_p. Simulations were performed using Eqs. (24) and (25). Table 1 presents the data used in the simulations.

Figures 7 and 8 illustrate, respectively, the results of the dimensionless average moisture content and dimensionless center temperature obtained in this study compared with experimental drying data to wheat [45] considering convective boundary condition at the surface of the solid. Based on the approach taken for this research and observing the drying and heating kinetics, it becomes apparent that a good agreement was obtained. We notice that, the higher the mass diffusion coefficient, the lower the drying time, i.e., the drying process becomes faster, fixed the convective mass transfer coefficient. For a fixed mass diffusion coefficient

Table 1 Physical and geometrical parameters used in the simulations [14, 45, 46]

Drying air					Wheat grain			
T (°C)	RH (%)	v (m/s)	a (cm)	b (cm)	M_0 (d.b.)	M_e (d.b.)	θ_e (°C)	θ_0 (°C)
67.5	13.3	1.61	0.1575	0.3276	0.2560	0.0362	67.5	26.0
			k (W/mK)	ρ (kg/m^3)	c_p (J/kgK)	h_c (W/m^{2}K)	$h_m \times 10^{+6}$ (m/s)	$D \times 10^{+11}$ (m^2/s)
			0.1400	790.00	2223.76	82.942	1.03823	5.73003

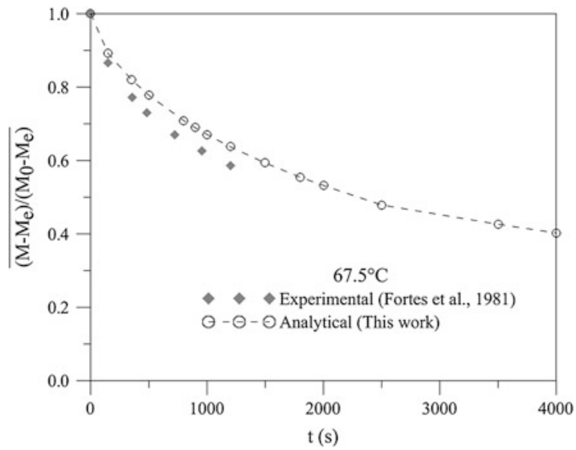


Fig. 7 Dimensionless average moisture content of the wheat grain as a function of time

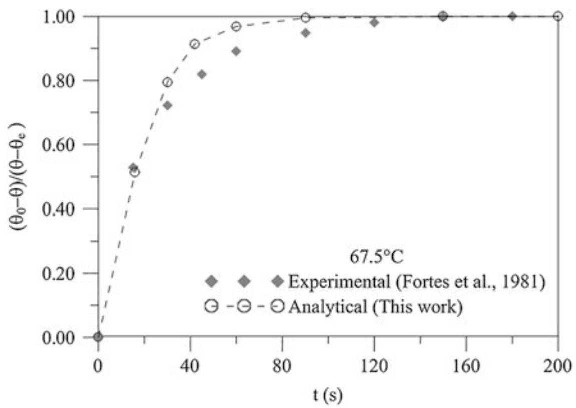


Fig. 8 Dimensionless temperature at the center of the wheat grain as a function of time

increasing the convective mass transfer coefficient we have increased drying rate, thus the solid dry faster.

Figures 9 and 10 illustrate the moisture content and temperature distributions inside the grain at two different elapsed times (1800 and 4000 s). Based on the analysis of this figure, we note the presence of concentration iso-lines, which has the same shape as prolate spheroid. It is also evident that the movement of liquid

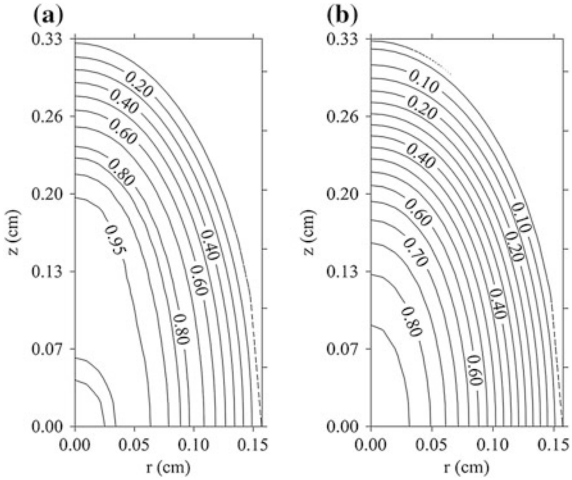


Fig. 9 Distribution of dimensionless moisture content during the wheat grain drying: **a** t = 1800 s and **b** t = 4000 s (T = 67.5 °C and RH = 13.3 %)

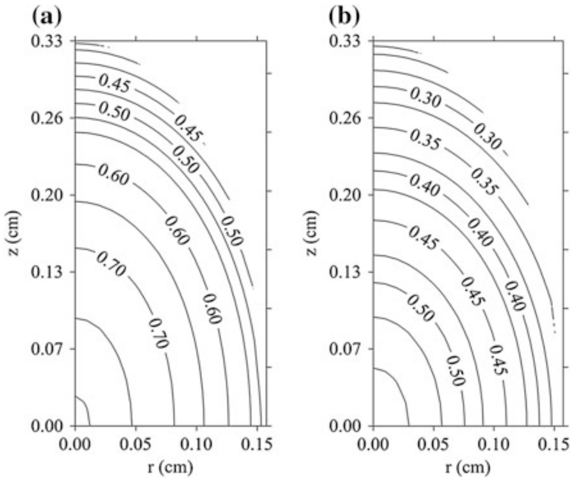


Fig. 10 Distribution of dimensionless temperature during the wheat grain drying: **a** t = 8 s and **b** t = 14 s (T = 67.5 °C and RH = 13.3 %)

takes place from the center of the wheat grain to the surface thereof meanwhile the heat flux occurred from the surface to center of the solid. The moisture content and temperature inside the wheat grain checked at all times occurred in a non-homogeneous form. Specifically for the instant 4000 s, it can be noted that the moisture content of the grain is close to reach the commercial moisture content, so, at this instant the drying is almost complete. The longer the drying time, the smaller the presence of moisture gradient inside the solid and thus it has a slower drying rate, tending to zero at the final of the drying, when the grain reach the equilibrium moisture content. Further, the time to grain reach the air temperature is very short, less than the 200 s, so, moisture migration has occurred at isothermal conditions after this time.

4 Concluding Remarks

Upon completion of this chapter and based on the drying data analysis, the following conclusions can be cited:

(a) The mathematical model and GBI technique used for obtaining the analytical solution was appropriate to describe drying problem. Thus it is also suitable for predicting transient problems such as cooling, heating or wetting. From the model solution, is possible to obtain the moisture content and temperature distributions within the solid as well as analyze the drying and heating kinetics.
(b) The body shape has a direct influence on the drying and heating kinetics, since this parameter is directly related to the area/volume of the body. For higher area/volume relationships the solid dry faster.
(c) During drying there was a difference in moisture content and temperature within the porous solid, so that the flux of moisture occurs from the center of the solid to the surface thereof, meanwhile the heat flux occurs from surface to center of the solid.
(d) It was found that the higher moisture and temperature gradients occur on the edge of the ellipsoid of revolution. This fact shows that this region is more affected by thermo-hydro-mechanical stresses, thus, being more susceptible to the appearance of defects such as cracks, fracture, warping and deformations.
(e) The diffusion coefficient affects the mass transfer process, reporting the velocity at which this process occurs. The greater the mass diffusivity the faster drying is found.

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