2

Fuel Cell System Model: Auxiliary Components

Models developed specifically for control studies have certain characteristics. Important characteristics such as dynamic (transient) effects are included and some other effects, such as spatial variation of parameters, are neglected or approximated. Furthermore, only dynamic effects that are related to the automobile control problem are integrated into the models. The relevant time constants for an automotive propulsion-sized PEM fuel cell stack system are summarized in [56] as

- Electrochemistry O(10^{-19} sec),
- Hydrogen and air manifolds O(10^{-1} sec),
- Membrane water content O(unclear),
- Flow control/supercharging devices O(10^{0} sec),
- Vehicle inertia dynamics O(10^{1} sec), and
- Cell and stack temperature O(10^{2} sec),

where O denotes the order of magnitude. The fast transient phenomena of electrochemical reactions have minimal effects in automobile performance and can be ignored. The transient behaviors due to manifold filling dynamics, membrane water content, supercharging devices, and temperature may have an impact on the behavior of the vehicle and, thus, must be included in the model. The response of the humidification and membrane water content cannot be easily decoupled from the temperature and flow dynamics and, thus, the associated time constant is listed as "unclear". Interactions between processes, when appropriate, are also included. However, with relatively slow responses, the cell and stack temperature may be viewed as a separate control system which is equipped with a separate controller. The temperature can then be considered as a constant for other faster subsystems.

The system block diagram showing the subsystem blocks along with input/output signals is illustrated in Figure 2.1. The thick arrow between two component blocks (marked "flow") represents flow rate as well as the condition of the gas (*e.g.*, pressure, humidity, and temperature). In this and the next chapters, the models of several components shown in the figure are explained.

We focus on the reactant supply subsystem and thus the models of the components related to this subsystem are developed. The component models for the heat management subsystem are left for future study. Figure 2.2 illustrates the components and flows related to the reactant supply subsystem. It is assumed that the cathode and anode volumes of the multiple fuel cells are lumped as a single stack cathode and anode volumes. The anode supply and return manifold volumes are very small. Their sizes allow us to lump all these volumes to one "anode" volume. The cathode supply manifold lumps all the volumes associated with pipes and connections between the compressor and the stack cathode flow field. The length, and thus volume, of the cathode supply manifold can be large depending on the physical location of the compressor with respect to the stack. The cathode return manifold represents the lumped volume of pipes downstream from the stack cathode.

In this chapter, the modeling of the auxiliary components is explained. The compressor dynamic model is explained in Section 2.1 followed by an explanation of the manifold filling model in Section 2.2. Static models of the air cooler and the air humidifier are explained in Sections 2.4 and 2.5. In the next chapter, the development of the fuel cell stack model, which consists of stack voltage, anode flow, cathode flow, and membrane hydration models, is presented.

