Sandglasses are simple devices, but you better think twice before crafting your own. A receipt from 1339 prescribes the sand to be made of “ground black marble, boiled nine times in wine” (presumably to etch away nicks and burrs and leave nicely rounded grains). Lead-containing sands are praised for their auto-lubricating features. Instructions on shaping and finishing the waist sections for smooth flow and least wear are more demanding still. Aspiring watchmakers had to prove their ability to correctly size and shape those sections for full hour, half-hour and quarter-hour models.

Like in Egyptian water clocks, “half the sand down” does not mean that half of the clock’s period has passed. Each sand-clock measures its particular time span, but no fractions of it. That led to brainteasers such as how to measure 16 minutes with a pair of hourglasses of, respectively, 7 and 12 minutes runtime? (Solution: Let both complete one run and turn them over. After 14 minutes, the 7 minutes glass has completed two turns, and the 12 minutes glass is 2 minutes into its second turn. Invert it right away and it will run for the two minutes left to complete the desired sixteen).

The world’s tallest sandglass is reportedly found in the Japanese Sand Museum in the city of Nima. It stands six meters tall and runs for an entire year. At least as interesting as the clock itself should be the physique of the man in charge of flipping it at year’s end.

The variety of time-keeping devices which preceded the event of a mechanical clock doesn’t necessarily mean that the idea didn’t come up earlier. Rather, as shown for the “Walgeuhr” of 1615, it was that weight-driven mechanisms tend to run at accelerated pace, because the energy, fed into the clockwork by a descending weight, accumulates and makes the weight gain speed. For a rate of gain of, say, one foot per second for every inch-pound of energy input, the final velocity at time t becomes $v = 1t$ and the distance covered $s = \frac{1}{2}t^2$, a parabolic progression. To make a weight descend at an unchanging speed, the inflow of energy must be throttled.

We could get rid of excess energy by consistent braking, such as by air friction acting on a fast rotating paddle wheel. But since their consumption of energy varies with air temperature and barometric pressure, such devices are found for controlling the striking mechanism in classical pendulum clocks rather than the clockwork itself.

The proverbial “quantum leap” in horology came with the control of clockworks by oscillating motion, commanded either by gravitational or inertial forces.

Mechanical clockworks

No other than Gerbert d’Aurillac, the later Pope Silvestre II, is credited with the introduction, in AD 996, of mechanical clockworks. But it took until 1344 for the Italian astronomer Giovanni Dondi to come up with a weight-driven, swing weight regulated clock; surprisingly, Dondi controlled the pace of his clock with the inertial forces of a horizontally swinging bar rather than the gravitational forces that keep a pendulum in motion.

Inertial forces surge in any piece of mass being accelerated or slowed down: in speeding up or braking a car; a test driver's maneuvering his car through a slalom; the kickoff of a dragster. Although inertial forces can have any direction in space, their vertical components are often overshadowed by gravitation and we perceive principally their effects in the horizontal plane.

In 1973, measurements of astronauts' body mass through inertial forces in the weightless environment inside Skylab, the first American space station, confirmed a host of so far earthbound discoveries, such as the proportionality of the period T of a torsion pendulum and the square root of its mass.

The foliot escapement

Giovanni Dondi's inertial force escapement (Fig. 3.11) employs the *foliot*, an end-weighted swing bar, to stabilize the pace of the clockwork. The center shaft of the foliot, called the *verge*, hangs suspended from the clock's frame by a rope, preferably spun from silk, to minimize friction. Torque from the weight driven pinwheel moves the foliot forward and backward by alternately pushing and releasing one or the other in a pair of dogs on the shaft. Thus, the pin that engages with the upper pallet induces the verge to move clockwise, and the one engaging the bottom pallet acts in the opposite direction. Set at a wide enough angle relative to each other (about 100°), only one pallet at a time is being driven, resulting in the alternate motion of the verge.

