Chapter 2
Flow Phenomena

The purpose of this chapter is to present some examples of flows in which there are significant differences between the behavior of Newtonian fluids and non-Newtonian fluids. In the figures to follow the Newtonian fluid is indicated with an “N” and the non-Newtonian fluid is marked with “nN”. These examples and some others are discussed in greater details in the book “Dynamics of Polymeric Liquids”, vol 1. Fluid Mechanics, by Bird, Armstrong and Hassager [3].

2.1 The Effect of Shear Thinning in Tube Flow

Figure 2.1 shows two vertical tubes, one filled with a Newtonian fluid (N) of viscosity $\eta$, and the other filled with a shear-thinning fluid (nN) with a viscosity function $\eta(\dot{\gamma})$. The tubes are open at the top but closed with a plate at the bottom. The two fluids are chosen to have the same density and such that they have approximately the same viscosity at low shear rates: $\eta(\dot{\gamma}) \approx \mu$ for small $\dot{\gamma}$. For example, the situation may be realized by using a glycerin-water solution as the Newtonian fluid and then adjust the viscosity by changing the glycerin content until two small identical spherical balls fall with the same velocity through the tubes, Fig. 2.1a.

Figure 2.1b indicates what happens after the plate has been removed. The tubes are emptied, but the shear-thinning fluid accelerates to higher velocities than the Newtonian fluid. At the relatively high shear rates $\dot{\gamma}$ that develop near the tube wall, the apparent viscosity $\eta(\dot{\gamma})$ is smaller than the constant viscosity $\mu$ of the Newtonian fluid, i.e., $\eta(\dot{\gamma}) < \mu$. The shear stress from the wall that counteracts the driving force of gravity is therefore smaller in the shear thinning fluid, leading to higher accelerations. The shear-thinning fluid leaves the tube faster than the Newtonian fluid.
2.2 Rod Climbing

Figure 2.2 illustrates two containers with fluids and with a vertical rod rotating at a constant angular velocity. The container in Fig. 2.2a is filled with a Newtonian fluid (N). The fluid sticks to the container wall and to the surface of the rod, and the fluid particles obtain a circular motion about the rod. Due to centrifugal effects the free surface of the fluid shows a depression near the rod. The container in Fig. 2.2b is filled with a non-Newtonian viscoelastic fluid (nN). This fluid will start to climb the rod until an equilibrium condition has been established. The phenomenon is explained as a consequence of tensile stresses in the circumferential direction that develop due to the shear strains in the fluid. The tensile stresses counteract the centrifugal forces and squeeze the fluid towards the rod and up the rod. Long, thread-like molecular structures are stretched in the directions of the circular stream lines and thus create the tensile stresses. The phenomenon may be observed in a food processor when mixing waffle dough.
2.3 Axial Annular Flow

We investigate the axial laminar flow of fluid in the *annular space* between two concentric circular cylindrical surfaces, Fig. 2.3. The pressure is measured at a point A at the inner surface and at a point B at the outer surface in the same cross section of the container. Measurements then show that the two pressures are the same when the fluid in Newtonian, while for a non-Newtonian fluid a small pressure difference is observed. The general result of this experiment is:

\[ p_A = p_B \text{ for Newtonian fluids, } \quad p_A > p_B \text{ for non-Newtonian fluids} \quad (2.1) \]

The measured pressure in this experiment is the difference between the *thermodynamic pressure* \( p \) in a compressible fluid, or any undetermined isotropic pressure \( p \) in an incompressible fluid, and the *viscous normal stress* \( \tau_{RR} \) in the radial direction. In Chap. 3 the difference between the pressure \( p \) and the pressure \( p/C_0 \tau_{RR} \) will be discussed in detail.

2.4 Extrudate Swell

A highly viscous fluid flows under pressure from a large reservoir and is extruded through a tube of diameter \( d \) and length \( L \) (Fig. 2.4). The extruded fluid exiting the tube swells and obtains a diameter \( d_e \) that is larger than the inner diameter \( d \) of the tube. A 200% increase in diameter is reported in tests. The ratio \( d_e/d \) is decreasing with increasing length \( L \) of the tube. A comparable Newtonian fluid, with viscosity \( \mu \) and density \( \rho \), will under similar conditions not exhibit any immediate change in diameter, i.e.: \( d_e/d = 1 \). For high *Reynolds numbers*, \( \text{Re} = \rho v d/\mu \), where \( v \) is the mean velocity in the tube, it may be shown that \( d_e \) is somewhat less than \( d \). This latter effect is of course due to gravity.
The swelling phenomenon may be explained based on two effects:

1. The non-Newtonian fluid is compressed elastically in the radial direction upon entering the relative narrow tube. In the tube the fluid is responding by expanding in the axial direction, while after leaving the tube the fluid is restituting by expanding radially. The fluid has a kind of memory of the deformation history it has experienced in passing into the tube, but this memory is fading with time. The longer back in time a deformation was introduced, the less of it is remembered. The fluid is said to possess a fading memory. The longer the tube is, the lesser will the restitution effect have for the swelling phenomenon.

