Preface

Almost Automorphic Type and Almost Periodic Type Functions in Abstract Spaces presents a comprehensive and reader-friendly introduction to the concepts of almost periodicity, asymptotic almost periodicity, almost automorphy, asymptotic almost automorphy, pseudo-almost periodicity, and pseudo-almost automorphy as well as their recent generalizations. Further, it presents a wide range of sufficient conditions for the boundedness, existence, uniqueness, and stability of solutions to various classes of first-, second-, third-, and higher-order abstract differential equations, difference equations, and integrodifferential equations whose coefficients belong to the above-mentioned classes of functions. It also offers various applications to partial differential equations including the beam boundary-value problem, the heat boundary-value problem, and some partial neutral integrodifferential equations with delay arising in control systems.

Some of the results presented are either new or else cannot be easily found in the mathematical literature. In order to establish the main results of the book, one makes extensive use of a myriad of tools ranging from functional analysis to operator theory.

Noteworthy progress has been made during the past 15 years in studying almost periodic, almost automorphic, and pseudo-almost periodic functions as well as their applications to abstract differential equations. In recent years, various generalizations of the above classes of functions have been introduced in the mathematical literature including, but are not limited to, the concepts of Stepanov-like almost automorphy, weighted pseudo-almost periodicity, pseudo-almost automorphy, weighted pseudo-almost automorphy, Stepanov-like pseudo-almost periodicity, Stepanov-like pseudo-almost automorphy, and many more. Despite the noticeable and rapid progress made on these important topics, the only standard references that currently exist on those new classes of functions and their applications are still scattered research articles. One of the main objectives of this book consists of closing that gap. Namely, the book collects and presents in a unified manner most of the recent material introduced in the literature upon the above-mentioned classes of functions as well as their applications to differential equations, difference equations, integrodifferential equations, and partial differential equations.

The book is divided into 12 main chapters and an appendix. It also offers some useful bibliographical notes at the end of each chapter.

The first chapter collects the basic background on metric, Banach, and Hilbert spaces. Proofs of some of the most important results presented will be given including that of the Banach fixed-point theorem.

Chapter 2 collects basic material in functional analysis and operator theory needed in the sequel including compact operators, self-adjoint operators, operators with compact resolvents, sectorial operators, analytic semigroups, abstract interpolation spaces, and various preliminary results on evolution families under Acquistapace–Terreni conditions and exponential dichotomy.

Chapter 3 gathers basic properties on almost periodic, Stepanov-like almost periodic, and asymptotically almost periodic functions, and then introduces and studies differentiable almost periodic and Stepanov-like almost periodic functions. Chapter 4 collects basic results on almost automorphic and asymptotically almost automorphic functions, and then introduces and studies Stepanov-like almost automorphic, differentiable almost automorphic and Stepanov-like almost automorphic functions. The main objective of Chap. 5 consists of reviewing basic properties of pseudo-almost periodic functions including some composition theorems as well as some of their recent extensions such as weighted pseudo-almost periodic, differentiable pseudo-almost periodic and Stepanov-like pseudo-almost periodic functions. Chapter 6 reviews basic properties of pseudo-almost automorphic functions as well as some of their recent extensions such as weighted pseudo-almost automorphic, differentiable pseudo-almost automorphic, and Stepanov-like pseudo-almost automorphic functions.

Chapter 7 studies the existence of almost periodic (respectively, almost automorphic) solutions to the classes of nonautonomous damped second-order differential equations

$$\frac{d^2u}{dt^2} + b(t)B\frac{du}{dt} + a(t)Au = h(t,u),$$

where A is a self-adjoint linear operator with compact resolvent, B is a closed linear operator satisfying some additional conditions, and the functions a,b,h are almost periodic (respectively, almost automorphic).