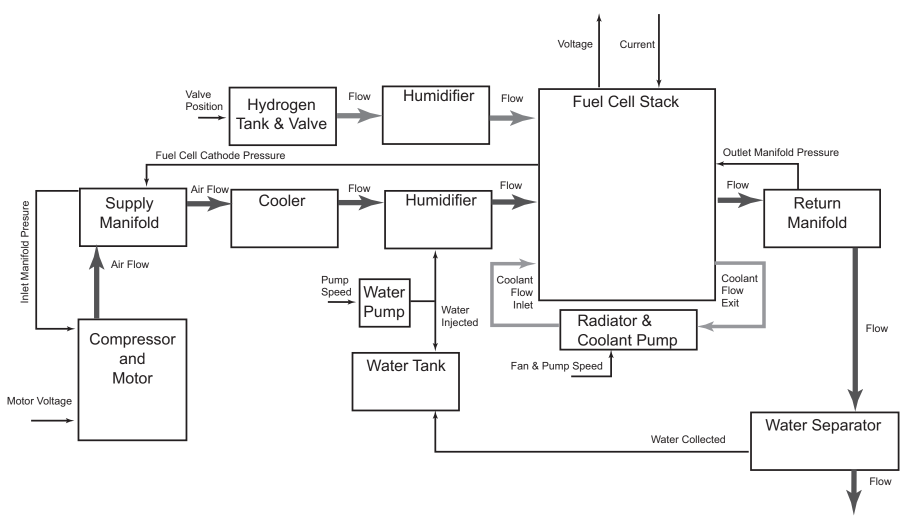

Fig. 2.1. System block diagram

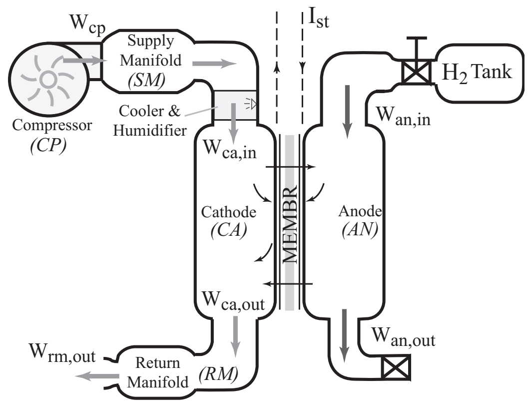

Fig. 2.2. Reactant supply subsystem model

2.1 Compressor Model

The compressor model is separated into two parts, as shown in Figure 2.3. The first part is a static compressor map which determines the air flow rate through the compressor. Thermodynamic equations are then used to calculate the exit air temperature and the required compressor power. The second part represents the compressor and motor inertia and defines the compressor speed. The speed is consequently used in the compressor map to find the air mass flow rate.

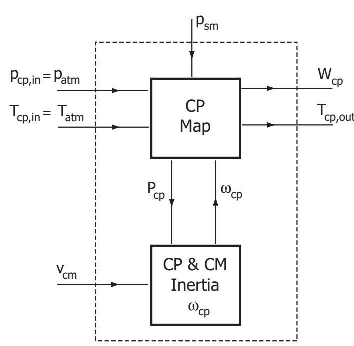

Fig. 2.3. Compressor block diagram

The only dynamic state in the model is the compressor speed ω_{cp}. The inputs to the model include inlet air pressure $p_{cp,in}$, its temperature $T_{cp,in}$, the voltage command to the compressor motor v_{cm}, and downstream pressure, which is the supply manifold pressure $p_{cp,out} = p_{sm}$. The inlet air is typically atmospheric and its pressure and temperature are assumed to be fixed at

$p_{atm} = 1$ atm and $T_{atm} = 25°$C, respectively. The motor command is one of the inputs to the fuel cell system. The downstream pressure is determined by the supply manifold model.

The compressor air mass flow rate W_{cp} (kg/sec) is determined, through a compressor flow map, from the pressure ratio across the compressor and the speed of the compressor. However, supplying the compressor flow map in the form of a look-up table is not well suited for dynamic system simulations [80]. Standard interpolation routines are not continuously differentiable and extrapolation is unreliable. Therefore, a nonlinear curve fitting method is used to model the compressor characteristics. The Jensen & Kristensen method, described in [80], is used in our model.

To reflect variations in the inlet condition of the compressor, which are the inlet flow pressure and temperature, the "corrected" values of mass flow rate and compressor speed are used in the compressor map. The corrected values [29] are the corrected compressor speed (rpm) $N_{cr} = N_{cp}/\sqrt{\theta}$, and the corrected mass flow $W_{cr} = W_{cp}\sqrt{\theta}/\delta$, where corrected temperature $\theta = T_{cp,in}/\ 288$ K and corrected pressure $\delta = p_{cp,in}/\ 1$ atm. Using the Jensen & Kristensen method, the dimensionless head parameter Ψ is first defined:

$$\Psi = C_p T_{cp,in} \left[\left(\frac{p_{cp,out}}{p_{cp,in}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] / \left(\frac{U_c^2}{2} \right) \tag{2.1}$$

where the inlet air temperature $T_{cp,in}$ is in Kelvin and U_c is the compressor blade tip speed (m/s),

$$U_c = \frac{\pi}{60} d_c N_{cr} \tag{2.2}$$

d_c is the compressor diameter (m), and γ is the ratio of the specific heats of the gas at constant pressure C_p/C_v, which is equal to 1.4 in the case of air. The normalized compressor flow rate Φ is defined by

