

Table of Contents

Preface	xi
Introduction	1
Chapter 1 The Prime Number Theorem and Generalizations	
§1 The Prime Number Theorem	5
§2 Primes in Arithmetic Progression	15
§3 Dedekind's zeta function	19
§4 Hecke's L -functions	21
Chapter 2 Artin L-Functions	
§1 Group-theoretic background	25
§2 Definition and basic properties of Artin L -functions	27
§3 The Artin-Schreier Theorem	30
§4 Dedekind's conjecture in the non-Galois case	32
§5 Zeros and poles of Artin L -functions	35
§6 Low order zeros of Dedekind zeta functions	37
§7 Chebotarev density theorem	41
§8 Consequences of Artin's conjecture	46
§9 The least prime in a conjugacy class	52
Chapter 3 Equidistribution and L-Functions	
§1 Compact groups and Haar measures	65
§2 Weyl's criterion for equidistribution	66
§3 L -functions on G	67
§4 Deligne's Prime Number Theorem	68

Chapter 4 Modular Forms and Dirichlet Series

§1	$SL_2(\mathbb{Z})$ and some of its subgroups	75
§2	The upper half-plane	76
§3	Modular forms and cusp forms	77
§4	L -functions and Hecke's theorem	81
§5	Hecke operators	82
§6	Oldforms and newforms	83
§7	The Sato-Tate conjecture	83
§8	Oscillations of Fourier coefficients of newforms	84
§9	Rankin's theorem	90

Chapter 5 Dirichlet L -functions

§1	Introduction	93
§2	Polya-Vinogradov estimate	95
§3	Jutila's character sum estimate	97
§4	Average value of $L(\frac{1}{2}, \chi_D)$	104
§5	Non-vanishing for a positive proportion of characters, I	110
§6	Non-vanishing for a positive proportion, II	119
§7	A conditional improvement	128

Chapter 6 Non-Vanishing of Quadratic Twists of Modular L -Functions

§1	Introduction	133
§2	The integrated Polya-Vinogradov estimate	141
§3	The main terms	142
§4	Estimates for real character sums	152
§5	Estimates for some weighted sums	158
§6	The statements $A^\pm(\alpha)$ and $C^\pm(\alpha)$	160
§7	Proof of main result	170

Chapter 7 Selberg's Conjectures

§1	Selberg's class of Dirichlet series	177
§2	Basic consequences	180
§3	Artin's conjecture and Selberg's conjectures	181

Chapter 8 Suggestions for Further Reading 187

Name Index	192
Subject Index	194



<http://www.springer.com/978-3-0348-0273-4>

Non-vanishing of L-Functions and Applications

Murty, M.R.; Murty, V.K.

1997, XI, 196 p. 1 illus., Softcover

ISBN: 978-3-0348-0273-4

A product of Birkhäuser Basel