Chapter 2
Theoretical Foundations

In this chapter the theoretical foundations of the Standard Model of electroweak interactions, an overview of flavor physics, the theory of the time evolution of neutral mesons and CP violating phenomena are summarised.

2.1 CP Violation in the Standard Model of Particle Physics

In the Standard Model of particle physics, the theories of electromagnetic and weak interactions are unified by the Glashow-Weinberg-Salam (GWS) model [1, 2]. The GWS model describes electroweak interactions by a gauge theory based on the spontaneously broken symmetry groups

\[
\text{SU}(2)_L \times \text{U}(1)_Y \xrightarrow{\text{SSB}} \text{U}(1)_{\text{em}}.
\]

The electroweak symmetry breaking is caused by the Higgs mechanism, which generates the \(W^\pm\) and \(Z^0\) gauge boson masses and the masses of the fundamental fermions [3–5].

In the following, the basic principles of the Standard Model of electroweak interactions and the spontaneous symmetry breaking by the Higgs mechanism are briefly described. In addition, it will be demonstrated, how in the Yukawa interactions the Higgs field introduces complex couplings to the quark fields, and how in the charged current interactions of quarks these couplings give rise to CP violation. The discussion neglects details of the leptonic sector and focuses on the electroweak interactions of quarks.

The fundamental fermions are organised into three families of quarks and leptons:

\[
\begin{pmatrix}
  u \\
  c \\
  t
\end{pmatrix}
\quad \begin{pmatrix}
  v_e \\
  v_\mu \\
  v_\tau
\end{pmatrix}
\quad \begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\quad \begin{pmatrix}
  e^- \\
  \mu^- \\
  \tau^-
\end{pmatrix}
\]

(2.2)
The families have identical gauge interactions, but differ in their flavor quantum numbers and masses. The quarks and leptons form doublets of left-handed and singlets of right-handed fields, denoted in the following by

\[
\begin{pmatrix}
u_j e_j \\
d_j e_j
\end{pmatrix} \rightarrow \begin{pmatrix}
u_{Lj} \\
d_{Lj}
\end{pmatrix}, \quad \begin{pmatrix}
u_{Lj} \\
l_{Lj}
\end{pmatrix}
\]

and \(u_{Rj}, d_{Rj}, l_{Rj}\) for \(j \in \{1, 2, 3\}\),

where the quark fields are given in the weak eigenstate basis and the index \(j\) specifies the family.

The Lagrangian invariant under gauge transformations involves dynamical terms for the quark fields and kinetic terms for the gauge fields \(B_\mu\) and \(W^i_\mu\),

\[
\mathcal{L} = \left(\bar{u}_{Lj} \gamma^\mu d_{Lj} + \bar{u}_{Rj} \gamma^\mu d_{Rj} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu}\right),
\]

where the symbol \(\gamma^\mu\) is defined as \(\gamma^\mu := \gamma^\mu D_\mu\). The covariant derivative \(D_\mu\) introduces couplings of the gauge fields \(B_\mu\) and \(W^i_\mu\) to the quark fields, and can be expressed as

\[
D_\mu = \partial_\mu - ig Y B_\mu - ig T^a W^a_\mu,
\]

where \(g'\) and \(g\) are the electroweak coupling constants, and \(T^a\) and \(Y\) are the generators of the SU(2)\(_L\) and the U(1)\(_Y\) symmetry groups that introduce the weak isospin and hypercharge.

In U(1), the field strength tensor \(B_{\mu\nu}\) is related to the gauge field by

\[
B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu},
\]

and in SU(2), the field strength tensors \(W^i_{\mu\nu}\) are related to the gauge fields by

\[
W^i_{\mu\nu} = \partial_{\mu} W^i_{\nu} - \partial_{\nu} W^i_{\mu} - g \epsilon^{ijk} W^j_{\mu} W^k_{\nu}
\]

for \(i \in \{1, 2, 3\}\).

The coupling constants \(g'\) and \(g\) are related to the Weinberg angle \(\theta_W\) and the electric charge \(e\) by

\[
g \sin \theta_W = g' \cos \theta_W = e,
\]

which provides the unification of weak and electromagnetic interactions.

In this form the vector bosons corresponding to the gauge fields \(B_\mu\) and \(W^i_\mu\) are massless and cause long range weak interactions. Furthermore, the couplings are defined as real preventing CP violation to emerge in the interactions.

The Standard Model contains a single SU(2) doublet of a scalar field, that is referred to as the Higgs field and defined by

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.
\]
The Lagrangian of the scalar field, given by

\[ \mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, \tag{2.10} \]

is invariant under local SU(2) × U(1) transformations. The covariant derivative \( D_\mu \) takes the same form as in Eq. 2.5 and introduces the scalar hypercharge in such a way that \( \phi^0 \) does not couple to the photon.

If the scalar field \( \phi \) acquires a non-vanishing vacuum expectation value, for example expressed by

\[ \langle 0 | \phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad \phi = \frac{1}{\sqrt{2}} e^{i \sigma_j \theta_j(\mathbf{x})} \begin{pmatrix} 0 \\ v + h(\mathbf{x}) \end{pmatrix} \quad \text{and} \quad v = \sqrt{-\frac{\mu^2}{\lambda}}, \tag{2.11} \]

then the SU(2)_L × U(1)_Y symmetry groups get spontaneously broken down to the U(1)_em subgroup, that remains symmetric with respect to the vacuum.

The non-vanishing vacuum expectation value causes the ground state to be degenerated and allows for massless scalar excitations, that can be identified as Nambu-Goldstone bosons [6–8]. The Nambu-Goldstone bosons can be eliminated by making use of the SU(2) invariance of the Lagrangian and removing the dependence on \( \theta^j \) in Eq. 2.11 by rotations. In the so-called unitarity gauge, the kinetic term of the Lagrangian in Eq. 2.10 adopts the form

\[ (D_\mu \phi)^\dagger D_\mu \phi = \frac{1}{2} \left( \partial_\mu h \right) \left( \partial^\mu h \right) + \frac{1}{8} \left[ \left( g'B_\mu - gW^3_\mu \right)^2 + g^2 \left( W^1_\mu \right)^2 + g^2 \left( W^2_\mu \right)^2 \right] (h + v)^2. \tag{2.12} \]

The non-vanishing vacuum expectation value of the scalar field gives rise to quadratic terms of the gauge fields and as a consequence the gauge bosons that correspond to these fields acquire a mass. The elimination of the Nambu-Goldstone bosons can be considered as absorbing the degrees of freedom from the Nambu-Goldstone bosons into the longitudinal components of the gauge bosons.

From the Higgs Lagrangian, the fields of three massive vector bosons can be constructed by combinations of the gauge fields \( B_\mu \) and \( W^i_\mu \):

\[ W^+_\mu = \frac{1}{\sqrt{2}} \left( W^1_\mu - i W^2_\mu \right) \]
[2.13]

\[ W^-_\mu = \frac{1}{\sqrt{2}} \left( W^1_\mu + i W^2_\mu \right) \]

\[ Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left( gW^3_\mu - g'B_\mu \right) \]
The fields $W^+, W^-$, and $Z\mu$ can be identified as the heavy vector bosons $W^+, W^-$ and $Z^0$ mediating the weak interactions. Their masses are related by

$$M_{W^\pm} = M_{Z^0} \cos \theta_W = \frac{1}{2} g v. \quad (2.14)$$

The combination orthogonal to the $Z\mu$ field,

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left( g' W^3_\mu + g B_\mu \right), \quad (2.15)$$

has no mass term and can be identified as the photon.

The couplings of the fermions to the vector bosons can be expressed as the interactions of neutral and charged currents described by the Lagrangian

$$L_{\text{int}} = e J_{\text{em}}^\mu A_\mu + \frac{g}{\cos \theta_W} \left( J_Z^\mu - \sin^2 \theta_W J_{\text{em}}^\mu \right) Z_\mu - \frac{1}{\sqrt{2}} g \left( J^\mu_{+} W^-_\mu + J^\mu_{-} W^+_\mu \right), \quad (2.16)$$

where the electromagnetic current $J_{\text{em}}^\mu$ is given by

$$J_{\text{em}}^\mu = \frac{2}{3} \bar{u}_L j^{\gamma \mu} u_L - \frac{1}{3} \bar{d}_L j^{\gamma \mu} d_L - \bar{l}_L j^{\gamma \mu} l_L, \quad (2.17)$$

the neutral weak current $J_Z^\mu$ is given by

$$J_Z^\mu = \frac{1}{2} \left( \bar{u}_L j^{\gamma \mu} u_L - \bar{d}_L j^{\gamma \mu} d_L - \bar{l}_L j^{\gamma \mu} l_L + \bar{\nu}_L j^{\gamma \mu} \nu_L \right), \quad (2.18)$$

and the weak charged currents $J_{\pm}^\mu$ are given by

$$J_+^\mu = \bar{u}_L j^{\gamma \mu} d_L + \bar{l}_L j^{\gamma \mu} \nu_L, \quad J_-^\mu = \bar{d}_L j^{\gamma \mu} u_L + \bar{\nu}_L j^{\gamma \mu} l_L. \quad (2.19)$$

The quark masses arise from the Yukawa interactions with the Higgs field. The Lagrangian of the Yukawa interaction involves the coupling of right-handed quark field singlets via the scalar field $\phi$ to left-handed quark field doublets and can be written as

$$L_{\text{Yukawa}} = -Y^u_{ij} \left( \bar{u}_{Lj}, \bar{d}_{Lj} \right) \left( \begin{array}{c} \phi^{0+} \\ \phi^{-} \end{array} \right) u_{Rj} - Y^d_{ij} \left( \bar{u}_{Lj}, \bar{d}_{Lj} \right) \left( \begin{array}{c} \phi^{+} \\ \phi^{0} \end{array} \right) d_{Rj}, \quad (2.20)$$

where the Yukawa couplings $Y^u$ and $Y^d$ are complex matrices for up-type and down-type quarks. If the scalar field $\phi$ acquires a non-vanishing vacuum expectation value, then the Yukawa interactions constitute mass terms for the quarks. In the unitarity
gauge, the mass matrices $M^u$ and $M^d$ for the up-type and down-type quarks are proportional to the Yukawa couplings with the scale given by the vacuum expectation value $v$:

$$(M^u)_{ij} = \frac{1}{\sqrt{2}} Y_{ij}^u v \quad \text{and} \quad (M^d)_{ij} = \frac{1}{\sqrt{2}} Y_{ij}^d v \quad (2.21)$$

The mass eigenstates of the quarks are determined by the diagonalisation of the mass matrices with the four unitary $3 \times 3$ matrices $U^u_L$, $U^u_R$, $U^d_L$ and $U^d_R$,

$$(\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}) = U^u_L M^u U^u_R^\dagger \quad \text{and} \quad (\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}) = U^d_L M^d U^d_R^\dagger. \quad (2.22)$$

The matrices $U^u_L$, $U^u_R$, $U^d_L$ and $U^d_R$ transform the left-handed and right-handed quark fields from the weak eigenstate basis, denoted by $u$ and $d$, to the mass eigenstate basis, denoted in the following by $u^m$ and $d^m$:

$$u^m_L = U^u_L u_L \quad \quad d^m_L = U^d_L d_L$$
$$u^m_R = U^u_R u_R \quad \quad d^m_R = U^d_R d_R \quad (2.23)$$

The Lagrangian in Eq. 2.16 describing the interactions of quarks with the gauge fields can be written in terms of the quark mass eigenstates. The expressions for the neutral currents retain their form when applying the above transformations. This invariance of the neutral currents with respect to transformations from the weak to the mass eigenstate basis is the reason that no flavor changing neutral currents occur on tree-level and is the fundament of the Glashow-Iliopoulos-Maiani (GIM) mechanism [9].