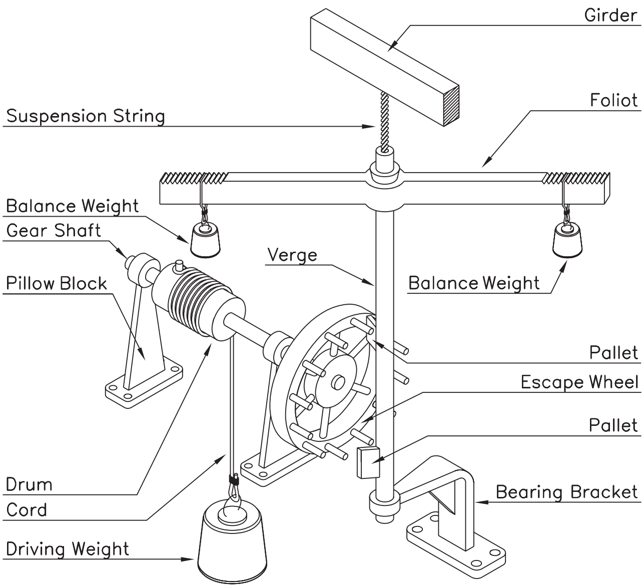

Fig. 3.11. Dondi's inertial escapement

The shortcoming of the foliot escapement is its swinging in forced mode, as the frequency of the balance bar is commanded by the torque of the pinwheel. By contrast, the swing-mass in modern clocks is spring loaded and allowed to oscillate at the system's natural frequency with the period of $T = 2\pi\sqrt{I/c}$, where I stands for the balance wheel's moment of inertia, and c for the torsional constant of the return spring.

Nevertheless, foliot clocks remained standard for about three hundred years. In 1577, Jost Bürgi's introduction of the minute hand in a clock for the Danish astronomer Tycho Brahe improved the display accuracy 12-fold. Even John Harrison (1693–1776), widely considered the most brilliant horologist of all times, still used a verge escapement with a balance wheel in his precision timepiece "H 4". Applied to the measurement of longitude, it won him part of a 20,000 pound sterling Longitude Prize from the British government.

Pendulum escapement

Real improvement came with Galileo Galilei's discovery of the laws of motion for the mathematical pendulum in 1582, and with them the "isochronism of the pendulum," i.e., the independence of a pendulum's period from the amplitude of its swing. Yet it took him until 1637 to model a pendulum-controlled clockwork.

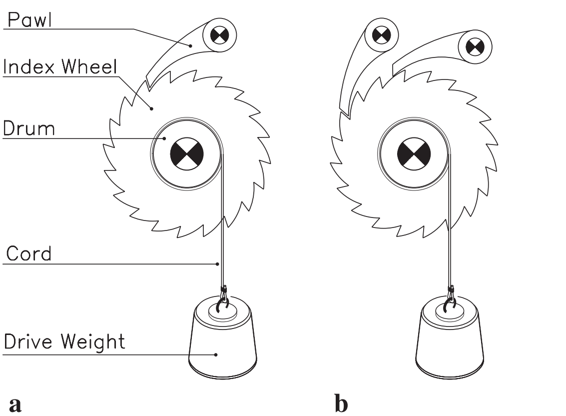

Fig. 3.12. Ratchet drive

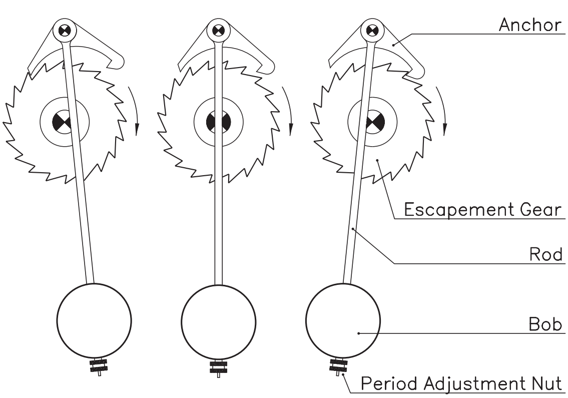

Fig. 3.13. Pendulum escapement

Ratchets, as used in hoisting mechanisms to keep the load from back-driving the drum (Fig. 3.12 a), may be seen as the harbingers of the pendulum escapement. A saw tooth gear with a pivoted pawl can turn in one direction only, but spins freely if the pawl is lifted. In the two pawls mechanism (Fig. 3.12 b), momentary lifting of the pawl on the left allows the gear to advance by half of a pitch, and then engage with the pawl on the right. A short disengagement of the latter causes another advance by half a pitch before the gear gets stopped by the left pawl. This basic mechanism resurfaced quite recently in tuning fork clockworks.

