Contents

Part I Geometric Algebra

New Tools for Computational Geometry and Rejuvenation of Screw Theory ................................... 3
David Hestenes
1 Introduction ........................................ 3
2 Universal Geometric Algebra ..................... 4
3 Group Theory with Geometric Algebra .......... 6
4 Euclidean Geometry with Conformal GA ....... 8
5 Invariant Euclidean Geometry .................... 10
6 Projective Geometry ................................ 13
7 Covariant Euclidean Geometry with Conformal Splits ........ 14
8 Rigid Displacements ................................ 18
9 Framing a Rigid Body ............................. 20
10 Rigid Body Kinematics ............................ 22
11 Rigid Body Dynamics ............................. 24
12 Screw Theory ....................................... 26
13 Conformal Split and Matrix Representation .... 28
14 Linked Rigid Bodies & Robotics ................. 31
References ........................................... 33

Tutorial: Structure-Preserving Representation of Euclidean Motions Through Conformal Geometric Algebra ........................................ 35
Leo Dorst
1 Introduction ......................................... 36
2 Conformal Geometric Algebra .................... 36
2.1 Trick 1: Representing Euclidean Points in Minkowski Space 36
2.2 Trick 2: Orthogonal Transformations as Multiple Reflections in a Sandwiching Representation .... 39
2.3 Trick 3: Constructing Elements by Anti-Symmetry .... 42
2.4 Trick 4: Dual Specification of Elements Permits Intersection 43
<table>
<thead>
<tr>
<th>Page</th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Bonus: The Elements of Euclidean Geometry as Blades</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Bonus: Euclidean Motions Through Sandwiching</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Bonus: Structure Preservation and the Transfer Principle</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Trick 5: Exponential Representation of Versors</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Trick 6: Sparse Implementation at Compiler Level</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Engineering Graphics in Geometric Algebra</strong></td>
<td>Alyn Rockwood and Dietmar Hildenbrand</td>
</tr>
<tr>
<td>1</td>
<td>Introduction</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Benefits of Geometric Algebra for Computational Engineering</td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Unification of Mathematical Systems</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>Uniform Handling of Different Geometric Primitives</td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>Simplified Rigid Body Motion</td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td>Curl, Vorticity and Rotation</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>More Efficient Implementations</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Some Applications</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>The Geometric Primitives in More Detail</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Planes as a Limit of Spheres</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Distances Based on the Inner Product</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Approximation of Points with the Help of Planes or Spheres</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Computational Efficiency of Geometric Algebra using Gaalop</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Parameterization of 3D Conformal Transformations in Conformal</strong></td>
<td>Hongbo Li</td>
</tr>
<tr>
<td>1</td>
<td>Terminology and Notations</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Exponential Map and Exterior Exponential Map</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Twisted Vahlen Matrices and Quaternionic Vahlen Matrices</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Cayley Transform</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Conclusion</td>
<td></td>
</tr>
<tr>
<td></td>
<td>References</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Part II Clifford Fourier Transform</strong></td>
<td>Mawardi Bahri, Eckhard M.S. Hitzer, and Sriwulan Adji</td>
</tr>
<tr>
<td>1</td>
<td>Two-Dimensional Clifford Windowed Fourier Transform</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Real Clifford Algebra $\mathbb{R}^2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Clifford Fourier Transform (CFT)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2D Clifford Windowed Fourier Transform</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Definition of the CWFT</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Properties of the CWFT</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Examples of the CWFT</td>
<td></td>
</tr>
</tbody>
</table>
Contents

5 Comparison of CFT and CWFT .................................. 105
6 Conclusion ..................................................... 105
References ........................................................ 106

The Cylindrical Fourier Transform ........................................ 107
Fred Brackx, Nele De Schepper, and Frank Sommen
1 Introduction .................................................... 107
2 The Clifford Analysis Toolkit ..................................... 108
3 The Clifford–Fourier Transform .................................. 110
4 The Cylindrical Fourier Transform ................................. 111
  4.1 Definition .................................................. 111
  4.2 Properties .................................................. 112
  4.3 Spectrum of the $L^2$-Basis Consisting of Generalized
        Clifford–Hermite Functions ................................ 114
5 Application Potential of the Cylindrical Fourier Transform .... 117
6 Conclusion ..................................................... 118
References ........................................................ 119