The charged currents expressed in the mass eigenstate basis are

$$J^{+}_\mu = \bar{u}_L \gamma^\mu d_L = \bar{u}^m_L \gamma^\mu U^u_L U^d_R^\dagger d^m_L \quad \text{and} \quad J^{-}_\mu = \bar{d}_L \gamma^\mu u_L = \bar{d}^m_L \gamma^\mu U^d_L U^u_R^\dagger u^m_L \quad (2.24)$$

where $V_{\text{CKM}} = U^u_L U^d_R^\dagger$ is the unitary $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. The CKM matrix defines the couplings of the $W^\pm$ bosons to the quarks with definite masses in charged current interactions.

For the charged current interactions of quarks, the Lagrangian and the corresponding CP-conjugated Lagrangian expressed by the mass eigenstates of quarks and using the chirality operator $1 - \gamma^5$ are
The above equations show, that the symmetry under CP-transformations is only conserved in the charged current interactions of quarks, if $(V_{\text{CKM}})^*_{ij} = (V_{\text{CKM}})_{ij}$ is satisfied. In 1973, Kobayashi and Maskawa demonstrated that the quark-mixing matrix contains an irreducible complex phase, if there are at least three families of quarks [10]. The complex phase causes $(V_{\text{CKM}})^*_{ij} \neq (V_{\text{CKM}})_{ij}$ and is the only source of CP violation in the Standard Model. Because of its importance, the properties and implications of the CKM matrix are explained in more detail in the next section.

### 2.2 The CKM Matrix and the Unitarity Triangle

In charged current interactions given by the Lagrangian in Eq. 2.25, the physical up- and down-type quarks with definite masses couple to the $W^\pm$ bosons according to the involved CKM matrix elements as illustrated in Fig. 2.1.

The CKM matrix relates the mass eigenstates of the down-type quarks to the corresponding weak eigenstates by

$$
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}_{\text{weak basis}} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}_{\text{mass basis}}.
$$

(2.26)

![Fig. 2.1](image-url) The couplings of the physical up- and down-type quarks to $W^\pm$ bosons in the mass eigenstate basis.
In general, a complex $N \times N$ matrix contains $2N^2$ real parameters. The unitarity of the CKM matrix, equivalent to

$$\sum_i (V_{\text{CKM}})_{ij} (V_{\text{CKM}})_{ik}^* = \delta_{jk} \quad \text{and} \quad \sum_j (V_{\text{CKM}})_{ij} (V_{\text{CKM}})_{kj}^* = \delta_{ik},$$

(2.27)

reduces the number of independent real parameters to $N^2$.

The phases of the up- and down-type quark fields can be arbitrarily rotated by the transformations

$$u_i \rightarrow e^{i\phi^u_i} u_i \quad \text{and} \quad d_j \rightarrow e^{i\phi^d_j} d_j.$$  

(2.28)

When applying the above transformations to the CKM matrix elements according to

$$(V_{\text{CKM}})_{ij} \rightarrow e^{-i\phi^u_i} (V_{\text{CKM}})_{ij} e^{i\phi^d_j},$$

(2.29)

then $2N - 1$ relative phases can be eliminated and therefore the matrix contains $(N - 1)^2$ independent physical parameters. In general, an orthogonal $N \times N$ matrix can be composed from $1/2 N (N - 1)$ independent rotation angles. Consequently, the CKM matrix can be constructed from

$$\frac{1}{2} N (N - 1) + \frac{1}{2} (N - 1) (N - 2) = (N - 1)^2$$


In the case of two fermion families, the CKM matrix contains only one angle and no phase. This case would not allow for CP violation and is equivalent to the two family quark-mixing described by the Cabibbo matrix [12],

$$V_{\text{Cabibbo}} = \left( \begin{array}{cc} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{array} \right),$$

(2.31)

where $\sin \theta_C = |V_{us}| \approx 0.23$ and $\theta_C \approx 13^\circ$ is the Cabibbo angle [13].

In the case of three fermion families as in the Standard Model, the CKM matrix is described by three angles and one complex phase. A representation in analogy to the Cabibbo matrix uses rotations with angles $\theta_{ij}$ between family i and j in the parameterisation

$$V_{\text{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} c_{13} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2.32)

$$= \begin{pmatrix} c_{12} c_{13} \\ s_{12} c_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} \\ c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} \\ -c_{23} s_{13} \\ s_{23} c_{13} \end{pmatrix},$$

(2.33)

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, and $\delta$ is the CP violating phase [14].
The experimental determination of the flavor structure revealed a strong hierarchy in the CKM matrix, which manifests itself in $s_{13} \ll s_{23} \ll s_{12} \ll 1$. The hierarchical structure becomes apparent in a representation in terms of the Wolfenstein parameters $\lambda, A, \rho$ and $\eta$ defined by the relations [15]

$$
s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \quad s_{23} = A \lambda^2 = \sqrt{\frac{|V_{cb}|}{V_{us}}}, \quad s_{13} e^{i\delta} = A \lambda^3 (\rho + i \eta) = V_{ub}^*.
$$

The expansion of the CKM matrix elements in terms of powers of the parameter $\lambda \approx 0.23$ up to the order of three results in

$$
V_{\text{CKM}} = \begin{pmatrix}
1 - \lambda^2/2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \lambda^2/2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix} + O(\lambda^4).
$$

This representation reveals the hierarchical structure of the CKM matrix. The diagonal elements represent transitions within the same family and are equal or close to one. The transitions between the first and the second family are suppressed by a factor $\lambda$, the transitions between the second and the third family by a factor $\lambda^2$, and the transitions between the first and the third family by a factor $\lambda^3$. Furthermore, at this order of expansion the complex phase manifests itself in the CKM matrix elements $V_{td}$ and $V_{ub}$. These elements are important for many physical processes such as the time evolution and the decay of B mesons. This implies CP violation to be possibly sizeable in the charged and neutral B meson system.

The unitarity of the CKM matrix as expressed by Eq. 2.27 results in the following six relations of vanishing CKM matrix combinations:

$$
\begin{align*}
V_{\text{ud}}^* V_{\text{us}} + V_{\text{cd}}^* V_{\text{cs}} + V_{\text{td}}^* V_{\text{ts}} &= \delta_{ds} = 0 \\
O(\lambda) &+ O(\delta) &+ O(\lambda^3)
\end{align*}
$$

$$
\begin{align*}
V_{\text{ud}} V_{\text{cd}}^* + V_{\text{us}} V_{\text{cs}}^* + V_{\text{ub}} V_{\text{cb}}^* &= \delta_{uc} = 0 \\
O(\lambda) &+ O(\delta) &+ O(\lambda^3)
\end{align*}
$$

$$
\begin{align*}
V_{\text{us}}^* V_{\text{ub}} + V_{\text{cs}}^* V_{\text{cb}} + V_{\text{ts}}^* V_{\text{tb}} &= \delta_{sb} = 0 \\
O(\lambda^4) &+ O(\lambda^2) &+ O(\lambda^3)
\end{align*}
$$

$$
\begin{align*}
V_{\text{td}} V_{\text{cd}}^* + V_{\text{ts}} V_{\text{cs}}^* + V_{\text{tb}} V_{\text{cb}}^* &= \delta_{ct} = 0 \\
O(\lambda^4) &+ O(\lambda^2) &+ O(\lambda^3)
\end{align*}
$$

$$
\begin{align*}
V_{\text{td}}^* V_{\text{ud}} + V_{\text{ts}} V_{\text{us}}^* + V_{\text{tb}} V_{\text{ub}}^* &= \delta_{ut} = 0 \\
O(\lambda^3) &+ O(\lambda^3) &+ O(\lambda^3)
\end{align*}
$$
\[ \mathcal{O}(\lambda^3) \]

\[ \mathcal{O}(\lambda^3) \]

\[ \mathcal{O}(\lambda^3) \]

\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = \delta_{db} = 0 \] (2.41)

The above relations can be interpreted geometrically as triangles in the complex plane. In the above equations, the order of side lengths in the triangles are specified in terms of the Wolfenstein parameter \( \lambda \). The first four triangles have sides of very different lengths with ratios of \( \lambda: \lambda^5 \approx 360 \) and \( \lambda^2: \lambda^4 \approx 19 \), and are therefore degenerate. The sides of the two last triangles all have lengths of the order of \( \lambda^3 \), and consequently the triangles have large internal angles of the order of 10°.

The six triangles corresponding to Eqs. 2.36–2.41 all have the same area which is half of the Jarlskog invariant \( J \) defined by [16]

\[ J = \pm \text{Im} \left[ V_{ij} V_{kl} V_{il}^* V_{kj}^* \right] \text{ for } i \neq k, \ 1 \neq j. \] (2.42)

The Jarlskog invariant is a dimensionless and phase-convention independent measure of the size of CP violation in the Standard Model. The experimentally determined value is \( J \approx 3 \times 10^{-5} \) [17]. In combination with the quark masses, it leads to a necessary condition for CP violation given by

\[ -2J(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_l^2)(m_l^2 - m_s^2)(m_s^2 - m_d^2)(m_d^2 - m_b^2) \neq 0. \] (2.43)

A non-vanishing value of the above relation is necessary for CP violation to emerge in the quark sector [11].