$$\Phi = \frac{W_{cr}}{\rho_a \frac{\pi}{4} d_c^2 U_c} \tag{2.3}$$

where ρ_a is the air density (kg/m^3). The normalized compressor flow rate Φ is then correlated with the head parameter Ψ by the equation

$$\Phi = \Phi_{max} \left[1 - \exp\left(\beta \left(\frac{\Psi}{\Psi_{max}} - 1 \right) \right) \right] \tag{2.4}$$

where Φ_{max}, β, and Ψ_{max} are polynomial functions of the Mach number M,

$$\begin{aligned} \Phi_{max} &= a_4 M^4 + a_3 M^3 + a_2 M^2 + a_1 M + a_0 \\ \beta &= b_2 M^2 + b_1 M + b_0 \\ \Psi_{max} &= c_5 M^5 + c_4 M^4 + c_3 M^3 + c_2 M^2 + c_1 M + c_0 \end{aligned} \tag{2.5}$$

The inlet Mach number M is defined by

Table 2.1. Compressor map parameters

Parameter	Value	Units
R_a	2.869×10^2	J/(kg·K)
ρ_a	1.23	kg/m^3
d_c	0.2286	m

$$M = \frac{U_c}{\sqrt{\gamma R_a T_{cp,in}}} \tag{2.6}$$

where R_a is the air gas constant. In Equation (2.5), a_i, b_i, and c_i are regression coefficients obtained by curve fitting of the compressor data. The air mass flow in kg/sec is then calculated using Equation (2.3):

$$W_{cr} = \Phi \rho_a \frac{\pi}{4} d_c^2 U_c \tag{2.7}$$

The parameters used in the model are given in Table 2.1. The compressor model used here is for an Allied Signal compressor. The data were obtained by digitizing the compressor map given in [29]. The regression coefficients obtained by curve fitting are given in Table 2.2. Figure 2.4 shows that the curve fitting scheme represents the compressor data very well.

Table 2.2. Compressor map regression coefficients

Parameter	Value
a_4	-3.69906×10^{-5}
a_3	2.70399×10^{-4}
a_2	-5.36235×10^{-4}
a_1	-4.63685×10^{-5}
a_0	2.21195×10^{-3}
b_2	1.76567
b_1	-1.34837
b_0	2.44419
c_5	-9.78755×10^{-3}
c_4	0.10581
c_3	-0.42937
c_2	0.80121
c_1	-0.68344
c_0	0.43331

A look-up table of the compressor efficiency η_{cp} is used to find the efficiency of the compressor from the mass flow rate and pressure ratio across the compressor. The maximum efficiency of the compressor is 80%. The temperature of the air leaving the compressor is calculated from the equation

$$T_{cp,out} = T_{cp,in} + \frac{T_{cp,in}}{\eta_{cp}} \left[\left(\frac{p_{cp,out}}{p_{cp,in}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= T_{atm} + \frac{T_{atm}}{\eta_{cp}} \left[\left(\frac{p_{sm}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \qquad (2.8)$$

The torque required to drive the compressor is calculated using thermodynamic equation:

$$\tau_{cp} = \frac{C_p}{\omega_{cp}} \frac{T_{atm}}{\eta_{cp}} \left[\left(\frac{p_{sm}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] W_{cp} \qquad (2.9)$$

where τ_{cp} is the torque needed to drive the compressor in N-m;
C_p is the specific heat capacity of air $= 1004 \quad \mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$;
γ is the ratio of the specific heats of air $= 1.4$.

Derivations of Equations (2.8) and (2.9) are standard and can be found in the thermodynamics or turbine literature [21, 53].

A lumped rotational parameter model with inertia is used to represent the dynamic behavior of the compressor speed:

$$J_{cp} \frac{d\omega_{cp}}{dt} = (\tau_{cm} - \tau_{cp}) \qquad (2.10)$$

where J_{cp} is the combined inertia of the compressor and the motor (kg·m^2);
ω_{cp} is the compressor speed (rad/sec);
τ_{cm} is the compressor motor torque input (N-m);
τ_{cp} is the torque required to drive the compressor (N-m).