Analyzing Real Vector Fields with Clifford Convolution and
Clifford–Fourier Transform ........................................... 121
Wieland Reich and Gerik Scheuermann
1 Fluid Flow Analysis ............................................. 121
2 Geometric Algebra ............................................... 122
3 Clifford Convolution ............................................ 124
4 Clifford–Fourier Transform ..................................... 124
5 Relation to Other Fourier Transforms ......................... 126
  5.1 $\mathbb{H}$-Holomorphic Functions and Fourier Series ........ 128
  5.2 Biquaternion Fourier Transform .......................... 130
  5.3 Two-Sided Quaternion Fourier Transform ................. 132
6 Conclusion ..................................................... 132
References ........................................................ 133

Clifford–Fourier Transform for Color Image Processing ............. 135
Thomas Batard, Michel Berthier, and Christophe Saint-Jean
1 Introduction .................................................... 135
2 Fourier Transform and Group Actions ................................ 136
3 Clifford–Fourier Transform in $L^2(\mathbb{R}^2, (\mathbb{R}^n, Q))$ ........ 138
  3.1 The Cases $n = 3, 4$: Group Morphisms from $\mathbb{R}^2$ to $\text{Spin}(4)$ 139
  3.2 The Cases $n = 3, 4$: The Clifford–Fourier Transform ........ 143
4 Application to Color Image Filtering ................................ 145
  4.1 Clifford–Fourier Transform of Color Images ............... 145
  4.2 Color Image Filtering ...................................... 147
5 Related Works .................................................. 151
  5.1 The Hypercomplex Fourier Transform of Sangwine et al. .... 152
  5.2 The Quaternionic Fourier Transform of Bülow ............ 153
6 Conclusion ..................................................... 155
Hilbert Transforms in Clifford Analysis

Fred Brackx, Bram De Knock, and Hennie De Schepper

1 Introduction: The Hilbert Transform on the Real Line
2 Hilbert Transforms in Euclidean Space
   2.1 Definition and Properties
   2.2 Analytic Signals
   2.3 Monogenic Extensions of Analytic Signals
   2.4 Example 1
   2.5 Example 2
3 Generalized Hilbert Transforms in Euclidean Space
   3.1 First generalization
   3.2 Second Generalization
4 The Anisotropic Hilbert Transform
   4.1 Definition of the Anisotropic Hilbert Transform
   4.2 Properties of the Anisotropic Hilbert Transform
   4.3 Example
5 Conclusion

References

Part III  Image Processing, Wavelets and Neurocomputing

Geometric Neural Computing for 2D Contour and 3D Surface Reconstruction

Jorge Rivera-Rovelo, Eduardo Bayro-Corrochano, and Ruediger Dillmann

1 Introduction
2 Geometric Algebra
   2.1 The OPNS and IPNS
   2.2 Conformal Geometric Algebra
   2.3 Rigid Body Motion
3 Determining the Shape of an Object
   3.1 Automatic Samples Selection Using GGVF
   3.2 Learning the Shape Using Versors
4 Experiments
5 Conclusion

References

Geometric Associative Memories and Their Applications to Pattern Classification

Benjamin Cruz, Ricardo Barron, and Humberto Sossa

1 Introduction
   1.1 Classic Associative Memory Models
<table>
<thead>
<tr>
<th>Contents</th>
<th>xiii</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Basics of Conformal Geometric Algebra</td>
<td>213</td>
</tr>
<tr>
<td>3 Geometric Algebra Classification Models</td>
<td>215</td>
</tr>
<tr>
<td>4 Geometric Associative Memories</td>
<td>216</td>
</tr>
<tr>
<td>4.1 Creating Spheres</td>
<td>216</td>
</tr>
<tr>
<td>4.2 Pattern Learning and Classification</td>
<td>221</td>
</tr>
<tr>
<td>4.3 Conditions for Perfect Classification</td>
<td>222</td>
</tr>
<tr>
<td>4.4 Conditions for Robust Classification</td>
<td>222</td>
</tr>
<tr>
<td>5 Numerical Examples</td>
<td>223</td>
</tr>
<tr>
<td>6 Real Examples</td>
<td>227</td>
</tr>
<tr>
<td>7 Conclusions and Future Work</td>
<td>228</td>
</tr>
<tr>
<td>References</td>
<td>229</td>
</tr>
</tbody>
</table>