In a pendulum escapement (Fig. 3.13), the left and right pawls are combined in a single unit, the pallet-arm or anchor, which releases one tooth while it blocks the advance of the other, half a pitch further on. Each swing of the pendulum to and fro allows for one tooth advance of the escapement gear.

But that tells only half of the story. While the weight's gravitational force sustains the gear's stepwise advance, the pendulum itself needs an impulse every time it changes direction, much like a swing needs a push every time it comes back if you want the fun to go on. But instead of an external driving force, such as the swing pusher, the pendulum clock derives such impulses from the gear's stop-and-go anchor that combines the two pawls in Fig. 3.12 b into one single unit, shaped and located so that inertial forces from the process of stopping the advance of the weight and the drum mount up to a jolt on the anchor and with it, the pendulum.

For higher accuracy, modern clockworks relieve the anchor bearings from carrying the pendulum's weight by hanging it separately from a thin strip of flat spring steel. The blade takes care of the load of anchor and pendulum, and virtually eliminates the inherent frictional losses of conventional bearings, since the energy for flexing the strip to one side is recovered by elastic forces on the way back.

The superior performance of pendulum clocks can be traced to the equations describing the "mathematical pendulum" – a bob on a weightless string. With L for the distance between swivel point and the center of mass of the bob, a mathematical pendulum's period, from left to right and back, is given by

$$T = 2\pi\sqrt{L/g}$$

for small amplitude oscillation, where the approximation $\sin\theta \approx \theta$ is applicable.

Of the three principal parameters of a pendulum, length, mass of the bob, and the amplitude θ of the pendulum's oscillations, this formula contains only one, namely, the length $L = (T^2 g)/(4\pi^2)$ as determinant for the pendulum's period. This independence from two out of three parameters makes the pendulum the ideal timekeeper, as long as it swings within narrowly set limits. Faithful to this condition, Grandfather's clocks' very slow swinging pendulums (usually $T = 2$ seconds) with length of $L = 2^2 \times 9.80665/4\pi^2 = 0.9936$ meter preclude wide swing angles anyway. 0.9936 m is known as the length of the "One second pendulum," which takes one second for the stroke from left to right or vice versa.

Nevertheless, other clock designs, such as the classical Black Forest Clock, use deflection angles far beyond the $\sin\theta \approx \theta$ limits. Under those conditions, any change in the pendulum's stroke causes a change of period, according to

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta}{2} + \frac{9}{64}\sin^4\frac{\theta}{2} + \dots\right)$$

The formula contains the basic amplitude formula, $T = 2\pi\sqrt{L/g}$, multiplied by the bracketed infinite progression, which accounts for the effects of greater amplitudes.

But there is always a better way. In 1673, the Dutch astronomer Christian

Huygens developed an amplitude-independent pendulum by sustaining the string between a mirrored pair of cycloid-shaped metal jaws. The wider the swing, the more of the string's length snuggles along the jaws' perimeter, and the effective length of the pendulum gets reduced.

The cycloid is easily imagined as the curve that a chalk mark on the perimeter of a tire describes with the vehicle rolling. But figuring the specific cycloid that reflects the infinite progression in the exact formula for *L* is something our mathematical forefathers deserve respect for.

Notwithstanding his advanced theories on free swinging pendulums, Huygens' first pendulum clock used a foliot type escapement to keep up the beat. Only the invention of the anchor escapement in 1660 by William Clement did away with the need for the sustained swings of the old foliots and opened the doors for a regulating element operating at its particular frequency of resonance.