Of particular interest for the physics of B mesons is the triangle given by Eq. 2.41, because each side is related to CKM matrix elements describing b-quark transitions as occurring in CKM-favored and CKM-suppressed B meson decays and in \( B^0 \bar{B}^0 \) oscillations. The so-called Unitarity Triangle arises by dividing each side by the experimentally best known side, which is \( V_{cd} V_{cb}^* \). An illustration of the Unitarity Triangle and the definitions of the angles and sides are shown in Fig. 2.2. The angles \( \alpha, \beta \) and \( \gamma \) of the Unitarity Triangle, also often referred to as \( \phi_2, \phi_1, \phi_3 \), are related to CKM matrix elements by

\[ \alpha = \phi_2 = \arg \left( -\frac{V_{ud} V_{tb}^*}{V_{ud} V_{ub}^*} \right), \] (2.44)

\[ \beta = \phi_1 = \arg \left( -\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right), \] (2.45)

\[ \gamma = \phi_3 = \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right). \] (2.46)

One key objective of flavor physics, in particular of the Belle experiment, is to experimentally overconstrain the Unitarity Triangle by a large variety of independent measurements. The experimental determination of the angles \( \alpha, \beta \) and \( \gamma \) of the Unitarity Triangle is closely related to the measurements of CP violation. The
angle $\beta$ is known at a precision better than $1^\circ$ from time-dependent measurements of mixing-induced CP violation in $b \rightarrow c\bar{c}s$ transitions such as in $B^0 \rightarrow J/\Psi K^0_s$ decays, see Sect. 2.6. The objective of the present analysis is the measurement of
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Time-dependent CP violation in $b \rightarrow c\bar{c}d$ transitions, in particular in $B^0 \rightarrow D^+D^-$ and $B^0 \rightarrow D^{*\pm}D^\mp$ decays. The mixing-induced CP violation in $b \rightarrow c\bar{c}d$ transitions is closely related to the angle $\beta$ and can be compared to the reference measurements in $b \rightarrow c\bar{c}s$ transitions.

The constraints on the Unitarity Triangle in the $\rho\eta$-plane at current experimental status are summarised in Fig. 2.3. In total, the flavor part of the Standard Model as determined by constraints from the Unitarity Triangle performs well. Tight constraints come from the mentioned measurements of the angle $\beta$, from $B^0\bar{B}^0$ and $B^0_s\bar{B}^0_s$ oscillation measurements and from measurements of $|V_{ub}|$ in semileptonic $b \rightarrow u$ transitions. In global fits, a few tensions persist which are related to measurements with large uncertainties and need to be confirmed.

The constraints on the Unitarity Triangle and on observables related to the CKM matrix from global fits of available measurements as obtained by the CKMFitter group and the UTfit Collaboration can be found in Refs. [18, 19]. A general summary covering the experimental knowledge about the CKM matrix and the Unitarity Triangle can be found in Ref. [17].

2.3 Time Evolution of Neutral Mesons

The time evolution of weakly decaying neutral mesons is characterised by the effect of particle-antiparticle oscillations. Particle-antiparticle oscillations emerge in different fields of high energy physics and have been experimentally established for neutral mesons and neutrinos. The theoretical explanations for the different kinds of oscillations have all in common, that particles once produced with a definite flavor content, e.g. as a $B^0$ meson or as a $\nu_e$ neutrino, are not the mass eigenstates of their particular dynamic quantum system and evolve over time into mixtures of $B^0$ and $\bar{B}^0$ mesons or of $\nu_e$, $\nu_\mu$ and $\nu_\tau$ neutrinos, respectively.

In the following, the theoretical formalism for the description of the time evolution and mixing of neutral mesons will be introduced with a focus on the $B^0$ system. The $B^0$ system is of special importance, because the mixing introduces a phase, that can give rise to large CP violating effects.

In the neutral meson sector, $K^0-K^0$, $D^0-D^0$, $B^0-B^0$ and $B^0_s-B^0_s$ oscillations are theoretically described as a 2-level quantum system coupled by higher order weak interactions.

A state that is initially a superposition of $P^0$ and $\bar{P}^0$ (with $P \in \{K, D, B, B_s\}$), e.g.

$$|\Psi(0)\rangle = a(0)|P^0\rangle + b(0)|\bar{P}^0\rangle,$$  \hspace{1cm} (2.47)

evolves over time into a different mixture of $P^0$ and $\bar{P}^0$ and additionally contains contributions of all possible final decay states,

$$|\Psi(t)\rangle = a(t)|P^0\rangle + b(t)|\bar{P}^0\rangle + \sum_i c_i(t)|f_i\rangle.$$  \hspace{1cm} (2.48)
The time scale of weak interactions that cause particle-antiparticle oscillations is much larger than that of strong or electromagnetic interactions. Following an approximation by Wigner and Weisskopf [20, 21], the time evolution is governed by a Schrödinger equation with an effective Hamiltonian, that does not contain explicitly the couplings to the final-states $f_i$:

$$i \frac{\partial}{\partial t} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{H} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$ (2.49)

The effective Hamiltonian $\mathbf{H}$ is not Hermitian due to the possibility of decays of the particles. It is composed of a dispersive and an absorptive part:

$$\mathbf{H} = \mathbf{M} - i \frac{\mathbf{\Gamma}}{2} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - i \frac{1}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$ (2.50)

The diagonal elements of the mass matrix $\mathbf{M}$ and the decay matrix $\mathbf{\Gamma}$ correspond to flavor-conserving $\nu^0 \leftrightarrow \nu^0$ and $\bar{\nu}^0 \leftrightarrow \bar{\nu}^0$ transitions and the off-diagonal elements to flavor-changing $\nu^0 \rightarrow \bar{\nu}^0$ and $\bar{\nu}^0 \rightarrow \nu^0$ transitions. The matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ are Hermitian inferring that $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$. The invariance under CPT-transformations requires $M_{11} = M_{22} =: \mathbf{M}$ and $\Gamma_{11} = \Gamma_{22} =: \mathbf{\Gamma}$.

The eigenstates of the Hamiltonian are

$$| \nu_{L,H} \rangle = p | \nu^0 \rangle \mp q | \bar{\nu}^0 \rangle,$$ (2.51)

where the complex coefficients $p$ and $q$ satisfy the normalisation condition $|p|^2 + |q|^2 = 1$ and are given by

$$\frac{q}{p} = \pm \left( \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \right)^{1/2} \approx \pm M_{12}^* \left( 1 - \frac{1}{2} \frac{\text{Im} \Gamma_{12}}{M_{12}} \right).$$ (2.52)

The corresponding eigenvalues are

$$\lambda_{L,H} = \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \pm \frac{q}{p} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right).$$ (2.53)

$| \nu_{L,H} \rangle$ represent the physical light (L) and heavy (H) mass eigenstates with well-defined masses $M_{L,H}$ and decay widths $\Gamma_{L,H}$:

$$M_L = \text{Re} (\lambda_L) \quad \text{and} \quad \Gamma_L = \text{Im} (\lambda_L)$$ (2.54)

$$M_H = \text{Re} (\lambda_H) \quad \text{and} \quad \Gamma_H = \text{Im} (\lambda_H)$$ (2.55)
2.3 Time Evolution of Neutral Mesons

The time evolution of the mass eigenstates can be inferred from the Schrödinger equation:

\[ |P_{L,H}(t)\rangle = e^{-iHt} |P_{L,H}\rangle \]

\[ = e^{-iM_{L,H}t}e^{-\frac{i}{2}\Gamma_{L,H}t} |P_{L,H}\rangle \]

(2.56)

(2.57)

The light and heavy eigenstate have a mass difference \( \Delta M = M_H - M_L \) and a decay width difference \( \Delta \Gamma = \Gamma_L - \Gamma_H \). Neglecting possible CP violation in the mixing by assuming \( |q/p| = 1 \) the differences are given by \( \Delta M = 2|M_{12}| \) and \( \Delta \Gamma = 2|\Gamma_{12}| \).

The time evolution for flavor eigenstates can be expressed by the above relations. At time \( t = 0 \) pure neutral meson states \( |P^0\rangle \) and \( |\bar{P}^0\rangle \) (with \( P \in \{K, D, B, B_s\} \)) evolve as

\[ |P^0(t)\rangle = g_+(t) |P^0\rangle + \frac{q}{p} g_-(t) |\bar{P}^0\rangle \]

(2.58)

\[ |\bar{P}^0(t)\rangle = g_+(t) |\bar{P}^0\rangle + \frac{p}{q} g_-(t) |P^0\rangle, \]

(2.59)

where

\[ g_+(t) = e^{-i\Delta t}e^{-\Gamma t/2} \left[ \cosh \left( \frac{\Delta \Gamma t}{4} \right) \cos \left( \frac{\Delta M t}{2} \right) - i \sinh \left( \frac{\Delta \Gamma t}{4} \right) \sin \left( \frac{\Delta M t}{2} \right) \right] \]

(2.60)

\[ g_-(t) = e^{-i\Delta t}e^{-\Gamma t/2} \left[ -\sinh \left( \frac{\Delta \Gamma t}{4} \right) \cos \left( \frac{\Delta M t}{2} \right) + i \cosh \left( \frac{\Delta \Gamma t}{4} \right) \sin \left( \frac{\Delta M t}{2} \right) \right]. \]

(2.61)

The functions \( g_+(t) \) and \( g_-(t) \) have the following properties:

\[ |g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2} \left[ \cosh \left( \frac{\Delta \Gamma t}{2} \right) \pm \cos \left( \Delta M t \right) \right] \]

(2.62)

\[ g_+^*(t)g_-(t) = \frac{e^{-\Gamma t}}{2} \left[ -\sinh \left( \frac{\Delta \Gamma t}{2} \right) + i \sin \left( \Delta M t \right) \right]. \]

(2.63)

From the above time evolution, the time-dependent decay rates of neutral mesons produced at \( t = 0 \) in a flavor eigenstate \( P^0 \) or \( \bar{P}^0 \) and decaying to the final-state \( f \) can be determined. The time-dependent decay rates are proportional to the absolute square of the involved decay amplitudes:

\[ \Gamma [P^0(t) \to f] \propto \left| \langle f|\mathcal{H}|P^0(t)\rangle \right|^2 \]