2. The shear strains introduced during the tube flow introduce elastic tensile stresses in the axial direction. We may imagine that these tensile stresses are due to long molecular structures in the fluid that are stretched elastically in the direction of the flow. Upon leaving the tube the fluid seeks to restitute itself in the axial direction. Due to the near incompressibility of the fluid, it will then swell in the radial direction.

### 2.5 Secondary Flow in a Plate/Cylinder System

Figure 2.5 illustrates a circular plate rotating on the surface of a fluid in a cylindrical container. The motion of the plate introduces a primary flow in the fluid in which the fluid particles move in circular paths. The particles closer to the plate move faster than the particles nearer the bottom of the cylinder. The effect of centrifugal forces therefore increases with the distance from the bottom. In a Newtonian fluid this effect introduces a secondary flow normal to the primary flow, as shown in Fig. 2.5a.
In a non-Newtonian fluid the secondary flow may be opposite of that in the Newtonian fluid, as shown in Fig. 2.5b. This phenomenon is a consequence of the tangential tensile stresses introduced by the primary flow, and related to the rod climbing phenomenon. The tensile stresses increase with the distance from the bottom of the container and counteract the centrifugal forces.

### 2.6 Restitution

Figure 2.6 shows a tube with a *visco-elastic fluid*. In Fig. 2.6a the fluid is at rest, the pressures at the ends of the tube are the same: \( p_B = p_A \). Using a colored fluid (black) a material diametrical line is marked in the fluid. The pressure \( p_A \) is increased and flow starts. The black material line deforms as shown in Fig. 2.6b. The pressure \( p_A \) is then reduced to \( p_B \). The flow is retarded, the fluid comes to rest, and then starts to move for a short while in the opposite direction. The black material line is seen to retract as the fluid is somewhat restituted, Fig. 2.6c.

The same phenomenon may be observed when a fluid is set in rotation in a container at rest. The fluid sticks to the container wall and bottom, and the flow of the fluid is slowed down. Eventually the fluid comes to rest and then starts to rotate slightly in the opposite direction. In this case the fluid motion and the restitution may be observed by introducing air bubbles into the fluid and study their motion. The bubbles will move in circles, stop, and then start to move in the reverse direction.

### 2.7 Tubeless Siphon

Figure 2.7a illustrates a vessel with fluid and a tube bent into a siphon. If the fluid is Newtonian the flow through the tube will stop as soon has the siphon has been lifted up such that the end of the tube stuck into fluid in the container has left the

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**Fig. 2.6** Restitution in a viscoelastic fluid

![Diagram](image-url)
surface of the fluid. A highly viscoelastic fluid, however, will continue to flow even after the tube end has left the fluid surface. It is also possible to empty the container without the siphon if the container is tilted to let the fluid start to flow over the edge. The elasticity of the fluid will then continue to lift the fluid up to the edge and over it. This is illustrated in Fig. 2.7b. Another way of starting the flow is to use a finger to draw the fluid up and over the edge.

2.8 Flow Through a Contraction

A low Reynolds number flow of a Newtonian fluid through a tube contraction, as illustrated in Fig. 2.8a, will have stream lines that all go from the region with the larger diameter to the region with smaller diameter. A non-Newtonian fluid may have stream lines as shown in Fig. 2.8b. Large eddies are formed and instabilities may occur, with the result that the main flow starts to oscillate back and forth across the axis of the tube.

2.9 Reduction of Drag in Turbulent Flow

Small amounts of polymer resolved in a Newtonian fluid in turbulent flow may reduce the shear stress at solid boundary surfaces dramatically. Figure 2.9 shows results from tests with pipe flow of water. The parameter $f$ is called the *Fanning friction number* and is defined by:

$$f = \frac{1D}{4L} \frac{\Delta p}{\rho v^2/2}$$

$D =$ pipe diameter, $\Delta p =$ the pressure difference over a pipe length $L$, and $v =$ the mean velocity in the pipe. The amounts of polymer, given in parts per million [ppm] by weight, are added to the water. The curves show that the drag reduction occurs in the turbulent regime. For the *Reynolds number* $Re = \frac{\rho vD}{\mu} = 10^5$, where $\rho$ is the density and $\mu$ is the viscosity of water, and a polymer concentration of 5 ppm, the Fanning number $f$ is reduced by 40 %. The viscosity in

![Fig. 2.7 Tubeless siphon.](image)

Non-Newtonian fluid
the fluid mixture is changed only slightly. For the present example the viscosity \( \mu \) is only increased by 1 % relative to that of water. The reason why very small amounts of polymer additives to a Newtonian fluid like water have such a large effect on drag, is not completely understood. What is known is that the effects of different types of polymers are very different. Polymers having long unbranched molecules and low molecular weight give the greatest drag reduction.

The applications of drag reduction using polymer additives are many. One example is in long distance transport of oil in pipes.

Figure 2.9 is adapted from Fig. 3.11-1 in Bird et al. [3]. The curves are based on original data from P.S.Virk, Sc.D. Thesis. Massachusetts Institute of Technology, 1961.
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