The compressor motor torque is calculated using a static motor equation:

$$\tau_{cm} = \eta_{cm} \frac{k_t}{R_{cm}} (v_{cm} - k_v \omega_{cp}) \qquad (2.11)$$

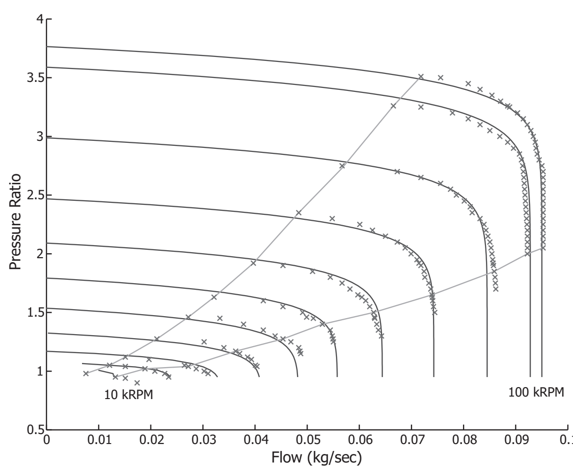

Fig. 2.4. Compressor map

where k_t, R_{cm}, and k_v are motor constants and η_{cm} is the motor mechanical efficiency. The values are given in Table 2.3.

Table 2.3. Compressor motor parameters

Parameter	Value
k_v	0.0153 V/(rad/sec)
k_t	0.0153 N-m/Amp
R_{cm}	0.82 Ω
η_{cm}	98%

2.2 Lumped Model of the Manifold Dynamics

The manifold model represents the lumped volume associated with pipes and connections between each device. The supply manifold volume includes the volume of the pipes between the compressor and the fuel cell stack including the volume of the cooler and the humidifier (Figure 1.5). The return manifold represents the pipeline at the fuel cell stack exhaust.

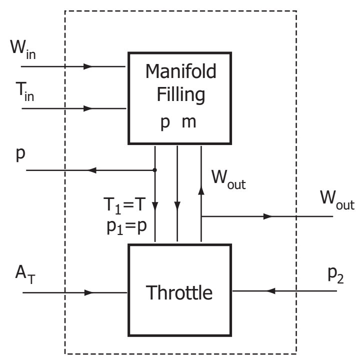

Fig. 2.5. Manifold block diagram

A block diagram of the manifold model is shown in Figure 2.5. The mass conservation principle is used to develop the manifold model. For any manifold,

$$\frac{dm}{dt} = W_{in} - W_{out} \tag{2.12}$$

where m is the mass of the gas accumulated in the manifold volume and W_{in} and W_{out} are mass flow rates into and out of the manifold. If we assume that the air temperature is constant in the manifold T and equal to the inlet flow temperature $T = T_{in}$, the manifold filling dynamics follow an isothermic relation:

$$\frac{dp}{dt} = \frac{R_a T}{V}(W_{in} - W_{out}) \tag{2.13}$$

where R_a is the gas constant of air and V is the manifold volume. If the air temperature is expected to change in the manifold, the pressure dynamic equation, which is derived from the energy conservation, the ideal gas law, and the air thermodynamic properties,

$$\frac{dp}{dt} = \frac{\gamma R_a}{V}\left(W_{in} T_{in} - W_{out} T\right) \tag{2.14}$$

is used in addition to the mass balance equation (2.12). The air temperature T in (2.14) is calculated from the air mass m in (2.12) and air pressure p in (2.14) using the ideal gas law. In summary, if the temperature of the air in the manifold is assumed constant, Equation (2.13) is used to model the manifold dynamics. If the temperature of the air is expected to change, Equations (2.12) and (2.14) are used.

The nozzle flow equation, derived in [58], is used to calculate the outlet flow of the manifold. The flow rate passing through a nozzle is a function of the upstream pressure p_1, the upstream temperature T_1, and the downstream pressure p_2, of the nozzle. The flow characteristic is divided into two regions by the critical pressure ratio:

$$\left(\frac{p_2}{p_1}\right)_{crit} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{2.15}$$

where γ is the ratio of the specific heat capacities of the gas C_p/C_v. In the case of air $\gamma = 1.4$ and the critical pressure ratio is equal to 0.528. For sub-critical flow where the pressure drop is less than the critical pressure ratio

$$\frac{p_2}{p_1} > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

the mass flow rate is calculated from

$$W = \frac{C_D A_T p_1}{\sqrt{\bar{R} T_1}} \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \left\{ \frac{2\gamma}{\gamma-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{\frac{1}{2}} \quad \text{for } \frac{p_2}{p_1} > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{2.16}$$

