Preface

The present volume arose from the conference on *"Quantum field theory and gravity – Conceptual and mathematical advances in the search for a unified framework"*, held at the University of Regensburg (Germany) from September 28 to October 1, 2010. This conference was the successor of similar conferences which took place at the *Heinrich Fabri Institut* in Blaubeuren in 2003 and 2005 and at the *Max Planck Institute for Mathematics in the Sciences* in Leipzig in 2007. The intention of this series of conferences is to bring together mathematicians and physicists to discuss profound questions within the non-empty intersection of mathematics and physics. More specifically, the series aims at discussing conceptual ideas behind different mathematical and physical approaches to quantum field theory and (quantum) gravity.

As its title states, the Regensburg conference was devoted to the search for a unified framework of quantum field theory and general relativity. On the one hand, the standard model of particle physics – which describes all physical interactions except gravitation – is formulated as a quantum field theory on a fixed Minkowski-space background. The affine structure of this background makes it possible for instance to interpret interacting quantum fields as asymptotically "free particles". On the other hand, the gravitational interaction has the peculiar property that all kinds of energy couple to it. Furthermore, since Einstein developed general relativity theory, gravity is considered as a dynamical property of space-time itself. Hence space-time does not provide a fixed background, and a back-reaction of quantum fields to gravity, i.e. to the curvature of space-time, must be taken into account. It is widely believed that such a back-reaction can be described consistently only by a (yet to be found) quantum version of general relativity, commonly called *quantum gravity*. Quantum gravity is expected to radically change our ideas about the structure of space-time. To find this theory, it might even be necessary to question the basic principles of quantum theory as well.

Similar to the third conference of this series, the intention of the conference held at the University of Regensburg was to provide a forum to discuss different mathematical and conceptual approaches to a quantum (field) theory including gravitational back-reactions. Besides the two well-known paths laid out by string theory and loop quantum gravity, also other ideas were presented. In particular, various functorial approaches were discussed, as well as the possibility that space-time emerges from discrete structures.

The present volume provides an appropriate cross-section of the conference. The refereed articles are intended to appeal to experts working in different fields of mathematics and physics who are interested in the subject of quantum field theory and (quantum) gravity. Together they give the reader some overview of new approaches to develop a quantum (field) theory taking a dynamical background into account.

As a complement to the invited talks which the articles in this volume are based on, discussion sessions were held on the second and the last day of the conference. We list some of the questions raised in these sessions:

1. **Can we expect to obtain a quantum theory of gravity by purely mathematical considerations?** What are the physical requirements to expect from a unified field theory? How can these be formulated mathematically? Are the present mathematical notions sufficient to formulate quantum gravity, or are new mathematical concepts needed? Are the criteria of mathematical consistency and simplicity promising guiding principles for finding a physical theory? Considering the wide variety of existing approaches, the use of gedanken experiments as guiding paradigms seems indispensable even for pure mathematicians in the field.
2. **Evolution or revolution?** Should we expect progress rather by small steps or by big steps? By "small steps" we mean a conservative approach towards a unified theory where one tries to keep the conventional terminology as far as possible. In contrast, proceeding in "big steps" often entails to replace the usual terminology and the conventional physical objects by completely new ones.

   In the discussion, the possibilities for giving up the following conventional structures were considered:
   - **Causality:** In what sense should it hold in quantum gravity?
   - **Superposition principle:** Should it hold in a unified field theory? More specifically, do we have to give up the Hilbert space formalism and its probabilistic interpretation?

   A related question is:
3. **Can we quantize gravity separately?** That is, does it make physical sense to formulate a quantum theory of pure gravity? Can such a formulation be mathematically consistent? Or is it necessary to include all other interactions to obtain a consistent theory?
4. **Background independence**: How essential is it, and which of the present approaches implement it? Which basic mathematical structure would be physically acceptable as implementing background independence?
5. **What are the relevant open problems in classical field theory?** One problem is the concept of charged point particles in classical electrodynamics (infinite self-energy). Other problems concern the notion of quasi-local mass in general relativity and the cosmic censorship conjectures.