We instinctively keep a play-swing in the state of resonance by applying a push at the very moment it changes direction. How far out the swing goes depends on the power we use, but we have little control on the time it takes for coming back for another push. By contrast, the pistons of an internal combustion engine, such as the one in your car, swing in forced mode, dictated by the rpm of the motor. Here is the difference: An engine can be run at any speed, while a swing operates at an unchangeable period.

Still, the use of the escapement to rekindle the pendulum's swing makes for an interdependence detriment to the accuracy of the mechanism. That led in 1721 to George Graham's "dead beat escapement", which moves the point of attack of the impulse maintaining the pendulum's motion from the reversal points to the point of passage through the vertical.

But even the dead beat escapement drains momentum by the amount needed to keep up the pendulum's oscillations. This introduces a certain degree of irregularity into the rhythm of the mechanism, which at the time was well known to clockmakers and made the search for "free swinging pendulum" mechanisms continue throughout the nineteenth century. Still in 1921, W. H. Shortt demonstrated a clockwork controlled by a master and a slave pendulum; the former for "keeping the beat," the latter for driving the clock's hands and providing the impulses that keep the master pendulum swinging.

Yet, atmospheric conditions, such as temperature and barometric pressure, still influenced the pace of pendulum clocks. For instance, a pendulum with a shaft of steel loses one second per day for every two degrees rise in temperature. That inspired George Graham in 1721 to use a jar of mercury for a bob. The mercury's upward expansion with rising temperatures compensated for the pendulum's downward increased length. For obvious reasons however, this idea never made it into the bestsellers list. The gridiron pendulum, invented in 1726 by Harrison, employs a shaft made from metals of markedly different coefficients of thermal expansion, such as steel and brass, mounted in such a pattern that the elongation of one leg compensates for that of the other.

Controlling the effects of barometric pressure proved a more daunting task. In principle, clocks could be sealed into pressure-controlled enclosures, but that remained a pipedream until the 19th century brought the availability of small enough electric motors for winding the clockwork within the confines of such an encasing.

Balance wheel escapement

The need for a portable timepiece led to the development of spring-driven clocks with balance wheel escapement, which work in any position and can be wound without stopping the clock. The balance wheel appeared around 1400. But no sooner than in 1674 introduced Christian Huygens the essential element, a spiral escapement return spring, often called hairspring, to step in for the role gravitation plays in pendulum clocks. The proper (resonance) frequency of spring-powered balance wheels occurs at $T = 2\pi\sqrt{I/c}$, compared to $T = 2\pi\sqrt{L/g}$ for the pendulum. By analogy, I, the moment of inertia of the balance wheel, substitutes for L, the length of the pendulum; and c, the torsion constant of the return spring, takes the place of gravitational acceleration g.

An essential step in the development of a free swinging balance wheel escapement was Robert Hooke's 1666 introduction of an intermediate element, the *lever*, as the link between escape wheel and the oscillating assembly. As shown in Fig. 3.14, the lever is kept immobile during most of its swing while its guide pin slides along the circular section of the hub of the balance wheel. Only when the guide pin dips into a recess in the hub can the lever flip and let the escape wheel advance by half a tooth. This intermediate period of stalling the anchor allows the balance wheel to swing through angles as high as 330° to 360° with periods of typically 1/4 to 1 seconds (Fig. 3.14 b). As a further innovation came pellets of low-friction metal inserted at the ends of the anchor.

For temperature compensation, the most simple of a number of possible approaches consists in making the rim of the balance wheel of steel and connect it to the hub with two spokes of brass. When temperature rises, the high thermal expansion of brass relative to steel (19 to 11) distorts the