For critical flow (or choked flow), the mass flow rate is given by

$$W_{choked} = \frac{C_D A_T p_1}{\sqrt{\bar{R} T_1}} \gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad \text{for } \frac{p_2}{p_1} \le \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{2.17}$$

Parameter C_D is the discharge coefficient of the nozzle, A_T is the opening area of the nozzle (m^2), and $\bar{R}$ is the universal gas constant. The plot of

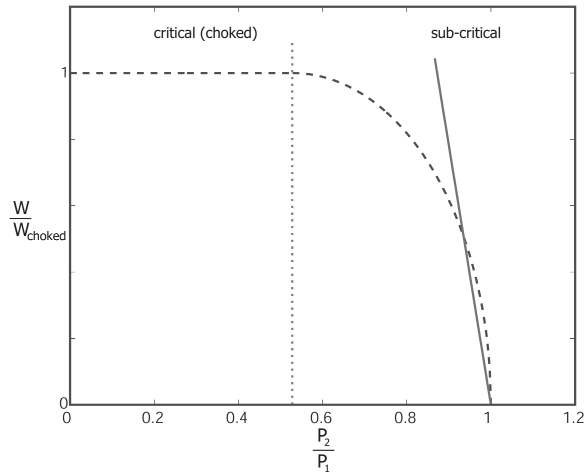

Fig. 2.6. Dashed line: relative mass flow rate as a function of nozzle pressure ratio (2.16)-(2.17); solid line: linearized mass flow rate at low pressure difference (2.18)

W/W_{choked} is shown as a dashed line in Figure 2.6. If the pressure difference between the manifold and the downstream volume is small and always falls into the subcritical flow region, the flow rate can be calculated by a linearized form of the subcritical nozzle flow equation (2.16),

$$W = k(p_1 - p_2) \tag{2.18}$$

where k is the nozzle constant. The plot of the linearized equation (2.18) for various manifold pressures is shown in Figure 2.7 as a solid line, compared to the plot of Equation (2.16) shown as a dashed line.

2.2.1 Supply Manifold

For the supply manifold, the inlet mass flow is the compressor flow W_{cp} and the outlet mass flow is $W_{sm,out}$. Because the pressure difference between the supply manifold and the cathode is relatively small,

$$W_{sm,out} = k_{sm,out}(p_{sm} - p_{ca}) \tag{2.19}$$

where $k_{sm,out}$ is the supply manifold outlet flow constant. Because the temperature of the air leaving the compressor is high, it is expected that the air temperature changes inside the supply manifold. Thus, Equations (2.12) and (2.14) are used to model the supply manifold

$$\frac{dm_{sm}}{dt} = W_{cp} - W_{sm,out} \tag{2.20}$$

$$\frac{dp_{sm}}{dt} = \frac{\gamma R_a}{V_{sm}} \left(W_{cp} T_{cp,out} - W_{sm,out} T_{sm} \right) \tag{2.21}$$

where V_{sm} is the supply manifold volume and T_{sm} is the supply manifold air temperature, which is calculated from m_{sm} and p_{sm} using the ideal gas law. A block diagram of the supply manifold is shown in Figure 2.8.

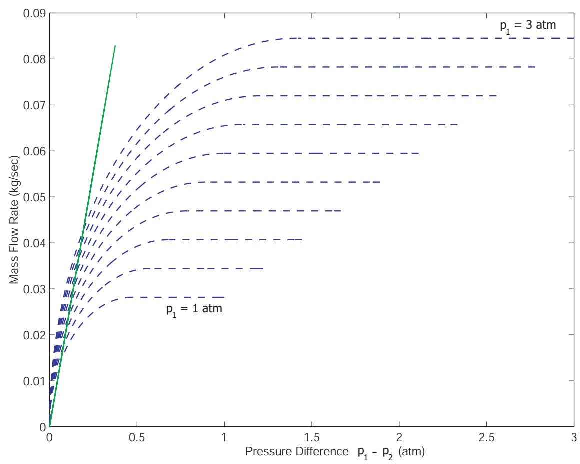

Fig. 2.7. Comparison of nozzle flow rate from nonlinear and linear nozzle equations