Chapter 8 studies the existence of asymptotically almost automorphic mild solutions to the abstract partial neutral integrodifferential equations

$$\begin{aligned} \frac{d}{dt}D(t,u_t) &= AD(t,u_t) + \int_0^t B(t-s)D(s,u_s)ds + g(t,u_t),\ t \in [\sigma, \sigma + a), \\ u_\sigma &= \varphi \in \mathscr{B}, \end{aligned}$$

where $A, B(t)$ are densely defined closed linear operators with a common domain $D(A)$, which is independent of t, the history u_t, is the function defined by $u_t(\theta) := u(t+\theta)$, which belongs to an abstract phase space $\mathscr{B}$ defined axiomatically, f, g are functions subject to some additional conditions, and

$$D(t,\varphi) := \varphi(0) + f(t,\varphi).$$

Chapter 9 studies the existence of differentiable pseudo-almost automorphic solutions to some general higher-order differential equations involving operator coefficients given by

$$\frac{d^n u}{dt^n} + \sum_{k=1}^{n-1} A_k \frac{d^k u}{dt^k} + Au = f,$$

where A and A_k ($k = 1,2,\ldots,n-1$) are densely defined closed linear operators on a Banach space $\mathscr{X}$, and $f : \mathbb{R} \mapsto D_A(\alpha+m,p)$ with $D_A(\alpha+m,p)$ being a real interpolation space. Our study concerns only the special case $A_k = A^{\alpha_k}$ (if it exists) with the exponents $0 \le \alpha_k < 1$ for $k = 1,2,\ldots,n-1$. A nonautonomous version of the above higher-order differential equation will also be considered.

In Chap. 10, using the Schauder and Banach fixed-point theorems, and exponential dichotomy techniques, we study and obtain, in various contexts, the existence of pseudo-almost periodic (respectively, weighted pseudo-almost periodic) solutions to the nonautonomous third-order differential equations

$$\frac{d}{dt}\left[\frac{d^2 u}{dt^2} + g(t,Bu)\right] = w(t)Au + f(t,Cu), \quad t \in \mathbb{R}$$

where A is a self-adjoint linear operator, B and C are linear operators such that their algebraic sum $B+C$ is nontrivial, the function w given by $w(t) = -\rho(t)$ is assumed to be almost periodic, there exist two constants $\rho_0, \rho_1 > 0$ satisfying the following conditions $\rho_0 \le \rho(t) \le \rho_1$, and the functions f,g are pseudo-almost periodic (respectively, weighted pseudo-almost periodic) in the first variable uniformly in the second one.

Chapter 11 studies the existence of pseudo-almost automorphic solutions to the class of Sobolev-type differential equations given by

$$\frac{d}{dt}[u + f(t,u)] = A(t)u + g(t,u),$$

where $A(t)$ is a family of densely defined closed linear operators on a domain D, independent of t, and f,g are Stepanov-like pseudo-almost automorphic functions.

In Chap. 12, one studies and obtains the existence of a globally asymptotically stable almost periodic (respectively, globally asymptotically stable almost automorphic) solutions to the higher-order difference equations

$$x(t+n) + \sum_{m=1}^{n-1} A_m(t)x(t+m) + A_0(t)x(t) = f(t,x(t)),$$

where $A_m(t)$ for $m = 0, 1, \ldots, n-1$ are sequences of bounded linear operators on a Banach space $\mathscr{X}$, and the forcing term f is almost periodic (respectively, almost automorphic) in the first variable uniformly in the second one, and satisfies some additional conditions.

The prerequisite for the book is the basic introductory course in real analysis. It is suitable for beginning graduate and/or advanced undergraduate students. Moreover, it may be of interest to researchers in mathematics as well as in engineering, physics, and related areas. Further, some parts of the book may be used for various graduate and advanced undergraduate courses. In particular, it contains lots of examples which might be used in graduate and advanced undergraduate lectures. Chapter 1 provides some basic material for a course in functional analysis. Chapter 2 provides the reader with some needed background for a course in operator theory. Chapters 3–6 may be used for the study of periodic and almost periodic functions and their extensions. Chapters 7–11 may be used for a course on abstract differential equations and their applications to partial differential equations.

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