Table 1.1 Common lifetime distributions used in reliability data analysis

Distribution	$f(t), \quad t>0$	$h(t)$	μ	σ^2	Parameter space
Exponential	$\theta\,\mathrm{e}^{-\theta t}$	θ	$1/\theta$	$1/\theta^2$	$\theta>0$
Weibull	$\lambda\kappa t^{\kappa-1}\,\mathrm{e}^{-\lambda t^{\kappa}}$	$\lambda\kappa t^{\kappa-1}$	$\lambda^{-1/\kappa}\Gamma\left(1+\frac{1}{\kappa}\right)$	$\lambda^{-2/\kappa}\left[\Gamma\left(1+\frac{2}{\kappa}\right) -\Gamma^2\left(1+\frac{1}{\kappa}\right)\right]$	$\kappa>0, \lambda>0$
Gamma	$\lambda^r\Gamma^{-1}(r)t^{r-1}\,\mathrm{e}^{-\lambda t}$	$\frac{\lambda^r t^{r-1}\,\mathrm{e}^{-\lambda t}}{\Gamma(r)\,[1-IG(r,\lambda t)]}$	r/λ	r/λ^2	$r>0, \lambda>0$
Log–normal	$\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{\frac{-(\log t-\mu)^2}{2\sigma^2}}$	$f(t)/R(t)$	$\mathrm{e}^{\mu+\sigma^2/2}$	$\mathrm{e}^{2\mu+2\sigma^2}-\mathrm{e}^{2\mu+\sigma^2}$	$-\infty<\mu<\infty, \sigma>0$
Logistic	$\frac{\mathrm{e}^{-(t-\lambda)/\beta}}{\beta\left(1+\mathrm{e}^{-(t-\lambda)/\beta}\right)^2}$	$\left[\beta\left(1+\mathrm{e}^{-(t-\lambda)/\beta}\right)\right]^{-1}$	λ	$(\beta\pi)^2/3$	$-\infty<\lambda<\infty, \beta>0$
Pareto	$\frac{m\theta^m}{t^{m+1}}$	$\frac{m}{t}$	$\frac{m\theta}{m-1}$	$\frac{m\theta^2}{(m-1)^2(m-2)}$	$t>\theta, m>0$
Extreme value	$\frac{\exp[-(t-a)/b]}{b\exp[-\exp(-(t-a)/b)]}$	$\frac{\exp[-(t-a)/b]}{b\exp[-\exp(-(t-a)/b)]-1}$	$a-b\Gamma'(1)$	$(b\pi)^2/6$	$-\infty<a<\infty, b>0$

A primary reason for its suitability to lifetime analysis is its flexible failure rate; unlike other distributions listed in Table 1.1, the Weibull failure rate is simple to model, easy to demonstrate and it can be either increasing or decreasing. A mixture of two Weibull distributions can be used to portray a bath-tub failure rate (as long as only one of the shape parameters is less than one). *Mudholkar*, et al. (1996) introduce a new shape parameter to a generalized Weibull distribution that allows bath-tub-shaped failure rates as well as a broader class of monotone failure rates.

For materials exposed to constant stress cycles with a given stress range, lifetime is measured in number of cycles until failure (N). The Whöler curve (or S–N curve) relates stress level (S) to N as $NS^b=k$, where b and k are material parameters (see [1.13] for examples). By taking logarithms of the S–N equation, we can express cycles to failure as a linear function: $Y=\log N=\log k-b\log S$. If N is log–normally distributed, then Y is normally distributed and regular regression models can be applied for predicting cycles to failure (at a given stress level). In many settings, the log–normal distribution is applied as the failure time distribution when the corresponding degradation process based on rates that combine multiplicatively. Despite having a concave-shaped (or upside-down bath-tub shape) failure rate, the log–normal is especially useful in modeling fatigue crack growth in metals and composites.

Birnbaum and *Saunders* [1.1] modeled the damage to a test item after n cycles as $B_n=\zeta_1+\cdots+\zeta_n$, where ζ_i represents the damage amassed in the i-th cycle. If failure is determined by B_n exceeding a fixed damage threshold value B^*, and if the ζ_i are identically and independently distributed,

$$P(N\le n)=P(B_n>B^*)\approx\Phi\left(\frac{B^*-n\mu}{\sigma\sqrt{n}}\right), \quad (1.3)$$

where Φ is the standard normal CDF. This results because B_n will be approximately normal if n is large enough. The reliability function for the test unit is

$$R(t)\approx\Phi\left(\frac{B^*-n\mu}{\sigma\sqrt{n}}\right) \quad (1.4)$$

which is called the *Birnbaum–Saunders* distribution. It follows that

$$W=\frac{\mu\sqrt{N}}{\sigma}-\frac{B^*}{\sigma\sqrt{N}} \quad (1.5)$$

has a normal distribution, which leads to accessible implementation in lifetime modeling (see [1.22] or [1.12] for more properties).

1.2.4 Lifetime Distributions from Degradation Modeling

These examples show how the product's lifetime distribution can be implied by knowledge of how it degrades in time. In general, degradation measurements have great potential to improve lifetime data analysis, but they also introduce new problems to the statistical inference. Lifetime models have been researched and refined for many manufactured products that are put on test. On the other hand, degradation models tend to be empirical

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