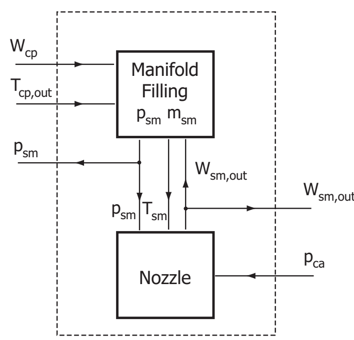

Fig. 2.8. Supply manifold block diagram

2.2.2 Return Manifold

The temperature of the air leaving the stack is relatively low. Therefore, the changes of air temperature in the return manifold are negligible, and the return manifold pressure is modeled by

$$\frac{dp_{rm}}{dt} = \frac{R_a T_{rm}}{V_{rm}}(W_{ca,out} - W_{rm,out}) \tag{2.22}$$

where V_{rm} is the return manifold volume and T_{rm} is the temperature of the gas in the return manifold. The flow entering the return manifold $W_{ca,out}$ is calculated in Equation (3.47), which is in the same form as Equation (2.19). The outlet mass flow of the return manifold is governed by nozzle (throttle) equations (2.16) to (2.17). The outlet mass flow is a function of the manifold pressure p_{rm} and the pressure downstream from the manifold, which is assumed to be fixed at p_{atm}. Because the pressure drop between the return manifold and the atmospheric is relatively large, the equations of the return manifold exit flow are

$$W_{rm,out} = \frac{C_{D,rm} A_{T,rm} p_{rm}}{\sqrt{\bar{R} T_{rm}}} \left(\frac{p_{atm}}{p_{rm}} \right)^{\frac{1}{\gamma}} \left\{ \frac{2\gamma}{\gamma - 1} \left[1 - \left(\frac{p_{atm}}{p_{rm}} \right)^{\frac{\gamma-1}{\gamma}} \right] \right\}^{\frac{1}{2}}$$
$$\text{for} \quad \frac{p_{atm}}{p_{rm}} > \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \tag{2.23}$$

and

$$W_{rm,out} = \frac{C_{D,rm} A_{T,rm} p_{rm}}{\sqrt{\bar{R} T_{rm}}} \gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}$$
$$\text{for} \quad \frac{p_{atm}}{p_{rm}} \leq \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \tag{2.24}$$

The throttle opening area $A_{T,rm}$ can be set constant or can be used as an extra control variable to regulate the return manifold pressure, and thus the cathode pressure [97]. The values of $C_{D,rm}$ and the nominal value of $A_{T,rm}$ used in the model are given in Table 4.1. A block diagram of the return manifold model is shown in Figure 2.9.

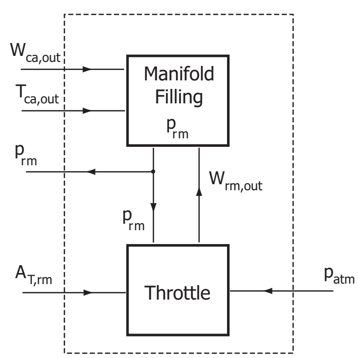

Fig. 2.9. Return manifold block diagram

The pressure calculated in the supply manifold model is used in the compressor model to determine the pressure ratio across the compressor. The return manifold pressure calculated in the return manifold model is used to determine the flow rate exiting the fuel cell cathode. The model of the cathode along with other parts of the fuel cell stack are described in the next chapter.

2.3 Review of the Thermodynamics of Gas Mixtures

In this section, we review the basic thermodynamic properties of gas mixtures that we use extensively in the model. Details can be found in [103]. We also focus on the mixture involving gases and water vapor.