(2.64)

\[ \Gamma [\bar{P}^0(t) \to f] \propto \left| \langle f|\mathcal{H}|\bar{P}^0(t)\rangle \right|^2 \]

(2.65)
The decay rates describing decays to the CP-conjugated state \( |\bar{f}\rangle = \text{CP}|f\rangle \) are:

\[
\Gamma [P^0(t) \to \bar{f}] \propto \left| \langle \bar{f}|\mathcal{H}|P^0(t)\rangle \right|^2 \tag{2.66}
\]
\[
\Gamma [\bar{P}^0(t) \to f] \propto \left| \langle f|\mathcal{H}|\bar{P}^0(t)\rangle \right|^2 \tag{2.67}
\]

For the following discussion, it is advantageous to define the decay amplitudes as

\[
A_f := \langle f|\mathcal{H}|P^0\rangle, \quad A_{\bar{f}} := \langle \bar{f}|\mathcal{H}|P^0\rangle, \quad \bar{A}_f := \langle f|\mathcal{H}|\bar{P}^0\rangle, \quad \bar{A}_{\bar{f}} := \langle \bar{f}|\mathcal{H}|\bar{P}^0\rangle,
\]

and to introduce the following ratios defined by

\[
\lambda_f := \frac{q\bar{A}_f}{pA_f} \quad \text{and} \quad \lambda_{\bar{f}} := \frac{q\bar{A}_{\bar{f}}}{pA_{\bar{f}}}, \tag{2.69}
\]

By inserting the time evolution of the flavor eigenstates given by Eqs. 2.58 and 2.59 into Eqs. 2.64–2.67 and making use of the functions \( g_+(t) \) and \( g_-(t) \) defined in Eqs. 2.60 and 2.61, the time-dependent decay rates are proportional to the following expressions [22]:

\[
\Gamma [P^0(t) \to f] \propto |A_f|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \frac{1 - |\lambda_f|^2}{2} \cos (\Delta \text{Mt}) \right. \\
- \left. \Re (\lambda_f) \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \Im (\lambda_f) \sin (\Delta \text{Mt}) \right\} \tag{2.70}
\]
\[
\Gamma [\bar{P}^0(t) \to f] \propto |A_{\bar{f}}|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_{\bar{f}}|^2}{2} \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \frac{1 - |\lambda_{\bar{f}}|^2}{2} \cos (\Delta \text{Mt}) \right. \\
- \left. \Re (\lambda_{\bar{f}}) \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \Im (\lambda_{\bar{f}}) \sin (\Delta \text{Mt}) \right\} \tag{2.71}
\]
\[
\Gamma [P^0(t) \to \bar{f}] \propto |\bar{A}_f|^2 \left| \frac{q}{p} \right|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \frac{1 - |\lambda_f|^2}{2} \cos (\Delta \text{Mt}) \right. \\
- \left. \Re \left( \frac{1}{\lambda_f} \right) \sinh \left( \frac{\Delta \Gamma}{2} t \right) + \Im \left( \frac{1}{\lambda_f} \right) \sin (\Delta \text{Mt}) \right\} \tag{2.72}
\]
\[
\Gamma [\bar{P}^0(t) \to f] \propto |\bar{A}_{\bar{f}}|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_{\bar{f}}|^2}{2} \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \frac{1 - |\lambda_{\bar{f}}|^2}{2} \cos (\Delta \text{Mt}) \right. \\
- \left. \Re \left( \frac{1}{\lambda_{\bar{f}}} \right) \sinh \left( \frac{\Delta \Gamma}{2} t \right) - \Im \left( \frac{1}{\lambda_{\bar{f}}} \right) \sin (\Delta \text{Mt}) \right\} \tag{2.73}
\]

The decay rates in Eqs. 2.70–2.73 allow to calculate the time evolution of any neutral weak decay process and are important not only for the mixing, but allow for interference effects that can give rise to CP violation as shown in Sect. 2.4. In the derivation of the decay rates no assumptions except the validity of the CPT theorem.
have been applied. Therefore the decay rates are applicable to any weakly decaying neutral meson $P^0$ or $\bar{P}^0$ (with $P \in \{ K, D, B, B_s \}$).

For the oscillations of neutral mesons, the probabilities to observe an initially pure neutral meson $P^0$ at time $t > 0$ without mixing as a $P^0$ or with mixing as a $\bar{P}^0$ are given by

$$P_{\text{unmixed}}(t) = \left| \langle P^0 | P^0(t) \rangle \right|^2 = \frac{1}{2} e^{-\Gamma t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) + \cos (\Delta M t) \right]$$

$$= \frac{1}{2} e^{-\Gamma t} \left[ \cosh (y \Gamma t) + \cos (x \Gamma t) \right]$$

(2.74)

$$P_{\text{mixed}}(t) = \left| \langle P^0 | \bar{P}^0(t) \rangle \right|^2 = \frac{1}{2} \frac{q}{p} e^{-\Gamma t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) - \cos (\Delta M t) \right]$$

$$= \frac{1}{2} \frac{q}{p} e^{-\Gamma t} \left[ \cosh (y \Gamma t) - \cos (x \Gamma t) \right],$$

(2.75)

where $x := \frac{\Delta M}{\Gamma}$ and $y := \frac{\Delta \Gamma}{2 \Gamma}$ are dimensionless parameters characterising the mixing of neutral mesons. In the case of a mass difference $\Delta M \neq 0$, the mixing is a consequence of $P^0 \leftrightarrow \bar{P}^0$ transitions. The case of a decay width difference $\Delta \Gamma \neq 0$ is equivalent to a difference in the lifetimes of the mass eigenstates and the mixing is a consequence of the disappearance of the shorter-lived eigenstate over time. Since the longer-lived eigenstate is a linear combination of $P^0$ and $\bar{P}^0$ states, an initially pure $P^0$ sample acquires a $\bar{P}^0$ component over time.

$K^0$-$\bar{K}^0$, $D^0$-$\bar{D}^0$, $B^0$-$\bar{B}^0$ and $B^0_s$-$\bar{B}^0_s$ oscillations have all been observed experimentally [23–25]. The current experimental status of the mixing parameters is summarised in Table 2.1 and the corresponding time-dependent mixing probabilities are shown in Fig. 2.4.

The $K^0$-$\bar{K}^0$ system is characterised by a large difference in lifetimes. The decay of the almost CP-odd $K_L^0$ to the CP-even $\pi \pi$ final-state is suppressed by $O \left( 10^{-3} \right)$. Additionally, since $M_{\pi \pi \pi} \approx M_{K^0}$, the decay to $\pi \pi \pi$ is kinematically suppressed and as a consequence $K_L^0$ mesons have a much larger lifetime than $K_S^0$ mesons.

### Table 2.1 Summary of properties and mixing parameters of $K^0$, $D^0$, $B^0$ and $B^0_s$ mesons at current experimental status [23–25]

<table>
<thead>
<tr>
<th></th>
<th>$K^0$</th>
<th>$D^0$</th>
<th>$B^0$</th>
<th>$B^0_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean mass M (MeV/$c^2$)</td>
<td>497</td>
<td>1865</td>
<td>5279</td>
<td>5366</td>
</tr>
<tr>
<td>Mass difference $\Delta M \left( \frac{h}{ps} \right)$</td>
<td>$5.27 \pm 0.03 \times 10^{-3}$</td>
<td>$5.27 \pm 0.03 \times 10^{-3}$</td>
<td>$5.04 \pm 0.004 \times 10^{-3}$</td>
<td>$17.77 \pm 0.12$</td>
</tr>
<tr>
<td>Lifetime $\tau_H$ (ps)</td>
<td>51160 $\pm$ 210</td>
<td>0.410 $\pm$ 0.002</td>
<td>1.519 $\pm$ 0.001</td>
<td>1.543 $\pm$ 0.060</td>
</tr>
<tr>
<td>$x = \frac{\Delta M}{\Delta \Gamma}$</td>
<td>0.946</td>
<td>0.01</td>
<td>0.776</td>
<td>26.1</td>
</tr>
<tr>
<td>$y = \frac{\Delta \Gamma}{2 \Gamma}$</td>
<td>0.997</td>
<td>0.01</td>
<td>&lt;0.01</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Mixing in the $D^0$-$
\bar{D}^0$ system is currently subject of intense experimental and theoretical research. The mixing can proceed via higher order box diagrams of weak interactions or via intermediate states accessible to both $D^0$ and $\bar{D}^0$ mesons that can have effects on large distances. In the box diagrams, down-type quarks are exchanged and the $b$-quark contributions, which are due to their large invariant mass typically expected to lead to enhancements, are suppressed by very small CKM matrix elements. The system is described in good approximation by only two generations and therefore GIM-cancellations contribute to further suppressions. As a consequence $D^0$-$\bar{D}^0$ mixing is a very small effect.

$B^0$-$\bar{B}^0$ and $B_s^0$-$\bar{B}_s^0$ oscillations are caused by higher order box diagrams mediated by the exchange of two $W$ bosons and two up-type quarks as shown in Fig. 2.5.

The off-diagonal elements $M_{12}^q$ ($q \in \{d, s\}$) calculated from the box diagrams are given by [26, 27]

$$M_{12}^q = -\frac{G_F^2 M_W^2}{12\pi^2} \eta_B M_{B_q} B_{B_q} f_{B_q}^2 S_0 \left( \frac{M_t^2}{M_W^2} \right) \left( V_{tq}^* V_{tb} \right)^2 e^{i(\pi - \phi_{B_q}^C)} , \quad (2.76)$$

where $G_F$ is the Fermi constant, $M_t$ the masses of the $B_q$ mesons and the $W$ bosons, $\eta$ a perturbative QCD correction, $B_{B_q}$ a non-perturbative parameter related to the involved hadronic matrix element $\langle \bar{B}_q | (\bar{b}q) | V-A (\bar{b}q) | V-A | B_q \rangle$, $f_{B_q}$ the $B_q$ decay con-
stant, $S_0\left(\frac{M^2_t}{M^2_W}\right)$ an Inami-Lin function describing the dependence on the top-quark mass, and $\phi_{CP}$ a convention dependent phase related to CP-transformations of the B flavor eigenstates.

