initial estimate in the Kalman prediction procedure. That is, the motor is judged to fail if the rotation speed drops to 95% of the normal value.
2. Mean time between failure (MTBF) for the motor is 100 000 h [40.21].
3. Sampling interval T is 1 h (this is the increment time for every step in Kalman prediction).
4. Disturbance $\boldsymbol{W}_k$ has mean 0 and variance 0.01 V [40.22].
5. Measurement error $\boldsymbol{V}_k$ for $\dot{\theta}$ has a zero mean and a standard deviation of 3.333 rad/s, which is 1% of the full-scale accuracy [40.23] of the measurement.
6. PM lead-time is set to $n \times 60$ min, where n is the number of steps ahead in the prediction. Accordingly, the alarm signal is activated (to indicate that PM should be executed) whenever the Kalman filter predicts that the motor speed will be lower than the prescribed threshold $n \times 60$ min later.

40.4.2 Monte Carlo Simulation and ARMA Model

Assuming that motor failures occur randomly, a Monte Carlo simulation (MCS) can be used to generate the failure times of the motor. The relationship between the failure rate $h(t)$ and the distribution function of the lifetime $f(t)$ is [40.5]

$$f(t) = h(t) \exp\left[-\int_0^t h(\tau)\,d\tau\right] . \quad (40.44)$$

Failures occur randomly during the useful lifetime, statistically conforming to a bathtub curve [40.5]. The failure rate is constant during this period. Let the failure rate in (40.44) be a constant λ, and so (40.44) becomes

$$f(t) = \lambda \exp\left(-\int_0^t \lambda\,d\tau\right) = \lambda\,e^{-\lambda t} , \quad (40.45)$$

which is an exponential distribution function. Let u_i, $i = 1, 2, 3, \ldots, m$, represent a set of standard uniformly distributed random numbers. The corresponding numbers t_i of the random variable t in (40.45) (in other words the simulated failure times), are written as [40.5]

$$t_i = -\frac{1}{\lambda} \ln u_i , \quad (40.46)$$

with exponential distributions.

The measurements needed for the recursive estimation loop of the Kalman filter, as depicted in Fig. 40.4, are generated by the ARMA model ((40.1) to (40.3)). Simulations in this section are performed using MATLAB [40.24]. All random numbers and white sequences with prescribed variances needed are obtained using the random number generator in MATLAB.

40.4.3 Exponential Attenuator

To account for the aging failure modes and the exponentially distributed failure times t_i, an exponential attenuator, represented by $e^{-t/\tau}$, is placed at the output ends of both the motor system and the Kalman filter. The block diagram of the simulation system is shown in Fig. 40.7. The symbol τ of the attenuator in Fig. 40.7 denotes the failure time constant of the motor, which varies with the failure times that are generated by the MCS.

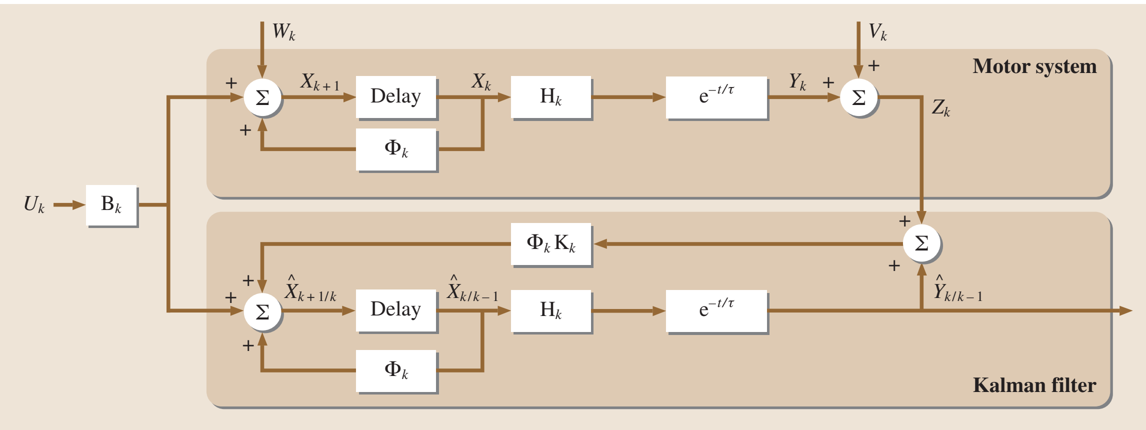

Fig. 40.7 Block diagram of the simulation system

40.4.4 Simulation Results

Two categories of simulation are conducted in this section, namely one-step-ahead prediction and two-steps-ahead prediction. According to the central limit theorem (CLT), estimators follow the normal distribution if the sample size is sufficiently large. A sample size of 30 is a reasonable number to use [40.25]. The larger the sample size, the smaller the estimated error, which tends to zero when the sample size approaches infinity. Hence, each simulation is executed 100 times. Simulation results for a lead-time of 60 min – one-step-ahead prediction – is shown in Fig. 40.8. Figure 40.8a shows the results for the 100 simulations of failure times generated by the MCS, the failure times predicted by the Kalman filter, and the associated alarm times. Figure 40.8b shows the results from one of the 100 simulations with properly scaled coordinates. The failure time differences between MCS and Kalman prediction are shown in Fig. 40.9. The mean value and the standard deviation of the differences in the 100 simulations are −34.71 min and 65.90 min, respectively. The negative sign of the mean value indicates that the failure time predicted by the Kalman filter occurs earlier than the time generated by MCS. According to the Z formula [40.25], the error in estimating the mean value of the sample population can be calculated by

