The book is devoted to the mathematical description of probabilistic conditional independence structures. The topic of conditional independence, which falls within both the scope of statistics and of artificial intelligence, has been at the center of my research activity for many years – since the late 1980s. I have been primarily influenced by researchers working in the area of graphical models but I gradually realized that the concept of conditional independence is not necessarily bound to the idea of graphical description and may have a broader impact. This observation led me to an attempt to develop a non-graphical method for describing probabilistic conditional independence structures which, in my view, overcomes an inherent limitation of graphical approaches. The method of structural imsets described in this book can be viewed as an algebraic approach to the description of conditional independence structures although it remains within the framework of discrete mathematics.

The basic idea of this approach was already presented in the middle of the 1990s in a series of papers [137]. However, I was not satisfied with the original presentation of the approach for several reasons. First, the series of papers only dealt with the discrete case, which is a kind of imperfection from the point of view of statistics. Second, the main message was dimmed by unnecessary mathematical peculiarities and important ideas were perhaps not pinpointed clearly. Third, the motivation was not explained in detail. I also think that the original series of papers was difficult for researchers in the area of artificial intelligence to read because “practical” implementation aspects of the presented approach were suppressed there. Another point is that the pictorial representation of considered mathematical objects, to which researchers interested in graphical models are accustomed, was omitted.

Within the next six years, further mathematical results were achieved which amended, supplemented and gave more precision to the original idea. I have also deliberated about suitable terminology and the way to present the method of structural imsets which would be acceptable to statisticians and researchers in the area of artificial intelligence, as well as exact from the mathematical point of view. I wrote it up in my DrSc thesis [146], which became
the basis of this monograph. After finishing the thesis, I realized the potential future practical application of the method to learning graphical models and decided to emphasize this by writing an additional chapter.

Thus, the aim of this monograph is to present the method of structural imsets in its full (present) extent: the motivation; the mathematical foundations, which I tried to present in a didactic form; indication of the area of application; and open problems. The motivation is explained in the first chapter. The second chapter recalls basic concepts in the area of probabilistic conditional independence structures. The third chapter is an overview of classic graphical methods for describing conditional independence structures. The core of the method of structural imsets is presented in the next four chapters. The eighth chapter shows application of the method to learning graphical models. Open problems are gathered in the ninth chapter and necessary elementary mathematical notions are provided in the Appendix for the reader’s convenience. Then the List of Notation follows. As there are many cross-references to elementary units of the text, like Lemmas, Remarks etc., they are listed with page numbers afterwards. The text is concluded by the References and the Index.

The book is intended for

- mathematicians who may be attracted by this particular application of mathematics in the area of artificial intelligence and statistics;
- researchers in statistics and informatics who may become interested in deeper understanding of the mathematical basis of the theory of (graphical) models of conditional independence structures;
- advanced PhD students in the fields of mathematics, probability, statistics, informatics and computer science who may find inspiration in the book and perhaps make some progress either by solving open problems or by applying the presented theory in practice.

In particular, I have in mind those PhD students who are thinking about an academic career. They are advised to read the book starting with the Appendix and to utilize the lists at the end of the book.

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