The off-diagonal elements $M_{12}$ are dominated by the contributions of the internal top-quarks. Due to the quadratic dependence on the involved CKM matrix element $V_{tq}$, the oscillation frequency $\Delta M$ is about 35-times higher in the $B^0_s$-$\bar{B}^0_s$ system with element $V_{ts}$ than in the $B^0$-$\bar{B}^0$ system with element $V_{td}$.

$M_{12}$ is related to $\Gamma_{12}$ by

$$\frac{\Gamma_{12}}{M_{12}} \approx -\frac{3\pi}{2S_0\left(\frac{M^2_t}{M^2_W}\right)} \left(\frac{M^2_b}{M^2_W}\right) = \mathcal{O}\left(\frac{M^2_b}{M^2_t}\right) \ll 1 \quad (2.77)$$

The absorptive parts of the Hamiltonian $\Gamma_{12}$ can give rise to differences in the decay width. The relative decay width differences $\frac{\Delta \Gamma}{\Gamma}$ of the mass eigenstates are given by
\[ \Delta \Gamma \approx 16\pi^2 \frac{f_{Bq}^2}{M_{Bq}^2} |V_{cq}|^2. \] (2.78)

In the $B^0$ system, the decay width difference is very small, \( \frac{\Delta \Gamma}{\Gamma} \lesssim \mathcal{O}(10^{-2}) \), and cannot be resolved at present experiments. In the $B^0_s$ system, the decay width difference is, with \( \frac{\Delta \Gamma}{\Gamma} \approx \mathcal{O}(10^{-1}) \), sizeable and results in significant different lifetimes of the mass eigenstates, which has been observed experimentally.

Comparing Eqs. 2.52 and 2.76, one finds that the phase of $q_p$ is given by the phase of $M^q_{12} = |M^q_{12}| e^{i\phi^q_M}$ with $\phi^q_M = \pi + 2 \text{arg}(V^*_{tq}V_{tb}) - \phi^\text{CP}_{Bq} = 2\beta_{(s)}$, (2.79)

where the convention dependent phase is set to $\phi^\text{CP}_{Bq} = \pi$. The phases of $q_p$ are directly related to the angle $\beta_s$ for $B^0_s$ mesons and to the angle $\beta$ of the Unitarity Triangle for $B^0$ mesons. The latter is, being approximately 21°, relatively large.

### 2.4 CP Violation in Meson Decays

As described in Sects. 2.1 and 2.2, the effect of CP violation in the Standard Model of electroweak interactions is caused by a single irreducible complex phase in the CKM matrix. In weak decays of charged and neutral mesons, CP violation can manifest itself in three different categories: in the decay, in the mixing, and in the interference between mixing and decay. The three categories of CP violation are illustrated in Fig. 2.6 and described in the following.

#### 2.4.1 CP Violation in the Decay

If the magnitudes of the amplitudes of CP-conjugated processes are different, equivalent to

\[ \left| \tilde{A}_f/A_f \right| \neq 1, \] (2.80)

then CP violation can occur in the decay. This category of CP violation is referred to as direct CP violation and is the only source of CP violation in decays of charged mesons. This category is illustrated in Fig. 2.6(a).

A necessary condition needed to be satisfied for direct CP violation to emerge is the existence of at least two contributing decay amplitudes with non-vanishing relative weak and strong phases. The weak phases originate from complex CKM matrix elements and enter with opposite sign in $A_f$ and $\tilde{A}_f$. The strong phases originate from hadronic effects, e.g. from rescattering of intermediate on-shell states. The strong
2.4 CP Violation in Meson Decays

(a) CP violation in the decay ("direct CP violation")

(b) CP violation in the mixing

(c) CP violation in the interference between mixing and decay ("mixing induced CP violation")

Fig. 2.6 Three categories of CP violation in meson decays. The illustration has been adapted from Ref. [11]

phases need to be different for the amplitudes contributing to the decay. Because strong interactions are invariant under CP-transformations, the strong phases are the same for $A_f$ and $\bar{A}_f$.

The total amplitude of a decay and of its CP-conjugated decay can be written as

$$A_f = \sum_j |a_j| e^{i(\delta_j + \phi_j)}$$  \hspace{1cm} (2.81)

and

$$\bar{A}_{\bar{f}} = \sum_j |a_j| e^{i(\delta_j - \phi_j)}.$$  \hspace{1cm} (2.82)

where $a_j$ denotes single contributing amplitudes, and $\delta_j$ and $\phi_j$ are the associated strong and weak phases. The necessary condition for direct CP violation becomes apparent in the difference of the squared amplitudes,

$$|A_f|^2 - |\bar{A}_{\bar{f}}|^2 = 2 \sum_{i,j} |a_i| |a_j| \sin (\delta_i - \delta_j) \sin (\phi_i - \phi_j).$$  \hspace{1cm} (2.83)
Due to the interference, a non-vanishing value in the difference only exists, if the contributing amplitudes provide both: a non-trivial strong phase difference $\delta_i - \delta_j$ and a non-trivial weak phase difference $\phi_i - \phi_j$.

For direct CP violation, an asymmetry can be defined by

$$A_{CP} = \frac{\Gamma(P \rightarrow f) - \Gamma(\bar{P} \rightarrow \bar{f})}{\Gamma(P \rightarrow f) + \Gamma(\bar{P} \rightarrow \bar{f})} = \frac{1 - |\bar{A}_f/A_f|^2}{1 + |\bar{A}_f/A_f|^2} \propto \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j).$$  \hspace{1cm} (2.84)

### 2.4.2 CP Violation in the Mixing

If the complex coefficients $q$ and $p$ differ by more than a phase, equivalent to

$$|q/p| \neq 1, \hspace{1cm} (2.85)$$

then CP violation occurs in the mixing. This category of CP violation is illustrated in Fig. 2.6(b).

Consider flavor-specific decays satisfying

$$P^0 \rightarrow f \leftrightarrow \bar{P}^0 \text{ and } P^0 \rightarrow \bar{f} \leftrightarrow \bar{P}^0, \hspace{1cm} (2.86)$$

equivalent to $|\bar{A}_f| = |A_f| = 0$, and assume no direct CP violation by requiring $|A_f| = |\bar{A}_f|$, then the forbidden $P^0 \rightarrow f$ and $P^0 \rightarrow \bar{f}$ decays are only possible due to $P^0$-$\bar{P}^0$ oscillations. The asymmetry of the forbidden decays calculated from the time-dependent decay rates in Eqs. 2.71 and 2.72 can be expressed as

$$A_{fs} = \frac{\Gamma(\bar{P}^0(t) \rightarrow f) - \Gamma(P^0(t) \rightarrow \bar{f})}{\Gamma(P^0(t) \rightarrow f) + \Gamma(P^0(t) \rightarrow \bar{f})} = \frac{1 - |q/p|^4}{1 + |q/p|^4}. \hspace{1cm} (2.87)$$

and is independent of time. A deviation of the asymmetry from zero and consequently a CP violation in the mixing corresponds to unequal mixing probabilities

$$|(\bar{P}^0|P(t))]^2 \neq |(P^0|\bar{P}(t))]^2. \hspace{1cm} (2.88)$$

### 2.4.3 CP Violation in the Interference Between Mixing and Decay

In decays to final-states $f$ that are common to $P^0$ and $\bar{P}^0$, an interference between the decay without mixing, $P^0 \rightarrow f$, and the decay with mixing, $P^0 \rightarrow \bar{P}^0 \rightarrow f$, arises.

CP violation in the interference between mixing and decay occurs, if
\[ \text{Im} \left( \lambda_f \right) \neq 0 \quad \text{with} \quad \lambda_f = \frac{q \tilde{A}_f}{p A_f}, \quad (2.89) \]

This category of CP violation is referred to as mixing-induced CP violation and is illustrated in Fig. 2.6(c). Mixing-induced CP violation can be observed by the time-dependent asymmetry \( A_{CP} (t) \) of neutral mesons decaying into CP eigenstates \( f_{CP} \), defined by

\[ A_{CP} (t) = \frac{\Gamma (\bar{P}^0 (t) \to f_{CP}) - \Gamma (P^0 (t) \to f_{CP})}{\Gamma (\bar{P}^0 (t) \to f_{CP}) + \Gamma (P^0 (t) \to f_{CP})}. \quad (2.90) \]

The asymmetry calculated from the time-dependent decay rates in Eqs. 2.70 and 2.71 can be expressed as

\[ A_{CP} (t) = \frac{S_{\text{mix}}^{\text{CP}} \sin (\Delta t) - C_{\text{dir}}^{\text{CP}} \cos (\Delta t)}{\cosh (\Delta \Gamma t/2)} - A_{\Delta \Gamma} \sinh (\Delta \Gamma t/2), \quad (2.91) \]

where

\[ S_{\text{mix}}^{\text{CP}} := \frac{2 \text{Im} \left( \lambda_{f_{CP}} \right)}{1 + \left| \lambda_{f_{CP}} \right|^2}, \quad C_{\text{dir}}^{\text{CP}} := \frac{1 - \left| \lambda_{f_{CP}} \right|^2}{1 + \left| \lambda_{f_{CP}} \right|^2}, \quad A_{\Delta \Gamma} := \frac{2 \text{Re} \left( \lambda_{f_{CP}} \right)}{1 + \left| \lambda_{f_{CP}} \right|^2}, \quad (2.92) \]

and

\[ \left| S_{\text{mix}}^{\text{CP}} \right|^2 + \left| C_{\text{dir}}^{\text{CP}} \right|^2 + |A_{\Delta \Gamma}|^2 = 1. \quad (2.93) \]

The parameters \( S_{\text{mix}}^{\text{CP}} \) and \( C_{\text{dir}}^{\text{CP}} \) measure mixing-induced and direct CP violation, respectively. The parameter \( A_{\Delta \Gamma} \) provides another observable in neutral meson systems with sizeable decay width difference \( \Delta \Gamma \).