6. **(How) can we test quantum gravity?** Can one hope to test quantum gravity in experiments whose initial conditions are controlled by humans, similar to tests of the standard model in particle accelerators? Or does one need to rely on astronomical observations (of events like supernovae or black hole mergers)?

Having listed some of the basic questions, we will now give brief summaries of the articles in this volume. They are presented in chronological order of the corresponding conference talks. Unfortunately, not all the topics discussed at the conference are covered in this volume, because a few speakers were unable to contribute; see also pp. xii–xiii below.

The volume begins with an overview by **Claus Kiefer** on the main roads towards quantum gravity. After a brief motivation why one should search for a quantum theory of gravitation, he discusses canonical approaches, covariant approaches like loop quantum gravity, and string theory. As two main problems that a theory of quantum gravity should solve, he singles out a statistical explanation of the Bekenstein–Hawking entropy and a description of the final stage of black-hole evaporation. He summarizes what the previously discussed approaches have found out about the first question so far.

Locally covariant quantum field theory is a framework proposed by Brunetti–Fredenhagen–Verch that replaces the Haag–Kastler axioms for a quantum field theory on a fixed Minkowski background, by axioms for a functor which describes the theory on a large class of curved backgrounds simultaneously. After reviewing this framework, **Klaus Fredenhagen*** [1] and **Katarzyna Rejzner** suggest that quantum gravity can be obtained from it via perturbative renormalization à la Epstein–Glaser of the Einstein–Hilbert action. One of the technical problems one encounters is the need for a global version of BRST cohomology related to diffeomorphism invariance. As a preliminary step, the authors discuss the classical analog of this quantum problem in terms of infinite-dimensional differential geometry.

Based on his work with Joel Smoller, **Blake Temple** suggests an alternative reason for the observed increase in the expansion rate of the universe, which in the standard model of cosmology is explained in terms of "dark energy" and usually assumed to be caused by a positive cosmological constant. He argues that since the moment when radiation decoupled from matter 379000 years after the big bang, the universe should be modelled by a wave-like perturbation of a Friedmann–Robertson–Walker space-time, according to the mathematical theory of Lax–Glimm on how solutions of conservation laws decay to self-similar wave patterns. The possible perturbations form a 1-parameter family. Temple proposes that a suitable member of this family describes the observed anomalous acceleration of the galaxies (without invoking a cosmological constant). He points out that his hypothesis makes testable predictions.

[1] In the cases where articles have several authors, the star marks the author who delivered the corresponding talk at the conference.

The term *“third quantization”* refers to the idea of quantum gravity as a quantum field theory on the space of geometries (rather than on space-time), which includes a dynamical description of topology change. **Steffen Gielen** and **Daniele Oriti*** explain how matrix models implement the third-quantization program for 2-dimensional Riemannian quantum gravity, via a rigorous continuum limit of discretized geometries. Group field theory (GFT) models, which originated in loop quantum gravity (LQG) but are also relevant in other contexts, implement third quantization for 3-dimensional Riemannian quantum gravity – but only in the discrete setting, without taking a continuum limit. The authors compare the GFT approach to the LQG-motivated idea of constructing, at least on a formal level, a continuum third quantization on the space of connections rather than geometries. They argue that the continuum situation should be regarded only as an effective description of a physically more fundamental GFT.

Andreas Döring* and **Rui Soares Barbosa** present the topos approach to quantum theory, an attempt to overcome some conceptual problems with the interpretation of quantum theory by using the language of category theory. One aspect is that physical quantities take their values not simply in the real numbers; rather, the values are families of real intervals. The authors describe a connection between the topos approach, noncommutative operator algebras and domain theory.

Many problems in general relativity, as well as the formulation of the AdS/CFT correspondence, involve assigning a suitable boundary to a given space-time. A popular choice is Penrose’s *conformal boundary*, but it does not always exist, and it depends on non-canonical data and is therefore not always unique. **José Luis Flores**, **Jónatan Herrera** and **Miguel Sánchez*** explain the construction of a *causal boundary of space-time* which does not suffer from these problems. They describe its properties and the relation to the conformal boundary. Several examples are discussed, in particular pp-waves.

