Preface

This book deals with a localization approach to the index problem for elliptic operators. Localization ideas for many years have been widely used for solving various specific index problems, but the fact that there is actually a fundamental localization principle underlying all these solutions has mostly passed unnoticed. The ignorance of this general principle has often necessitated using various artificial tricks and hindered the solution of new important problems in index theory. So far, the general localization principle has been only scarcely covered in journal papers and not covered at all in monographs. The present book is intended to fill the gap. We explain the general localization principle and illustrate it by examples. The book is intended for working mathematicians as well as graduate and postgraduate university students specializing in differential equations and related topics.

In the construction of index formulas for elliptic operators on manifolds with boundary, singular manifolds, or noncompact manifolds with a special structure at infinity ("cylindrical ends"), the problem of separating index contributions from the "interior" part of the manifold and from the boundary, singular points, or a neighborhood of infinity is often important. Putting forward this problem is justified by the "locality" of the index. The fact that the index of an elliptic operator on a smooth compact manifold without boundary possesses some locality property was known in elliptic theory at least since the so-called "local index formulas" had emerged. A more careful consideration shows that locality property is actually a property not of the index itself, but of the *relative index*, i.e., the difference of indices of two operators differing on some subset of the manifold and coinciding elsewhere. For the case in which local index formulas are not known *a priori*, the proof of the locality property for the relative index is more complicated and has mostly been carried out on a case-by-case basis. For example, the locality property for the case of Dirac operators on complete noncompact Riemannian manifolds was proved by Gromov and Lawson, whose result was later generalized in various directions.

We consider a general functional-analytic model in which the locality principle holds for the relative index. One can refer to this principle more precisely as the *superposition principle for the relative index*. Note that the derivation of the superposition principle in our model is not based on any index formula, and hence the model applies in situations where index formulas are yet to be obtained. This abstract model serves as a source of relative index formulas (and, under additional assumptions like symmetry conditions, of index formulas) in various specific cases. By way of example, we present applications to the index of elliptic operators on noncompact manifolds and the index of elliptic boundary value problems and also briefly mention the index problem for elliptic operators (pseudodifferential operators and Fourier integral operators) on manifolds with singularities (which is treated in detail in another book by the authors). Furthermore, we include some recent results where the localization principle is used to compute the spectral flow of a family of Dirac operators on a manifold with boundary and this computation is applied to the description of the Aharonov–Bohm effect for massless Dirac fermions in graphene.

The outline of the book is as follows. The introduction exposes the superposition principle for the relative index at the most elementary level and briefly presents its applications, some of which are covered in detail in the main body of the book. It concludes with very brief bibliographical remarks, which are by no means exhaustive but do give some insight into the history of the subject and also indicate possible further reading.

Part I deals with the theory of the superposition principle. Chapter 1 introduces the general superposition principle for the relative index in the widest setting, and Chapters 2 and 3 provide a generalization of this principle to K-homology and Kasparov's KK-theory, thus putting the topic into the context of noncommutative geometry.

Part II contains examples of applications of the superposition principle to various specific problems, including those to elliptic operators on smooth manifolds (Chapter 4), boundary value problems (Chapter 5), and the spectral flow (Chapter 6).

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