In the \( B^0 \) system, the decay width difference \( \Delta \Gamma \) is negligible and the time-dependent CP asymmetry \( A_{CP} (t) \) in Eq. 2.91 can be rewritten as

\[ A_{CP} (t) = S_{\text{mix}}^{\text{CP}} \sin (\Delta t) - C_{\text{dir}}^{\text{CP}} \cos (\Delta t). \quad (2.94) \]

The parameters \( S_{\text{mix}}^{\text{CP}} \) and \( C_{\text{dir}}^{\text{CP}} \) are experimentally accessible from the amplitudes of the sine and cosine oscillations in the proper decay times of flavor-tagged decays, see Sect. 2.5. If the amplitudes contributing to a decay all carry the same weak phase, then \( |A_{f_{CP}}| = |\tilde{A}_{f_{CP}}| \) and \( |\lambda_{f_{CP}}| = 1 \). As a consequence, no direct CP violation occurs and the mixing-induced CP violation,

\[ S_{\text{mix}}^{\text{CP}} = \text{Im} (\lambda_f), \quad (2.95) \]

is directly related to the phases of the CKM matrix elements involved in the mixing. These decays are referred to as golden modes. Because of its importance, the CP violation in \( b \to c \bar{c} s \) transitions in the golden mode of \( B^0 \to J/\psi K^0_S \) decays is discussed in Sect. 2.6.
The objective of the present analysis is the measurement of time-dependent CP violation in \( B^0 \to D^+ D^- \) and \( B^0 \to D^{*+} D^- \) decays. These decays are mediated by \( b \to c \bar{c} d \) transitions. As it will be discussed in Sect. 2.7, \( b \to d \) penguin amplitudes that may have different weak phases can contribute to the decays and possibly result in \( |\lambda_{f\text{CP}}| \neq 1 \). Consequently, in addition to mixing-induced CP violation that is expected to be similar to that in \( b \to c \bar{c} s \) transitions, the penguin amplitudes might give rise to direct CP violation.

### 2.5 Coherent \( B^0 \bar{B}^0 \) Mixing

At the Belle experiment, B mesons are produced by the asymmetric-energy \( e^+ e^- \)-collider KEKB via the decay of the \( \Upsilon(4S) \)-resonance. Neutral \( B \bar{B} \) mesons originating from \( \Upsilon(4S) \) decays are produced in an entangled quantum state and evolve coherently. The decay flavor of one of the B mesons determines the flavor content of the second B meson to be opposite at this particular instant of time. This is a manifestation of the Einstein-Podolsky-Rosen (EPR) effect [28] and is important for the time-dependent CP violation measurements at the Belle experiment.

Starting from the two-body wave function describing the entangled \( B^0 \bar{B}^0 \) state and applying the theory of the time evolution of neutral mesons from Sect. 2.3, the rate of the joint decay of both B mesons, the resulting CP asymmetries and the probability distributions needed for the time-dependent CP violation measurements in the present analysis are derived.

The mechanism of coherent \( B \bar{B} \) pair production via decays of the \( \Upsilon(4S) \)-resonance is shown in Fig. 2.7. The \( \Upsilon(4S) \) has the quantum numbers \( j^{PC} = 1^{--} \). The decay of the \( \Upsilon(4S) \) into \( B \bar{B} \) pairs is a strong interaction process that conserves the quantum
numbers. B mesons are pseudo-scalars and the $B^0 \bar{B}^0$ state is produced with orbital angular momentum $L = 1$ in a P-wave configuration. The parity of the system is $P = (-1)^L = -1$ and requires the spatial part of the wave function of the $B^0 \bar{B}^0$ state to be antisymmetric. The Bose-Einstein statistics requires the overall wave function to be symmetric. Consequently, the flavor part of the wave function, denoted in the following by $\Psi$, needs also to be antisymmetric and can be written as

$$|\Psi(t_1, t_2)\rangle = \frac{1}{\sqrt{2}} \left( |B^0(t_1)\rangle |\bar{B}^0(t_2)\rangle - |\bar{B}^0(t_1)\rangle |B^0(t_2)\rangle \right).$$  \hspace{1cm} (2.96)$$

Consider one of the B mesons decays at time $t_1$ to the final-state $f_a$ and assume the second B meson decays at time $t_2$ to the final-state $f_b$, then the joint amplitude can be expressed as

$$\langle f_a f_b | \Psi(t_1, t_2) \rangle = \frac{1}{\sqrt{2}} \left( \langle f_a | B^0(t_1) \rangle \langle f_b | \bar{B}^0(t_2) \rangle - \langle f_a | \bar{B}^0(t_1) \rangle \langle f_b | B^0(t_2) \rangle \right).$$  \hspace{1cm} (2.97)$$

By defining the amplitudes

$$A_i := \langle f_i | B^0 \rangle \quad \text{and} \quad \bar{A}_i := \langle f_i | \bar{B}^0 \rangle,$$

and introducing the time evolution of the flavor eigenstates from Eqs. 2.58 and 2.59, the joint amplitude can be written as

$$\langle f_a f_b | \Psi(t_1, t_2) \rangle = \frac{1}{\sqrt{2}} \left\{ g_+(t_1) g_+(t_2) - g_-(t_1) g_-(t_2) \right\} \left( A_a \bar{A}_b - \bar{A}_a A_b \right)$$

$$+ \left\{ g_+(t_1) g_-(t_2) - g_-(t_1) g_+(t_2) \right\} \left( \frac{P}{q} A_a A_b - \frac{q}{p} \bar{A}_a \bar{A}_b \right) \right\}. \hspace{1cm} (2.99)$$

The time-dependent decay rate is obtained from the absolute square of the joint amplitude. By substituting the functions $g_+(t)$ and $g_-(t)$ defined in Eqs. 2.60 and 2.61, the decay rate is given by [29]

$$\Gamma[\Psi(t_1, t_2) \rightarrow f_a f_b] = \left| \langle f_a f_b | \Psi(t_1, t_2) \rangle \right|^2$$

$$= \frac{1}{2} e^{-\Gamma(t_1 + t_2)} \left\{ \left( |A_a|^2 + |\bar{A}_a|^2 \right) \left( |A_b|^2 + |\bar{A}_b|^2 \right) - 4 \Re \left( \frac{q}{p} A_a^* \bar{A}_a \right) \Re \left( \frac{q}{p} A_b^* \bar{A}_b \right) \right.$$

$$- 2 \sin[\Delta M(t_1 - t_2)] \left[ \Im \left( \frac{q}{p} A_a^* \bar{A}_a \right) \left( |A_b|^2 - |\bar{A}_b|^2 \right) - \left( |A_a|^2 - |\bar{A}_a|^2 \right) \Im \left( \frac{q}{p} A_b^* \bar{A}_b \right) \right]$$

$$\left. - \cos[\Delta M(t_1 - t_2)] \left[ \left( |A_a|^2 - |\bar{A}_a|^2 \right) \left( |A_b|^2 - |\bar{A}_b|^2 \right) + 4 \Im \left( \frac{q}{p} A_a^* \bar{A}_a \right) \Im \left( \frac{q}{p} A_b^* \bar{A}_b \right) \right] \right\}. \hspace{1cm} (2.100)$$
Equation 2.100 describes the time-dependent decay rate of an entangled $B^0 \bar{B}^0$ pair produced by the $\Upsilon(4S)$-resonance and decaying to the final-states $f_a$ and $f_b$. Assuming CPT invariance and no CP violation in the mixing equivalent to $|q/p| = 1$, the decay rate can be used to calculate the time-dependence of any neutral weak decay process.

The dynamics in the time evolution depends on the proper decay time difference of both B mesons, $t_1 - t_2$, and not on their sum, $t_1 + t_2$. This dependence is already implicitly included in the decay rates in the classic paper by I.I. Bigi and A.I. Sanda [30], predicting CP violation in the B meson system in 1987 (see the emphasised minus signs in Eq. 2.6 in Fig. 1.1(b)) and is the reason for the asymmetry in the energy of the beams at the $e^+e^-$-collider KEKB. The asymmetry causes a Lorentz boost of the $\Upsilon(4S)$, and thereby increases the spatial separation of the B meson decay vertices. The translation of measurements of the decay flight length differences with the known Lorentz boost into proper decay time differences allows to observe time-dependent CP violation in the B meson system. The principle of the time-dependent CP violation measurements is illustrated in Fig. 2.8.

Consider one of the neutral B mesons decays at time $t_{CP}$ to a CP eigenstate $f_{CP}$, for example $B^0 \to D^+D^-$. Assuming the second B meson decays at time $t_{tag}$ flavor-specific as a $B^0$ to $f_{tag}$, corresponding to $A_{tag} := A_{b} = \langle f_{tag}|B^0 \rangle$ and $A_{b} = 0$. By introducing $\lambda_{f_{CP}}$ defined in Eq. 2.69, the time-dependent decay rate can be written as
\[
\Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} f_{\text{tag}} \right] = \left| \langle f_{CP} f_{\text{tag}} | \Psi \left( t_{CP}, t_{tag} \right) \rangle \right|^2
\]

\[
= \frac{1}{2} e^{-\Gamma (t_{CP} + t_{tag})} |A_{CP}|^2 |A_{tag}|^2 \left[ (1 + |\lambda_{fCP}|^2) - 2 \text{Im}(\lambda_{fCP}) \sin \left( (\Delta M(t_{CP} - t_{tag})) - (1 - |\lambda_{fCP}|^2) \cos \left( (\Delta M(t_{CP} - t_{tag})) \right) \right] .
\]

(2.101)

In the opposite case, if the second B meson decays at time \( t_{tag} \) flavor-specific as a \( \bar{B}^0 \) to \( f_{\text{tag}} \), corresponding to \( A_{tag} := \bar{A}_h = \langle f_{\text{tag}} | \bar{B}^0 \rangle \) and \( A_p = 0 \), then the time-dependent decay rate can be written as

\[
\Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} \bar{f}_{\text{tag}} \right] = \left| \langle f_{CP} \bar{f}_{\text{tag}} | \Psi \left( t_{CP}, t_{tag} \right) \rangle \right|^2
\]

\[
= \frac{1}{2} e^{-\Gamma (t_{CP} + t_{tag})} |A_{CP}|^2 |A_{tag}|^2 \left[ (1 + |\lambda_{fCP}|^2) + 2 \text{Im}(\lambda_{fCP}) \sin \left( (\Delta M(t_{CP} - t_{tag})) + (1 - |\lambda_{fCP}|^2) \cos \left( (\Delta M(t_{CP} - t_{tag})) \right) \right] .
\]

(2.102)

Again in the decay rates the time-dependence of the dynamics originates from the proper decay time differences, denoted in the following by \( \Delta t = t_{CP} - t_{tag} \). The sine and cosine oscillations in the decay rates have opposite signs depending on whether the second B meson, referred to as \( B_{tag} \), decays as a \( B^0 \) or as a \( \bar{B}^0 \). Furthermore, the amplitudes of the sine and cosine oscillations provide observables sensitive to mixing-induced and direct CP violation according to the definitions in Eq. 2.92.