Dietrich Häfner gives a mathematically rigorous description of the Hawking effect for second-quantized spin-$\frac{1}{2}$ fields in the setting of the collapse of a rotating charged star. The result, which confirms physical expectations, is stated and proved using the language and methods of scattering theory.

One problem in constructing a background-free quantum theory is that the standard quantum formalism depends on a background metric: its operational meaning involves a background time, and its ability to describe physics *locally* in field theory arises dynamically, via metric concepts like causality and cluster decomposition. In his *general boundary formulation (GBF)* of quantum theory, **Robert Oeckl** tries to overcome this problem by using, instead of spacelike hypersurfaces, boundaries of arbitrary spacetime regions as carriers of quantum states. His article lists the basic GBF objects and the axioms they have to satisfy, and describes how the usual quantum states, observables and probabilities are recovered from a GBF setting. He proposes various quantization schemes to produce GBF theories from classical theories.

Felix Finster, **Andreas Grotz*** and **Daniela Schiefeneder** introduce causal fermion systems as a general mathematical framework for formulating relativistic quantum theory. A particular feature is that space-time is a secondary object which emerges by minimizing an action for the so-called universal measure. The setup provides a proposal for a "quantum geometry" in the Lorentzian setting. Moreover, numerical and analytical results on the support of minimizers of causal variational principles are reviewed which reveal a "quantization effect" resulting in a discreteness of space-time. A brief survey is given on the correspondence to quantum field theory and gauge theories.

Christian Bär* and **Nicolas Ginoux** present a systematic construction of bosonic and fermionic locally covariant quantum field theories on curved backgrounds in the case of free fields. In particular, they give precise mathematical conditions under which bosonic resp. fermionic quantization is possible. It turns out that fermionic quantization requires much more restrictive assumptions than bosonic quantization.

Christopher J. Fewster asks whether every locally covariant quantum field theory (cf. the article by Fredenhagen and Rejzner described above) represents "the same physics in all space-times". In order to give this phrase a rigorous meaning, he defines the "SPASs" property for families of locally covariant QFTs, which intuitively should hold whenever each member of the family represents the same physics in all space-times. But not every family of locally covariant QFTs has the SPASs property. However, for a "dynamical locality" condition saying that kinematical and dynamical descriptions of local physics coincide, every family of dynamically local locally covariant QFTs has SPASs.

Rainer Verch extends the concept of *local thermal equilibrium (LTE) states*, i.e. quantum states which are not in global thermal equilibrium but possess local thermodynamical parameters like temperature, to quantum field theory on curved space-times. He describes the ambiguities and anomalies that afflict the definition of the stress-energy tensor of QFT on curved space-times and reviews the work of Dappiaggi–Fredenhagen–Pinamonti which, in the setting of the semi-classical Einstein equation, relates a certain fixing of these ambiguities to cosmology. In this context, he applies LTE states and shows that the temperature behavior of a massless scalar quantum field in the very early history of the universe is more singular than the behavior of the usually considered model of classical radiation.

Inspired by a version of Mach's principle, **Julian Barbour** presents a framework for the construction of background-independent theories which aims at quantum gravity, but whose present culmination is a theory of classical gravitation called *shape dynamics*. Its dynamical variables are the elements of the set of compact 3-dimensional Riemannian manifolds divided by isometries and volume-preserving conformal transformations. It "eliminates time", involves a procedure called *conformal best matching*, and is equivalent to general relativity for space-times which admit a foliation by compact spacelike hypersurfaces of constant mean curvature.

Michael K.-H. Kiessling considers the old problem of finding the correct laws of motion for a joint evolution of electromagnetic fields and their point-charge sources. After reviewing the long history of proposals, he reports on recent steps towards a solution by coupling the Einstein–Maxwell–Born–Infeld theory for an electromagnetic space-time with point defects to a Hamilton–Jacobi theory of motion for these defects. He also discusses how to construct a "first quantization with spin" of the sources in this classical theory by replacing the Hamilton–Jacobi law with a de Broglie–Bohm–Dirac quantum law of motion.