Classification and Clustering of Spatial Patterns with Geometric Algebra 231
Minh Tuan Pham, Kanta Tachibana, Eckhard M.S. Hiter,
Tomohiro Yoshikawa, and Takeshi Furuhashi

1 Introduction                                                          231
2 Method                                                                232
2.1 Feature Extraction for Geometric Data                               233
2.2 Distribution Learning and Its Mixture for Classification            235
2.3 GA Kernel and Alignment and Semi-Supervised Learning for Clustering 237
3 Experimental Results and Discussion                                   239
3.1 Classification of Handwritten Digits                                240
3.2 Kernel Alignment and Web Questionnaire Analysis Results             242
4 Conclusions                                                           244
References                                                              246

QWT: Retrospective and New Applications                                249
Yi Xu, Xiaokang Yang, Li Song, Leonardo Traversoni, and Wei Lu

1 Introduction                                                          249
2 Evolution of Qwt and Principles of Quaternion Wavelet Construction    251
2.1 Evolution of QWT                                                    251
2.2 Principles of Quaternion Wavelet Construction                       254
3 The Mechanism of Adaptive Scale Representation in QWT                 257
4 The Potential Use of QWT in Image Registration                        261
5 The Potential Use of QWT in Image Fusion                              265
6 The Potential Use of QWT in Color Image Recognition                   268
7 Conclusion                                                            272
References                                                              272
# Part IV  Computer Vision

## Image Sensor Model Using Geometric Algebra: From Calibration to Motion Estimation

Thibaud Debaecker, Ryad Benosman, and Sio H. Ieng

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
</tr>
<tr>
<td>2</td>
<td>Introduction to Conformal Geometric Algebra</td>
</tr>
<tr>
<td>2.1</td>
<td>Geometric Algebras</td>
</tr>
<tr>
<td>2.2</td>
<td>Conformal Geometric Algebra (CGA)</td>
</tr>
<tr>
<td>3</td>
<td>General Model of a Cone-Pixels Camera</td>
</tr>
<tr>
<td>3.1</td>
<td>Geometric Settings</td>
</tr>
<tr>
<td>3.2</td>
<td>The General Model of a Central Cone-Pixel Camera</td>
</tr>
<tr>
<td>3.3</td>
<td>Intersection of Cones</td>
</tr>
<tr>
<td>4</td>
<td>General Cone-Pixel Camera Calibration</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental Protocol</td>
</tr>
<tr>
<td>4.2</td>
<td>Calibration Experimental Results</td>
</tr>
<tr>
<td>5</td>
<td>Motion Estimation</td>
</tr>
<tr>
<td>5.1</td>
<td>Problem Formulation</td>
</tr>
<tr>
<td>5.2</td>
<td>Cone Intersection Score Functions</td>
</tr>
<tr>
<td>5.3</td>
<td>Simulation Experiments for Motion Estimation Using Cone Intersection Criterion</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion and Future Works</td>
</tr>
</tbody>
</table>

## Model-Based Visual Self-localization Using Gaussian Spheres

David Gonzalez-Aguirre, Tamim Asfour, Eduardo Bayro-Corrochano, and Ruediger Dillmann

