# Preface

This book contains selected peer-reviewed contributions submitted by the plenary speakers to the workshop "Reduced Basis, POD and Reduced Order Methods for model and computational reduction: towards real-time computing and visualization?" funded by CECAM (European Center for Atomistic and Molecular Computing) hosted at Ecole Polytechnique Fédérale de Lausanne, Switzerland on 14–16 May 2012 (More info: http://www.cecam.org/workshop-681.html).

This book addresses a wide range of model reduction strategies with applications in various fields.

The increasing complexity of mathematical models used to predict real-world systems, such as climate or the human cardiovascular system, calls for the development of model reduction strategies, that is computationally cheaper algorithms that however still accurately capture the most important features of the phenomena being modelled. Model reduction strategies can be classified according to two main approaches: "reduce-then-model" and "discretize-then-reduce". In the former approach the continuous equations representing the underlying physics are first reduced, e.g. by symmetry assumptions that allow us to consider 1D or 2D equations instead of the full 3D equations, before a computational model is derived. In the latter approach a computational model is obtained by discretizing the continuous equations and only then a reduced model is sought. Some subtopics include spatial dimensionality reduction and multiscale modelling frameworks in the "reduce-then-model" category; state space and parameter space reduction – with a special accent on reduced basis and proper orthogonal decomposition – in the "discretize-then-reduce" category. This monograph focuses more on this second aspect.

It can be regarded as a state of the art survey and integration of several contributions on model order reduction developed in the last few years in different fields and with different purposes, in order to:

1. facilitate a stronger interaction between scientists doing model order reduction on ordinary and partial differential equations (both theory and applications);
2. enhance the state of the art in model reduction making it possible to perform real-time computing for complex systems;

3. address the reliability of reduced order models when compared to more classical high fidelity discretization techniques (and discuss the trade-off between accuracy and costs);
4. improve and generalize parametrization techniques from both a physical and a geometrical point of view, in order to better deal with realistic parametrized geometries and complex parametrized systems;
5. propose and analyze novel sampling and parameter space exploration techniques;
6. certify reduced order modelling for time-dependent problems by simulating long-time phenomena;
7. explore several possible combinations of reduction strategies (like POD, RB, PGD).

The monograph emphasizes model reduction topics in several areas:

1. design, optimization, and control theory in real-time with applications in engineering;
2. data assimilation, geometry registration, and parameter estimation with a special attention to real-time computing in biomedical engineering and computational physics;
3. the treatment of high-dimensional problems in state space, physical space or parameter space;
4. the interactions between different model reduction and dimensionality reduction approaches;
5. the development of general error estimation frameworks which accommodate both model and discretization effects.

The book deals with mathematical models based on both ordinary and partial differential equations with emphasis on engineering and life-sciences applications, including continuum mechanics, fluid dynamics, and transport problems with a methodological focus.

We anticipate a wide range of both academic and industrial problems of high complexity to motivate, stimulate, and ultimately demonstrate the meaningfulness and efficiency of the selected approaches.

The proposed topics open new perspectives in the development of efficient methodologies related with new frontiers in computational science and engineering in order to assist scientists and engineers during design, construction, manufacturing or production phases, and even medical doctors during surgery or diagnosis.

The methodologies we consider are motivated by, optimized for, and applied within two particular contexts: real-time (e.g., parameter estimation or control) and many query (e.g., design, optimization or multimodel/scale simulation). Both contexts are crucial to computational science and engineering and to more widespread adoption and application of numerical methods for partial and/or ordinary differential equations in engineering practice and education.

The real-time context can be found in engineering situations dealing with in-the-field robust parameter estimation (or inverse problems, or nondestructive evaluation), design and optimization, and control. On the other side the many-query context

involves multiscale (temporal or spatial) or multiphysics models in which behavior at a larger scale must "invoke" many spatial or temporal realizations of parametrized behavior at a smaller scale.

Both the real-time and many-query contexts present a significant and often unsurmountable challenge to "classical" numerical techniques such as the finite element method (or finite difference, finite volume, spectral methods). These contexts are often much better served by the reduced order modelling techniques (even associated with a posteriori error estimation techniques).

The development of reduced order modelling can perhaps be viewed as a response to the considerations and imperatives described above. In particular, the parametric real-time and many query contexts represent not only computational challenges, but also computational opportunities.

The state of the art is currently moving towards interaction between different reduced order modelling techniques. For example, reduced basis method and proper orthogonal decomposition are combined to solve time-dependent parametrized diffusion-reaction problems with certification of accuracy for the reduced model provided by a posteriori error bounds.

Theoretical studies are being carried out to ensure a better understanding of model order reduction and the reliability and the applicability of the methodologies proposed. Parametrization of systems is advancing by proposing new techniques to deal with more complex configurations and more parameters. Techniques to improve the exploration of parameter space (sampling procedures, greedy algorithms) have been refined, combined, and specialized.

Advances made in computer graphics and physics-based simulation communities can be adapted to produce new methodologies satisfying the real-time needs of applications.

The book is organized as it follows. Chapter 1 deals with model order reduction techniques for coupled multiphysics problems, then Chap. 2 introduces a case study to compare reduced basis method in a time domain and the Loewner rational interpolation in a frequency domain. Chapter 3 focuses on the comparison with some reduced representation approximations by showing different features, in Chap. 4 the emphasis is on reduction techniques for nonlinear parametrized problems. Then Chap. 5 deals with efficient sampling techniques using nonlinear optimization; Chapter 6 introduces parametrized model order reduction by implicit moment matching. In Chap. 7 the focus is on reduced basis method for *parareal* time integration. Stability of reduced order linearized models in computational fluid dynamics is discussed in Chap. 8, followed by Chap. 9 with some more challenges and perspectives for model order reduction in fluid dynamics; window proper orthogonal decomposition and applications is the content of Chap. 10, followed by Chap. 11 with applications of reduced order modeling in aeronautics and medicine.

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Lausanne, Milano and Trieste                                                       Alfio Quarteroni
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