The time-dependent CP asymmetry \( A_{CP} (t) \) constructed from the time-dependent decay rates in Eqs. 2.101 and 2.102 is

\[
A_{CP} (\Delta t) = \frac{\Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} \bar{f}_{\text{tag}} \right] - \Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} f_{\text{tag}} \right]}{\Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} \bar{f}_{\text{tag}} \right] + \Gamma \left[ \Psi \left( t_{CP}, t_{tag} \right) \rightarrow f_{CP} f_{\text{tag}} \right]}
\]

\[
= S_{CP}^{\text{mix}} \sin (\Delta M \Delta t) - c_{CP}^{\text{dir}} \cos (\Delta M \Delta t) .
\]

(2.103)

Compared to the time-dependent CP asymmetry introduced in Eq. 2.94, one finds, that \( A_{CP} (t) \) has the same form, but depends on the proper decay time difference \( \Delta t \) of two B mesons in an \( \Upsilon (4S) \) event instead of the proper decay time of one B meson.

By integration over the experimentally not accessible sum of proper decay times, \( t_{CP} + t_{tag} \), and by normalisation to the possible proper decay time difference interval of \(-\infty < \Delta t < +\infty\), in an \( \Upsilon (4S) \) event the probability to find one B meson decaying to a CP eigenstate and the second B meson decaying flavor-specific with a proper decay time difference \( \Delta t \) is
Fig. 2.9  Theoretical (dashed lines) and experimental (solid lines) proper decay time difference distributions and the corresponding CP asymmetries for three different scenarios of mixing-induced CP violation $S$ and direct CP violation $C$

\[
P(\Delta t, q) = \frac{1}{4\tau_{B^0}} e^{-|\Delta t|/\tau_{B^0}} \left[ 1 + q \left( S_{CP}^{mix} \sin (\Delta m \Delta t) - C_{CP}^{dir} \cos (\Delta m \Delta t) \right) \right].
\]

where $q = +1 (-1)$ represents the $b$-flavor charge, when the second $B$ meson is tagged as $B^0$ ($\bar{B}^0$).

In Fig. 2.9 the distributions for $B^0$ and $\bar{B}^0$ tags and the corresponding CP asymmetries for three different scenarios of CP violation are shown.

- In the first scenario, a non-vanishing mixing-induced CP violation ($S \neq 0$) with no direct CP violation ($C = 0$) shifts and deforms the $\Delta t$ distributions for $B^0$ (red) and $\bar{B}^0$ (blue) tags horizontally. The resulting time-dependent CP asymmetry follows a sine oscillation.
- In the second scenario, a non-vanishing direct CP violation ($C \neq 0$) with no mixing-induced CP violation ($S = 0$) shifts the $\Delta t$ distributions vertically. The direct CP violation causes one decay to occur more or less frequently than that of its CP-conjugated process. The resulting CP asymmetry follows a cosine oscillation.
- In the third scenario, a non-vanishing mixing-induced and a non-vanishing direct CP violation, ($S \neq 0$ and $C \neq 0$) are present. The $\Delta t$ distributions are shifted and deformed both, horizontally and vertically, and the resulting CP asymmetry is a superposition of sine and cosine oscillations.

The CP violation measurements are performed at the Belle experiment as illustrated in Fig. 2.8. One experimental difficulty is to resolve the proper decay time differences. Due to the low Lorentz boost, the resolution in the measurements of the decay flight length difference of the two $B$ mesons is in the same order as the average
flight length difference. The effect of the resolution is visible when comparing the theoretical (dashed lines) and experimental (solid lines) $\Delta t$ distributions in Fig. 2.9. In addition, the experimental assignment of the flavor of the $B_{tag}$ meson from its decay products cannot be perfect. The uncertainties in the flavor tagging decision and possible mistags affect the measurement of the sine and cosine time-dependence and effectively result in a reduction of the measured amplitudes. This effect is visible in the time-dependent CP asymmetries in Fig. 2.9: the amplitudes of the experimental oscillations are lower than the theoretical ones.

### 2.6 CP Violation in $b \rightarrow c\bar{c}s$ Transitions

In the $B$ meson system CP violation has been observed for the first time in $b \rightarrow c\bar{c}s$ transitions such as in $B^0 \rightarrow J/\Psi K_S^0$ decays [31, 32]. This decay is referred to as the golden mode for the determination of the angle $\beta$ of the Unitarity Triangle and is discussed in the following.

In $B^0 \rightarrow J/\Psi K_S^0$ decays the $J/\Psi$ and the $K_S^0$ are produced with orbital angular momentum of $L = 1$ in a P-wave configuration. Neglecting the small CP violation in $K^0-K^0$ mixing, the $K_S^0$ has like the $J/\Psi$ a CP eigenvalue of $+1$. In combination with the $(-1)^{L=1} = -1$ contribution from the angular momentum, the $J/\Psi K_S^0$ final-state has the CP eigenvalue $\eta_{CP} = -1$.

The tree-level and penguin diagrams contributing to $B^0 \rightarrow J/\Psi K_S^0$ decays are shown in Fig. 2.10. On tree-level, the decay is mediated by $b \rightarrow c\bar{c}s$ transitions that are color-suppressed due to the internal emission of a $W$ boson. The penguin diagrams have an internal loop with virtual up-type quarks. The loops involving virtual $\bar{c}$ and $\bar{t}$ quarks have CKM matrix elements of the same order as the transitions in the tree diagram and carry the same weak phase. In contrast, the loop involving virtual $\bar{u}$ quarks, that could introduce a different weak phase via $V_{ub}$, is suppressed by a factor of $\lambda^2$, where $\lambda \approx 0.23$ refers to the expansion parameter in the Wolfenstein parameterisation. In the penguin diagrams, the $c\bar{c}$ pair forming the $J/\Psi$ is created from gluons. Since the $c\bar{c}$ pair has to be created in a color singlet state, it cannot be created from a single gluon. All above effects result in a high suppression of the penguin amplitudes. No so-called penguin pollution and hence no direct CP violation are expected to emerge in $b \rightarrow c\bar{c}s$ transitions.

The mixing-induced CP violation in $B^0 \rightarrow J/\Psi K_S^0$ decays can be estimated from the involved decay amplitudes and mixing-phases. The parameter $\lambda_{J/\Psi K_S^0}$ defined in Eq. 2.69 characterises the mixing-induced CP violation explained in Sect. 2.4.3. It depends on the ratio $q/p$ which introduces the $B$ mixing phase and the amplitudes of the CP-conjugated decays. Since $B^0$ mesons decay to $J/\Psi K^0$, but $\bar{B}^0$ mesons decay to $J/\Psi \bar{K}^0$, the $J/\Psi K_S^0$ final-state involves $K^0-\bar{K}^0$ mixing and $\lambda_{J/\Psi K_S^0}$ includes also the neutral kaon mixing phase. The parameter $\lambda_{J/\Psi K_S^0}$ can then be expressed as
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\[ \lambda_{J/\Psi K^0_S} = \left( \frac{q}{p} \right) \frac{\bar{A}}{A} \frac{q}{p} \frac{q}{p} \]

\[ = \eta_{CP} \frac{V_{ub}^* V_{td}}{V_{ub}^* V_{td}} \frac{V_{cb} V_{cs}^*}{V_{cb} V_{cs}^*} \frac{V_{cd} V_{cs}}{V_{cd} V_{cs}} \]

\[ = \eta_{CP} e^{-2i\beta}, \quad (2.105) \]

where \( \beta \) refers to the angle of the Unitarity Triangle defined in Eq. 2.45.

Because of \( S_{J/\Psi K^0_S}^{mix} = -\eta_{CP} \sin (2\beta) \), the resulting time-dependent CP asymmetry in \( B^0 \rightarrow J/\Psi K^0_S \) decays is

\[ A_{CP} (\Delta t) = -\eta_{CP} \sin (2\beta) \sin (\Delta M \Delta t). \quad (2.106) \]

Thus the angle \( \beta \) of the Unitarity Triangle can be determined by time-dependent CP asymmetry measurements of \( B^0 \rightarrow J/\Psi K^0_S \) decays. The same arguments also hold for other \( b \rightarrow c \bar{c}s \) transitions, for example \( B^0 \rightarrow J/\Psi K^0_L \) decays with \( \eta_{CP} = +1 \).

The current world average of \( \sin (2\beta) \) from \( b \rightarrow c \bar{c}s \) transitions is

\[ \sin (2\beta) = 0.68 \pm 0.02, \quad (2.107) \]

---

**Fig. 2.10** Tree-level (top) and penguin diagrams (bottom) contributing to \( B^0 \rightarrow J/\Psi K^0_S \) decays.
which corresponds to a value of $\beta = (21.4 \pm 0.8)^\circ$ [33]. Due to the high precision in $b \to c\bar{c}s$ transitions achieved by the B-factory experiments BaBar and Belle, the $\sin(2\beta)$ result provides a Standard Model reference and contributes to put tight constraints on the CKM metrology, e.g. on the Unitarity Triangle shown in Fig. 2.3.

### 2.7 CP Violation in $b \to c\bar{c}d$ Transitions

The Feynman diagrams contributing to $B^0 \to D^{+}D^-$, $D^{*+}D^-$, $D^{*-}D^+$ and $D^{*+}D^{*-}$ decays, referred to as neutral B meson to double-charm decays, are shown in Fig. 2.11. On tree level, the decays are mediated by Cabibbo-disfavored, but color-allowed $b \to c\bar{c}d$ transitions. In addition to the tree-level diagrams, $b \to d$ penguin diagrams can contribute to the decays. The CKM matrix elements of the penguin amplitudes are of same order of magnitude as the elements of the tree-level amplitudes, but can carry different weak phases.

In contrast to $B^0 \to D^{+}D^-$, $D^{*+}D^-$ and $D^{*-}D^+$ decays, $B^0 \to D^{*+}D^{*-}$ is the decay of a pseudo-scalar meson to two vector mesons and involves a mixing of CP-even and CP-odd states which can be disentangled by an angular analysis. The measurements of $B^0 \to D^{*+}D^{*-}$ decays are not part of this work and have been carried out in parallel by the Belle Collaboration [34, 35]. The present analysis covers the measurement of branching fractions and time-dependent CP violation in $B^0 \to D^{+}D^-$, $D^{*+}D^-$ and $D^{*-}D^+$ decays. These decays are discussed in the following.