Here, we consider properties of ideal gases. Specifically, each component of the mixture is independent of the presence of other components and each component can be treated as an ideal gas. Consider the mixture of gas A and gas B. From the ideal gas law, we have

$$pV = n\bar{R}T = mRT \tag{2.25}$$

where p is the gas pressure, V is the gas volume, n is the number of moles of the gas, m is the mass of the gas, $\bar{R}$ is the universal gas constant, R is the gas constant, and T is the gas temperature. The total number of moles of the mixture is equal to the sum of the number of moles of each component:

$$n = n_A + n_B \tag{2.26}$$

If we treat each component as an ideal gas, the law in (2.25) holds for each component:

$$\begin{aligned} p_A V &= n_A \bar{R} T \\ p_B V &= n_B \bar{R} T \end{aligned} \tag{2.27}$$

where p_A and p_B are the partial pressures. By substitution of Equations (2.25) and (2.27) into Equation (2.26), we get

$$p = p_A + p_B \tag{2.28}$$

Thus, for a mixture of ideal gases, the pressure of the mixture is the sum of the partial pressures of the individual components.

Let us now consider a mixture of air and water vapor. The humidity ratio ω is defined as the ratio of the mass of water vapor m_v to the mass of dry air m_a:

$$\omega = \frac{m_v}{m_a} \tag{2.29}$$

The total mass of the mixture is $m_a + m_v$. The humidity ratio does not give a good representation of the humidity of the mixture because the maximum amount of water vapor that the air can hold (saturation) depends on the temperature and pressure of the air. The relative humidity, which represents the amount of water in the air relative to the maximum possible amount, is therefore more widely used. The relative humidity ϕ is defined as the ratio of the mole fraction of the water vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and pressure. With the assumption of ideal gases, the definition reduces to the ratio of the partial pressure of the water vapor p_v in the mixture to the saturation pressure of the vapor at the temperature of the mixture p_{sat}:

$$\phi = \frac{p_v}{p_{sat}} \tag{2.30}$$

The saturation pressure p_{sat} depends on the temperature and is easily obtained from a thermodynamic table of vapor [103]. In the model, the saturation pressure is calculated from an equation of the form given in [83]. The saturation pressure data in [103] is used to obtain the coefficients in the equation:

$$\begin{aligned}\log_{10}(P_{sat}) = -1.69 \times 10^{-10}T^4 + 3.85 \times 10^{-7}T^3 - 3.39 \times 10^{-4}T^2 \\ + 0.143\,T - 20.92\end{aligned} \tag{2.31}$$

where the saturation pressure p_{sat} is in kPa and the temperature T is in Kelvin.

The relation between the humidity ratio and the relative humidity can be derived from the ideal gas law:

$$\omega = \frac{m_v}{m_a} = \frac{p_v V/R_v T}{p_a V/R_a T} = \frac{R_a p_v}{R_v p_a} = \frac{M_v}{M_a}\frac{p_v}{p_a} \tag{2.32}$$

where M_v and M_a, both in kg/mol, are the molar mass of vapor and dry air, respectively. By using Equations (2.28) and (2.30), the relative humidity can be calculated from dry air pressure and the humidity ratio

$$\phi = \omega \frac{M_a}{M_v}\frac{p_a}{p_{sat}} \tag{2.33}$$

There are some details that should be pointed out. First, relative humidity having a value of one means that the mixture is saturated or fully humidified. If there is more water content in the mixture, the extra amount of water will condense into a liquid form. Second, with the ideal gas assumption, various components in the mixture can be treated separately when performing the internal energy and enthalpy calculations.

2.4 Air Cooler (Static) Model

The temperature of the air in the supply manifold is typically high due to the high temperature of air leaving the compressor. To prevent any damage to the fuel cell membrane, the air needs to be cooled down to the stack operating temperature. In this study, we do not address heat transfer effects and thus we assume that an ideal air cooler maintains the temperature of the air entering the stack at $T_{cl} = 80°\text{C}$. It is assumed that there is no pressure drop across the cooler, $p_{cl} = p_{sm}$. Because temperature change affects gas humidity, the humidity of the gas exiting the cooler is calculated as

$$\phi_{cl} = \frac{p_{v,cl}}{p_{sat}(T_{cl})} = \frac{p_{cl} p_{v,atm}}{p_{atm} p_{sat}(T_{cl})} = \frac{p_{cl} \phi_{atm} p_{sat}(T_{atm})}{p_{atm} p_{sat}(T_{cl})} \tag{2.34}$$

where $\phi_{atm} = 0.5$ is the nominal ambient air relative humidity and $p_{sat}(T_i)$ is the vapor saturation pressure that is a function of temperature T_i. The change in temperature does not affect the mass of the gas; thus, the mass flow rate does not change in the cooler model; that is, $W_{cl} = W_{sm,out}$.