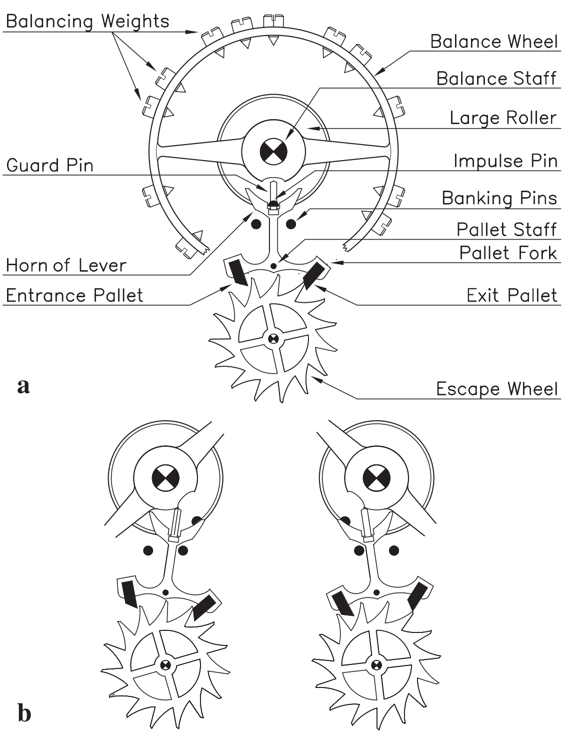

Fig. 3.14. Balance wheel escapement

rim into an oval, which lesser moment of inertia compensates for the balance wheel's expansion. The balance screws along the perimeter of the balance wheel allow for fine tuning the oscillations.

To understand why the moment of inertia of an elliptic ring is less than that of a circular one, we consider the extreme case of a ring of radius R and circumference $2\pi R$ being compressed into a straight piece of length L and circumference $2L$. This gives $L = 2\pi R/2 = \pi R$ for the length of the squeezed ring.

The moment of inertia of a ring of mass m is mR^2, and for a bar $(mL)^2/12$, in our case $(\pi^2/12)mR^2$. From this, we get the ratio of the moments of inertia of circular to compressed ring as $1 : \pi^2/12 = 1 : 0.822$, indicating an up to 18% reduction in the ring's moment of inertia.

Spring-driven clockworks

Once the design of the balance wheel escapement had been established, further development went in step with the advance of technology in general. The story of Nuremberg's machinist Peter Henlein (1480–1542) and his first portable timepieces in 1504 is well known. Unlike later periods' pocket watches, which had to be wound every 24 hours, Henlein's kept going for forty hours on one winding. They had only one hand, showing the hour.

Henlein had to forge the mainspring by hand, a job that in later times went to hot and cold rolling mills. In the process, he must have dealt with a host of metallurgical problems, such as slag and sand inclusions and inconsistencies in the steel's chemical composition.

The principal difference between iron and steel is in carbon content, 2% to 5% in cast iron, but only 0.2% in soft steel. Steels with 0.33% C and up are heat temperable, and carbon contents between 0.9% and 1.25% yield spring steel. Nowadays, chemical and spectral analyses allow for controlling the minute changes in carbon content crucial for the steels' properties, but sixteenth-century machinists and locksmiths had to rely on their instinct in selecting, heat treating, and forging the right slabs for their delicate work.

Heat treatment is not the only way of hardening steel. Work hardening occurs by repeatedly cold rolling or persistent hammering of steel and nonferrous metals not temperable otherwise. Spring makers could choose between the risks of heat treatment, including accidental overheating, and cracks and distortion from chilling; or they could rely on work hardening, which then involved long hours of swinging the hammer.

Understandably, Henlein's spiral springs weren't as thin and long as those in modern clocks, and their torque varied according to their state of unwinding. So much so, that a special element, the *Stackfreed*, was developed to compensate for such discrepancies. Most chronicles credit Peter Henlein with the invention of this device, which consists of an eccentric cam and a spring-loaded roller follower. The torque of the mainspring was supposed to ebb down in step with the pressure exerted by the roller and the resultant frictional braking – strongest at the high point of the cam and gradually decreasing with the cam's edging toward the center.

Electric clock

While mechanical clockworks improved with the availability of increasingly purer raw materials and refined methods of metal finishing, the age of electricity bore a new type of clock with the 60 cycles per second period of alternating current as timebase, much like the synchronous electric motor, which in its basic layout turns at $60 \times 60 = 3600$ rotations per minute.