$$E_r^2 = \frac{Z_{\alpha/2}^2 \sigma^2}{n} \, .$$

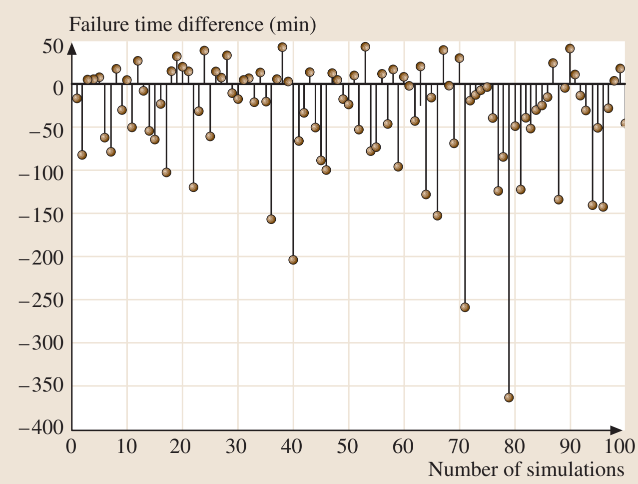

Fig. 40.9 Differences between the failure times given by Monte Carlo simulation results and Kalman filter predictions when the lead-time = 60 min

The Z value for a 99% confidence level is 2.575 [40.25]. Solving for E_r gives

$$E_r = \frac{(2.575)(65.8954)}{\sqrt{100}} = 16.97(\text{min}) \, .$$

According to the above data, we can say with 99% confidence that the mean value of the time difference between the MCS and the Kalman prediction is -34.71 ± 16.97 min, in other words from −17.74 min to −51.68 min. Taking the time difference into account,

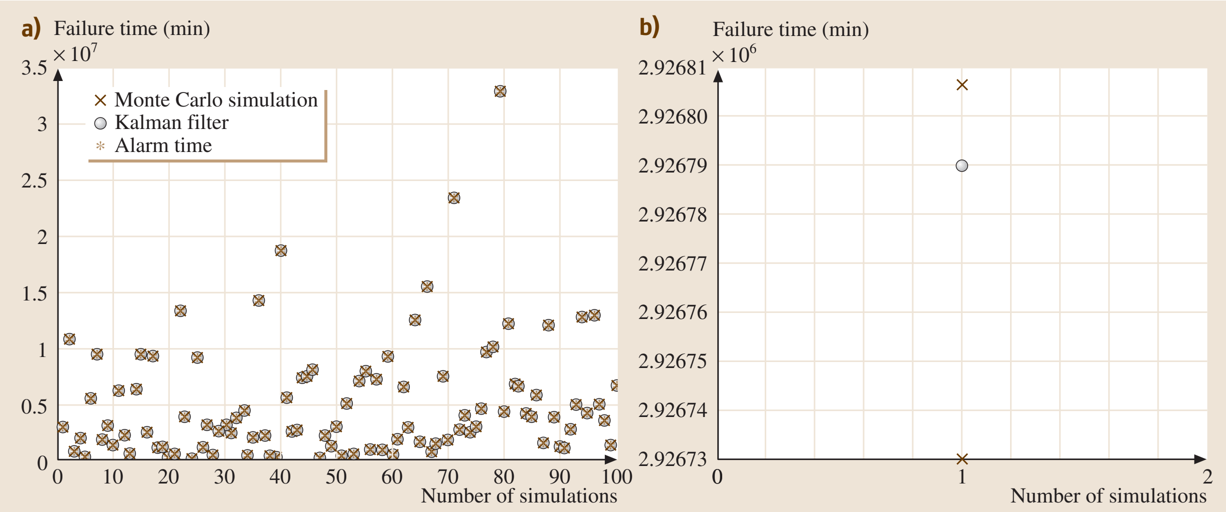

Fig. 40.8 Failure times generated by Monte Carlo simulation and predicted by Kalman filter when lead-time = 60 min

http://www.springer.com/978-1-85233-806-0

Springer Handbook of Engineering Statistics
Pham, H. (Ed.)
2006, XLIV, 1120 p. With CD-ROM., Hardcover
ISBN: 978-1-85233-806-0