2.5 Humidifier (Static) Model

Air flow from the cooler is humidified before entering the stack by injecting water into the air stream in the humidifier. Here, the volume of the humidifier is small and hence it can be considered as part of the supply manifold volume. A static model of the humidifier is used to calculate the change in air humidity due to the additional injected water. The temperature of the flow is assumed to be constant; thus, $T_{hm} = T_{cl}$. The injected water is assumed to be in the form of vapor or the latent heat of vaporization is assumed to be taken into account in the air cooler. Based on the condition of the flow exiting the cooler ($W_{cl} = W_{sm,out}$, p_{cl}, T_{cl}, ϕ_{cl}), the dry air mass flow rate $W_{a,cl}$, the vapor mass flow rate $W_{v,cl}$, and the dry air pressure $p_{a,cl}$, can be calculated using the thermodynamic properties discussed in Section 2.3. The vapor saturation pressure is calculated from the flow temperature using Equation (2.31). Then, the vapor pressure is determined using Equation (2.30):

$$p_{v,cl} = \phi_{cl} p_{sat}(T_{cl}) \tag{2.35}$$

Because humid air is a mixture of dry air and vapor, dry air partial pressure is the difference between the total pressure and the vapor pressure:

$$p_{a,cl} = p_{cl} - p_{v,cl} \tag{2.36}$$

The humidity ratio can then be calculated from

$$\omega_{cl} = \frac{M_v}{M_a} \frac{p_{v,cl}}{p_{a,cl}} \tag{2.37}$$

where M_a is the molar mass of dry air (28.84×10^{-3} kg/mol). The mass flow rate of dry air and vapor from the cooler is

$$W_{a,cl} = \frac{1}{(1 + \omega_{cl})} W_{cl} \tag{2.38}$$

$$W_{v,cl} = W_{cl} - W_{a,cl} \tag{2.39}$$

The mass flow rate of dry air remains the same for the inlet and outlet of the humidifier, $W_{a,hm} = W_{a,cl}$. The vapor flow rate increases by the amount of water injected:

$$W_{v,hm} = W_{v,cl} + W_{v,inj} \tag{2.40}$$

The vapor pressure also changes and can be calculated using Equation (2.32):

$$p_{v,hm} = \omega_{cl} \frac{M_a}{M_v} p_{a,cl} = \frac{W_{v,hm}}{W_{a,cl}} \frac{M_a}{M_v} p_{a,cl} \tag{2.41}$$

The vapor pressure $p_{v,hm}$ can then be used to determine the exit flow relative humidity

$$\phi_{hm} = \frac{p_{v,hm}}{p_{sat}(T_{hm})} = \frac{p_{v,hm}}{p_{sat}(T_{cl})} \tag{2.42}$$

Because the vapor pressure increases, the total pressure also increases:

$$p_{hm} = p_{a,cl} + p_{v,hm} \tag{2.43}$$

The humidifier exit flow rate is governed by the mass continuity

$$W_{hm} = W_{a,cl} + W_{v,hm} = W_{a,cl} + W_{v,cl} + W_{v,inj} \tag{2.44}$$

The flow leaving the humidifier enters the fuel cell cathode and thus, in the next chapter, the humidifier exit flow is referred to as cathode inlet (ca, in) flow; for example, $W_{ca,in} = W_{hm}$ and $\phi_{ca,in} = \phi_{hm}$.

The models of auxiliary components in the fuel cell system are developed in this chapter. These models will interact with the fuel cell stack model. In the next chapter, the fuel cell stack model and its submodels are described.

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Control of Fuel Cell Power Systems
Principles, Modeling, Analysis and Feedback Design
Pukrushpan, J.T.; Stefanopoulou, A.G.; Peng, H.
2004, XVII, 161 p., Hardcover
ISBN: 978-1-85233-816-9