Likewise, electric clockworks are driven by small synchronous gear motors. A 1 : 3600 gear reduction gives the 1 turn per minute for driving the seconds hand, and a subsequent 1 : 60 reduction makes the minutes hand advance by one full turn per hour. And finally, a 1 : 12 reduction controls the hour hand.

An alternative is in the use of four and eight pole motors with 1800 and 900 rpm respectively, combined with gear trains of proportionally lesser rates of reduction.

What made synchronous clocks practical is the close control power plants keep on the frequency of their AC electricity. However, they must be reset after power failures, and American made clocks cannot be used on European standard 50 Hz networks.

Battery powered clocks would be free of such problems, but the speed of rotation of DC motors changes with variations of load and voltage. Thus, it took the invention of the transistor to make motor controllers for network independent clockworks feasible.

Tuning fork ratchet control

Tuning forks, traditionally used for attuning musical instruments, such as pianos, produce sinusoidal sound waves at their highly stable resonance frequency, while most musical instruments blend their fundamental frequency with harmonics.

For instance, striking an "A" on a violin produces sound of 440 Hz (2.5 feet of wavelength) along with overtones of 880, 1320 Hz, etc. The human auditory system hears each of those frequencies separately, but the brain recombines the overtones into what we call timbre – a unique characteristic of each particular category of instrument. On the screen of an oscilloscope, this mélange shows as a complex waveform.

The electrical clockwork in Fig. 3.15 shows a tuning fork with the core of a solenoid mounted to the ends of its legs. Unusual is the cores' tapered shape, selected to make the air gap between core and coil the narrower the deeper the core moves into the coil. This compensates for the loss of the solenoid's drag when the magnetic center of the core approaches the magnetic center of the coil. The solenoid's pull would drop to zero when they coincide.

Further, the narrowing air gap balances the leg's mechanical resistance to bending, which increases with amplitude.

Magnet cores are pressed and subsequently sintered from powdered metals, such as iron, cobalt, nickel, vanadium, and rare metals (niobium), which makes them many times more powerful than conventional horseshoe magnets of soft steel. Associated with the core is the ferromagnetic mantle that guides the magnetic flux lines into enclosing the coil all around.