Several theories related to quantum gravity postulate (large- or small-sized) extra dimensions of space-time. **Stefan Hollands**' contribution investigates a consequence of such scenarios, the possible existence of higher-dimensional black holes, in particular of stationary ones. Because of their large number, the possible types of such stationary black holes are much harder to classify than their 4-dimensional analogs. Hollands reviews some partial uniqueness results.

Since properties of general relativity, for instance the *Einstein equivalence principle (EEP)*, could conceivably fail to apply to quantum systems, experimental tests of these properties are important. **Domenico Giulini**'s article explains carefully which subprinciples constitute the EEP, how they apply to quantum systems, and to which accuracy they have been tested. In 2010, Müller–Peters–Chu claimed that the least well-tested of the EEP subprinciples, the *universality of gravitational redshift*, had already been verified with very high precision in some older atom-interferometry experiments. Giulini argues that this claim is unwarranted.

Besides the talks summarized above there were also presentations covering the "main roads" to quantum gravity and other topics related to quantum theory and gravity. PDF files of these presentations can be found at *www.uni-regensburg.de/qft2010.*

Dieter Lüst (LMU München) gave a talk with the title *The landscape of multiverses and strings: Is string theory testable?*. He argued that, despite the huge number of vacua that superstring/M-theory produces after compactification, it might still yield experimentally testable predictions. If the string mass scale, which can a priori assume arbitrary values in brane-world scenarios, is not much larger than 5 TeV, then effects like string Regge excitations will be seen at the Large Hadron Collider.

Christian Fleischhack from the University of Paderborn gave an overview of loop quantum gravity, emphasizing its achievements – e.g. the construction of geometric operators for area and volume, and the derivation of black hole entropy – but also its problems, in particular the still widely unknown dynamics of the quantum theory.

In her talk *New 'best hope' for quantum gravity*, **Renate Loll** from the University of Utrecht presented the motivation, the status and perspectives of "Quantum Gravity from Causal Dynamical Triangulation (CDT)" and how it is related to other approaches to a non-perturbative and mathematically rigorous formulation of quantum gravity.

Mu-Tao Wang from Columbia University gave a talk *On the notion of quasilocal mass in general relativity.* After explaining why it is difficult to define a satisfying notion of quasilocal mass, he presented a new proposal due to him and Shing-Tung Yau. This mass is defined via isometric embeddings into Minkowski space and has several desired properties, in particular a vanishing property that previous definitions were lacking.

Motivated by the question – asked by 't Hooft and others – whether quantum mechanics could be an emergent phenomenon that occurs on length scales sufficiently larger than the Planck scale but arises from different dynamics at shorter scales, **Thomas Elze** from the University of Pisa discussed in the talk *General linear dynamics: quantum, classical or hybrid* a path-integral representation of classical Hamiltonian dynamics which allows to consider direct couplings of classical and quantum objects. Quantum dynamics turns out to be rather special within the class of such general linear evolution laws.

In his talk on *Massive quantum gauge models without Higgs mechanism*, **Michael Dütsch** explained how to construct the S-matrix of a non-abelian gauge theory in Epstein–Glaser style, via the requirements of renormalizability and causal gauge invariance. These properties imply already the occurrence of Higgs fields in massive non-abelian models; the Higgs fields do not have to be put in by hand. He discussed the relation of this approach to model building via spontaneous symmetry breaking.

Jerzy Kijowski from the University of Warszawa spoke about *Field quantization via discrete approximations: problems and perspectives.* He explained how the set of discrete approximations of a physical theory is partially ordered, and that the observable algebras form an inductive system for this partially ordered set, whereas the states form a projective system. Then he argued that loop quantum gravity is the best existing proposal for a quantum gravity theory, but suffers from the unphysical property that its states form instead an inductive system.

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