![Feynman diagrams](image)

**Fig. 2.11** Tree-level (top) and penguin diagrams (bottom) contributing to $B^0 \to D^{(*)+}D^{(*)-}$ decays
2.7.1 CP Violation in B^0 → D^+D^- Decays

The D^+D^- configuration is a CP eigenstate with eigenvalue \( \eta_{\text{CP}} = +1 \). The total decay amplitude of the \( B^0 \rightarrow D^+D^- \) decay involves contributions from tree-level amplitudes \( (A_T) \) and penguin amplitudes \( (A_p^q \text{ with } q \in \{u, c, t\}) \) and can be written as [36, 37]

\[
A \left( B^0 \rightarrow D^+D^- \right) = \lambda_c (A_T + A_p^c) + \lambda_u (A_p^u + \lambda_t (A_p^t), \tag{2.108}
\]

where \( \lambda_q = V_{qd}V_{qg}^* \).

By introducing the relative strong phase \( \delta \) and the relative weak phase \( \gamma \) between the decay amplitudes, the total decay amplitudes can be rewritten as

\[
\bar{A} \left( B^0 \rightarrow D^+D^- \right) = A \left[ 1 - r \cos (\delta + \gamma) \right], \tag{2.109}
\]

\[
\tilde{A} \left( B^0 \rightarrow D^+D^- \right) = A \left[ 1 - r \sin (\delta - \gamma) \right], \tag{2.110}
\]

where \( A \) refers to the CP conserving amplitude governed by the tree-level contributions and the ratio \( r \) measures the suppression of the penguin amplitudes with respect to the tree amplitude.

According to

\[
\lambda_{D^+D^-} = e^{-2i\beta} \frac{\tilde{A} \left( B^0 \rightarrow D^+D^- \right)}{A \left( B^0 \rightarrow D^+D^- \right)}, \tag{2.111}
\]

the CP violation in \( B^0 \rightarrow D^+D^- \) decays is determined by the expressions

\[
S_{\text{mix}}^{D^+D^-} = \frac{2 \text{Im} (\lambda_{D^+D^-})}{1 + |\lambda_{D^+D^-}|^2} = - \frac{\sin (2\beta) + 2r \cos (\delta) \sin (2\beta + \gamma) + r^2 \sin [2(\beta + \gamma)]}{1 + 2r \cos (\delta) \cos (\gamma) + r^2} \approx - \sin (2\beta) - 2r \cos (2\beta) \cos (\delta) \sin (\gamma), \tag{2.112}
\]

\[
S_{\text{dir}}^{D^+D^-} = \frac{1 - |\lambda_{D^+D^-}|^2}{1 + |\lambda_{D^+D^-}|^2} = - \frac{2r \sin (\delta) \sin (\gamma)}{1 + 2r \cos (\delta) \cos (\gamma) + r^2} \approx -2r \sin (\delta) \sin (\gamma). \tag{2.113}
\]

In the case of negligible penguin contributions, the direct CP violation vanishes and the mixing-induced CP violation in \( B^0 \rightarrow D^+D^- \) decays is exactly \( S_{\text{mix}}^{D^+D^-} = - \sin (2\beta) \). This case is equivalent to the CP violation in the golden modes involving \( b \rightarrow c \bar{c} s \bar{s} \) transitions such as in \( B^0 \rightarrow J/\psi K_s^0 \) decays discussed in Sect. 2.6.

In the case of sizeable penguin amplitudes, the mixing-induced CP violation is shifted with respect to \( - \sin (2\beta) \) depending on the magnitude of the ratio \( r \). Additionally, in combination with a non-vanishing relative strong phase a non-zero direct CP violation can emerge.
2.7 CP Violation in $b \to c\bar{c}d$ Transitions

The calculation of the contributions of penguin amplitudes are associated with difficulties due to hadronic effects. The decays involve charm quarks in intermediate and final-states which give rise to long-range interactions. This complicates certain theoretical approaches such as calculations based on factorization approximations. The current theoretical predictions on mixing-induced and direct CP violation in $B^0 \to D^+D^-$ decays are associated with large uncertainties. Standard Model conform model-dependent and model-independent predictions for the ratio $r$ corresponding to the suppression of the penguin contributions range from values as low as 0.03 [38] to upper bounds of 0.3 [39–41]. Even larger contributions can be accommodated in extensions of the Standard Model [42].

CP violation in $b \to c\bar{c}d$ transitions has been previously studied by both, the Belle and the BaBar Collaborations. In 2007, Belle using a data set of $535 \times 10^6$ $B\bar{B}$ pairs has found evidence of a large direct CP violation in $B^0 \to D^+D^-$ decays:

$$C_{D^+D^-} = -0.91 \pm 0.23 \text{ (stat.)} \pm 0.06 \text{ (syst.),}$$

(corresponding to a $3.2 \sigma$ deviation from zero [43]. This deviation has not been confirmed by BaBar and has not been observed in other $B^0 \to D^{*\pm}D^{(*)\mp}$ decay modes [44–46].

The objective of the present analysis is the measurement of the branching fraction and time-dependent CP violation in $B^0 \to D^+D^-$ decays using the final Belle data set of $772 \times 10^6$ $B\bar{B}$ pairs. Furthermore, to provide a comparison to a directly related decay, the analysis covers additionally the measurement of $B^0 \to D^{*\pm}D^{(*)\mp}$ decays.

2.7.2 CP Violation in $B^0 \to D^{*\pm}D^{\mp}$ Decays

The $B^0 \to D^{*\pm}D^{\mp}$ decays allow to probe independently from $B^0 \to D^+D^-$ decays for penguin effects in double-charm decays. Unlike $D^+D^-$, the $D^{*+}D^-$ and $D^{*-}D^+$ configurations are not CP eigenstates. Both states are accessible from $B^0$ and $\bar{B}^0$ with decay amplitudes of comparable magnitudes. Therefore an interference between direct decays and decays following $B^0-\bar{B}^0$ oscillations emerges, and effects of mixing-induced and direct CP violation in analogy to those in $B^0 \to D^+D^-$ decays are expected.

The time-dependent decay rate for $B^0 \to D^{*\pm}D^{\mp}$ has four flavor-charge configurations and can be written as

$$f_{D^*\pm D^\mp}(\Delta t) = \left(1 \pm A_{D^*D} \right) \frac{e^{-|\Delta t|/\tau_{B^0}}}{8 \tau_{B^0}} \times \left\{ 1 + q \left[ S_{D^*\pm D^\mp} \sin(\Delta m \Delta t) - C_{D^*\pm D^\mp} \cos(\Delta m \Delta t) \right] \right\}.$$ (2.114)

Generally, the parameters $S_{D^*\pm D^\mp}$ and $C_{D^*\pm D^\mp}$ for the $D^{*+}D^-$ and $D^{*-}D^+$ configurations are not independent, but related by $S_{D^*\pm D^\mp} = -\sqrt{1 - C_{D^*\pm D^\mp}^2} \sin(2\beta_{\text{eff}} \pm \delta)$ [47]. The expression includes the sensitivity on the angle $\beta$, which can be modified
by penguin contributions to the effective angle $\beta_{\text{eff}}$, and on the relative strong phase $\delta$ between the $B^0 \to D^{*+}D^-$ and $B^0 \to D^{*-}D^+$ decay amplitudes.

The time- and flavor-integrated asymmetry $A_{D^*D}$ is defined by

$$A_{D^*D} = \frac{N_{D^{*+}D^-} - N_{D^{*-}D^+}}{N_{D^{*+}D^-} + N_{D^{*-}D^+}}, \quad (2.115)$$

and measures direct CP violation in $B^0 \to D^{*\pm}D^{\mp}$ decays.

The decay rate in Eq. 2.114 can be expressed in an equivalent parameterisation given by [48]

$$S_{D^*D} = \frac{1}{2} (S_{D^{*+}D^-} + S_{D^{*-}D^+}),$$

$$C_{D^*D} = \frac{1}{2} (C_{D^{*+}D^-} + C_{D^{*-}D^+}),$$

$$\Delta S_{D^*D} = \frac{1}{2} (S_{D^{*+}D^-} - S_{D^{*-}D^+}),$$

$$\Delta C_{D^*D} = \frac{1}{2} (C_{D^{*+}D^-} - C_{D^{*-}D^+}). \quad (2.116)$$

In this parameterisation, the decay rate of $B^0 \to D^{*\pm}D^{\mp}$ decays can be written as

$$f_{D^{*\pm}D^{\mp}}(\Delta t) = (1 \pm A_{D^*D}) \frac{e^{-|\Delta t|/\tau_{B^0}}}{8 \tau_{B^0}} \left[ 1 + q \left( (S_{D^*D} \pm \Delta S_{D^*D}) \sin(\Delta m\Delta t) 
- (C_{D^*D} \pm \Delta C_{D^*D}) \cos(\Delta m\Delta t) \right) \right], \quad (2.117)$$

where $S_{D^*D}$ parameterises mixing-induced and $C_{D^*D}$ flavor-dependent direct CP violation. The parameters $\Delta S_{D^*D}$ and $\Delta C_{D^*D}$ are not sensitive to CP violation. The parameter $\Delta S_{D^*D}$ is related to the relative strong phase $\delta$ between the decay amplitudes and $\Delta C_{D^*D}$ describes the asymmetry between the rates $\Gamma(B^0 \to D^{*+}D^-) + \Gamma(B^0 \to D^{*-}D^+)$ and $\Gamma(B^0 \to D^{*+}D^-) + \Gamma(B^0 \to D^{*+}D^-)$.

If the contributions of penguin amplitudes to $B^0 \to D^{*\pm}D^{\mp}$ decays are negligible, and the relative strong phase between the $B^0 \to D^{*+}D^-$ and $B^0 \to D^{*-}D^+$ decay amplitudes is zero and their magnitudes are the same, then

$$A_{D^*D} = 0, \quad S_{D^{*+}D^-} = S_{D^{*-}D^+} = -\sin(2\beta), \quad C_{D^{*+}D^-} = C_{D^{*-}D^+} = 0, \quad (2.118)$$

or equivalent

$$A_{D^*D} = 0, \quad S_{D^*D} = -\sin(2\beta), \quad C_{D^*D} = \Delta S_{D^*D} = \Delta C_{D^*D} = 0. \quad (2.119)$$

In this case, the time-dependent CP violating asymmetry in $B^0 \to D^{*\pm}D^{\mp}$ decays measures directly $-\sin(2\beta)$. 

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