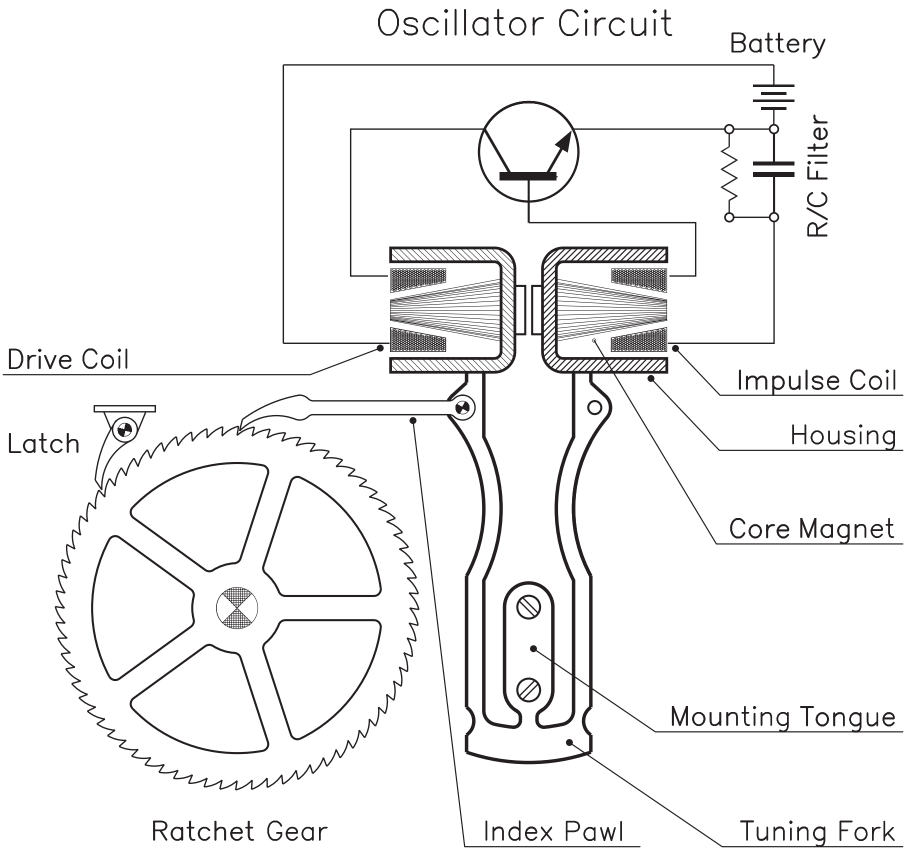

Fig. 3.15. Tuning fork controlled clockwork

The device is powered by a transistorized L/C oscillator circuit with feedback from the coil, shown in Fig. 3.15, right. This leg swings freely at the fork's resonance frequency and induces in its coil the electric pulses that keep the left side leg playing along and drive the ratchet wheel.

The conversion of the fork's vibrations into the motion of the hands happens "the old fashioned way," similar to Fig. 3.12. Not surprisingly, the sawtooth gear is of very fine pitch as to accommodate the minute amplitude of the fork's oscillations. Tuning fork controlled watches have reportedly been made to hold time within two seconds per day.

Crystal-controlled watches

The idea of crystal-controlled oscillator circuits reaches back to the early days of radio technology, when they became the basic elements for stabilizing the frequency of transmitters. If it weren't for their outstanding stability, the noise from our radios and the images on our television screens would fade in and out time again. In watch making, the quest for higher accuracy led to the replacement of the moving elements, such as pendulum, balance wheel, and tuning fork, by a pulsating piezoelectric crystal.

The so-called L/C oscillator circuit includes a capacitance in parallel with an inductance – read coil –, while the piezoelectric crystal makes part of the feed back loop. Oscillations occur as electric energy stored in the capacitor flows through the windings of the coil and builds a magnetic field until the charge of the capacitor is used up. Subsequently, a surge of current generated by the coil's degenerating magnetic field recharges the capacitor.

This flip-flop between electric and magnetic field would go on forever, weren't it for the power drain of the emitted electromagnetic waves and the energy converted to heat in the coil's copper wire. Transistor amplified positive feedback makes up for these losses and maintains oscillations. Radio transmission and time measurement have in common the need for a stabilizing element, which in both cases is a piezoelectric crystal.

Piezoelectricity is a material's characteristic to generate a voltage in response to strain from compression or elongation. Conversely, an applied voltage causes a piezoelectric crystal to shrink or to swell, depending on the applied voltage's polarity.

The natural frequency of a bar of silicon dioxide, SiO_2, vulgo quartz, depends – not surprisingly – on its dimensions, and second, on the direction of its cut relative to the direction of the quartz's crystalline structure. Crystals for quartz watches are usually sized to oscillate at 32 kHz[1], and digital frequency splitters either on discrete chips or as part of clock chips bring that down to better manageable levels. The basic type outputs one electrical pulse for every other input pulse, and thus divides by two. Others divide by ten. For instance, splitting the 32 kHz crystal frequency consecutively five times leaves you with a 1000 Hz oscillation, followed by an electronic counter which emits one output pulse per 1000 inputs.

Digital clocks hitting the market in the early 1970s were expected to become the prototypes of the ultimate timepiece. But when the dust settled, most people found it more convenient to grasp the positions of hour and minute hands rather than interpreting a four-digit luminous or sometimes shadowy readout. This led to a different kind of quartz clock, where a stepping motor, powered by the pulses of the oscillator, drives the hands over one of the usual gear trains.

Stepping motors resemble synchronous motors insofar as their speed of rotation keeps step with the frequency of their power source. As synchronous motors of 2, 4, 6, 8 poles, respectively, run at 3600, 1800, 1200, and 900 rpm, stepping motors come with much higher numbers of poles and run as slow as needed.

Although laboratory quality quartz clocks may keep time within 10^{-5} seconds per day, they still rely on mechanical oscillators, namely, resonating crystals.

1 32,000 oscillations per second.

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