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Motivation</td>
</tr>
<tr>
<td>2</td>
<td>Outline of Visual Self-localization</td>
</tr>
<tr>
<td>2.1</td>
<td>Visual Acquisition of Landmarks</td>
</tr>
<tr>
<td>2.2</td>
<td>Data Association for Model Matching</td>
</tr>
<tr>
<td>2.3</td>
<td>Pose-Estimation Optimization</td>
</tr>
<tr>
<td>3</td>
<td>Uncertainty</td>
</tr>
<tr>
<td>3.1</td>
<td>Image-to-Space Uncertainty</td>
</tr>
<tr>
<td>3.2</td>
<td>Space-to-Ego Uncertainty</td>
</tr>
<tr>
<td>4</td>
<td>Geometry and Uncertainty Model</td>
</tr>
<tr>
<td>4.1</td>
<td>Gaussian Spheres</td>
</tr>
<tr>
<td>4.2</td>
<td>Radial Space</td>
</tr>
<tr>
<td>4.3</td>
<td>Restriction Lines</td>
</tr>
<tr>
<td>4.4</td>
<td>Restriction Hyperplanes</td>
</tr>
<tr>
<td>4.5</td>
<td>Duality and Uniqueness</td>
</tr>
<tr>
<td>5</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>

**Contents**

<table>
<thead>
<tr>
<th>Part</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part IV</td>
<td>Computer Vision</td>
</tr>
</tbody>
</table>

**Part IV  Computer Vision**

**Image Sensor Model Using Geometric Algebra: From Calibration to Motion Estimation**

Thibaud Debaecker, Ryad Benosman, and Sio H. Ieng

1. Introduction

2. Introduction to Conformal Geometric Algebra

   2.1 Geometric Algebras
   2.2 Conformal Geometric Algebra (CGA)

3. General Model of a Cone-Pixels Camera

   3.1 Geometric Settings
   3.2 The General Model of a Central Cone-Pixel Camera
   3.3 Intersection of Cones

4. General Cone-Pixel Camera Calibration

   4.1 Experimental Protocol
   4.2 Calibration Experimental Results

5. Motion Estimation

   5.1 Problem Formulation
   5.2 Cone Intersection Score Functions
   5.3 Simulation Experiments for Motion Estimation Using Cone Intersection Criterion

6. Conclusion and Future Works

**References**

**Model-Based Visual Self-localization Using Gaussian Spheres**

David Gonzalez-Aguirre, Tamim Asfour, Eduardo Bayro-Corrochano, and Ruediger Dillmann

1. Motivation

2. Outline of Visual Self-localization

   2.1 Visual Acquisition of Landmarks
   2.2 Data Association for Model Matching
   2.3 Pose-Estimation Optimization

3. Uncertainty

   3.1 Image-to-Space Uncertainty
   3.2 Space-to-Ego Uncertainty

4. Geometry and Uncertainty Model

   4.1 Gaussian Spheres
   4.2 Radial Space
   4.3 Restriction Lines
   4.4 Restriction Hyperplanes
   4.5 Duality and Uniqueness

5. Conclusion

**References**

**Contents**

<table>
<thead>
<tr>
<th>Part</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part IV</td>
<td>Computer Vision</td>
</tr>
</tbody>
</table>

**Part IV  Computer Vision**

**Image Sensor Model Using Geometric Algebra: From Calibration to Motion Estimation**

Thibaud Debaecker, Ryad Benosman, and Sio H. Ieng

1. Introduction

2. Introduction to Conformal Geometric Algebra

   2.1 Geometric Algebras
   2.2 Conformal Geometric Algebra (CGA)

3. General Model of a Cone-Pixels Camera

   3.1 Geometric Settings
   3.2 The General Model of a Central Cone-Pixel Camera
   3.3 Intersection of Cones

4. General Cone-Pixel Camera Calibration

   4.1 Experimental Protocol
   4.2 Calibration Experimental Results

5. Motion Estimation

   5.1 Problem Formulation
   5.2 Cone Intersection Score Functions
   5.3 Simulation Experiments for Motion Estimation Using Cone Intersection Criterion

6. Conclusion and Future Works

**References**

**Model-Based Visual Self-localization Using Gaussian Spheres**

David Gonzalez-Aguirre, Tamim Asfour, Eduardo Bayro-Corrochano, and Ruediger Dillmann

1. Motivation

2. Outline of Visual Self-localization

   2.1 Visual Acquisition of Landmarks
   2.2 Data Association for Model Matching
   2.3 Pose-Estimation Optimization

3. Uncertainty

   3.1 Image-to-Space Uncertainty
   3.2 Space-to-Ego Uncertainty

4. Geometry and Uncertainty Model

   4.1 Gaussian Spheres
   4.2 Radial Space
   4.3 Restriction Lines
   4.4 Restriction Hyperplanes
   4.5 Duality and Uniqueness

5. Conclusion

**References**
Part V Conformal Mapping and Fluid Analysis

Geometric Characterization of $M$-Conformal Mappings .......................... 327
   K. Gürlebeck and J. Morais
1 Introduction ................................ 327
2 Preliminaries ................................ 329
3 Monogenic-Conformal Mappings and Their Relation to Monogenic Functions .... 331
4 Influence of the Linear Part of a Monogenic Function ......................... 332
5 Observations and Perspectives ........................................ 340
References ................................................ 342

Fluid Flow Problems with Quaternionic Analysis—An Alternative Conception ........ 345
   K. Gürlebeck and W. Sprößig
1 Introduction ........................................ 345
2 Operator Calculus ................................... 346
   2.1 Operator Triple .................................. 347
   2.2 Plemelj-Type Projections ...................... 347
   2.3 Examples of $L$-Holomorphy .................... 348
   2.4 Quaternionic Analysis .......................... 348
   2.5 Bergman–Hodge Decomposition ............... 350
   2.6 Quaternionic Operator Calculus ............... 351
   2.7 Discrete Quaternionic Analysis ................ 352
3 Fluid Flow Problems .................................. 353
   3.1 A Brief History of Fluid Dynamics .......... 353
   3.2 Stationary Linear Stokes Problem ............ 354
   3.3 Nonlinear Stokes Problem ..................... 355
   3.4 Stationary Navier–Stokes Problem ............ 356
   3.5 Navier–Stokes Equations with Heat Conduction .... 357
   3.6 Continuous and Discrete Teodorescu Transforms .... 359
   3.7 Discrete Version of Navier–Stokes Equations .... 360
   3.8 Stationary Magneto-Hydromechanics .......... 360
4 Time-Dependent Fluid Flow Problems .................................. 366
   4.1 Characterization of Fluids .................... 366
5 Rothe’s Method of Semi-Discretization ................................ 368
   5.1 Time-Dependent Stokes Problem ............... 368
   5.2 Oseen’s Equation ................................ 369
   5.3 A Special Discretization Method ............... 369
   5.4 $(D + ia)$-Holomorphic Functions .............. 371
6 Approximation and Stability ......................................... 373
   6.1 Approximation Property ......................... 373
   6.2 Stability ......................................... 374
   6.3 Representation Formulae ......................... 374
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>More Relevant Problems in Fluid Dynamics</td>
<td>375</td>
</tr>
<tr>
<td>7.1</td>
<td>Magnetic Benard’s Problem</td>
<td>375</td>
</tr>
<tr>
<td>7.2</td>
<td>Boussinesq’s Formulation of Poisson–Stokes’ Problem</td>
<td>377</td>
</tr>
<tr>
<td>7.3</td>
<td>Shallow Water Equations</td>
<td>378</td>
</tr>
<tr>
<td>7.4</td>
<td>Forecasting Equations</td>
<td>378</td>
</tr>
<tr>
<td>8</td>
<td>Numerical Examples</td>
<td>379</td>
</tr>
<tr>
<td>9</td>
<td>Conclusions</td>
<td>380</td>
</tr>
</tbody>
</table>

**Part VI Crystallography, Holography and Complexity**

Interactive 3D Space Group Visualization with CLUCalc and Crystallographic Subperiodic Groups in Geometric Algebra

Eckhard M.S. Hitzer, Christian Perwass, and Daisuke Ichikawa

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td>385</td>
</tr>
<tr>
<td>2</td>
<td>Point Groups and Space Groups in Clifford Geometric Algebra</td>
<td>386</td>
</tr>
<tr>
<td>2.1</td>
<td>Cartan–Dieudonné and Geometric Algebra</td>
<td>386</td>
</tr>
<tr>
<td>2.2</td>
<td>Two-Dimensional Point Groups</td>
<td>387</td>
</tr>
<tr>
<td>2.3</td>
<td>Three-Dimensional Point Groups</td>
<td>387</td>
</tr>
<tr>
<td>2.4</td>
<td>Space Groups</td>
<td>388</td>
</tr>
<tr>
<td>3</td>
<td>Interactive Software Implementation</td>
<td>390</td>
</tr>
<tr>
<td>3.1</td>
<td>The Space Group Visualizer GUI</td>
<td>390</td>
</tr>
<tr>
<td>3.2</td>
<td>Space Group and Symmetry Selection</td>
<td>390</td>
</tr>
<tr>
<td>3.3</td>
<td>Mouse Pointer Interactivity</td>
<td>391</td>
</tr>
<tr>
<td>3.4</td>
<td>Visualization Options in Detail</td>
<td>393</td>
</tr>
<tr>
<td>3.5</td>
<td>Integration with the Online International Tables of Crystallography</td>
<td>393</td>
</tr>
<tr>
<td>4</td>
<td>Subperiodic Groups Represented in Clifford Geometric Algebra</td>
<td>394</td>
</tr>
<tr>
<td>4.1</td>
<td>Frieze Groups</td>
<td>395</td>
</tr>
<tr>
<td>4.2</td>
<td>Rod Groups</td>
<td>396</td>
</tr>
<tr>
<td>4.3</td>
<td>Layer Groups</td>
<td>396</td>
</tr>
<tr>
<td>5</td>
<td>Conclusion</td>
<td>397</td>
</tr>
</tbody>
</table>

**Geometric Algebra Model of Distributed Representations**

Agnieszka Patyk

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td>401</td>
</tr>
<tr>
<td>2</td>
<td>Geometric Algebra Model</td>
<td>402</td>
</tr>
<tr>
<td>3</td>
<td>Recognition</td>
<td>406</td>
</tr>
<tr>
<td>3.1</td>
<td>Right-Hand-Side Questions</td>
<td>407</td>
</tr>
<tr>
<td>3.2</td>
<td>Appropriate-Hand-Side Reversed Questions</td>
<td>412</td>
</tr>
<tr>
<td>4</td>
<td>Other Measures of Similarity</td>
<td>413</td>
</tr>
<tr>
<td>4.1</td>
<td>Matrix Representation</td>
<td>413</td>
</tr>
<tr>
<td>4.2</td>
<td>The Hamming Measure of Similarity</td>
<td>416</td>
</tr>
<tr>
<td>4.3</td>
<td>The Euclidean Measure of Similarity</td>
<td>416</td>
</tr>
<tr>
<td>4.4</td>
<td>Performance of Hamming and Euclidean Measures</td>
<td>417</td>
</tr>
</tbody>
</table>
Contents

5  The Average Number of Potential Answers ........................................... 418
6  Comparison with Previously Developed Models ................................. 425
7  Conclusion ....................................................................................... 429
References ......................................................................................... 429

Computational Complexity Reductions Using Clifford Algebras ............ 431
René Schott and G. Stacey Staples
1  Introduction .................................................................................... 431
2  Preliminaries .................................................................................. 432
  2.1  Graph Preliminaries ................................................................. 435
3  Complexity Reduction for Graph Problems: Nilpotent Adjacency
   Matrix Approach ............................................................................. 437
4  Matrix-Free Approach to Representing Graphs ................................ 440
5  Conclusion ....................................................................................... 451
References ......................................................................................... 452

Part VII Efficient Computing with Clifford (Geometric) Algebra

Efficient Algorithms for Factorization and Join of Blades ................. 457
Daniel Fontijne and Leo Dorst
1  Introduction .................................................................................... 457
2  Blade Factorization ......................................................................... 459
  2.1  New Algorithm for Blade Factorization .................................... 459
3  Algorithms for Computing the Join of Blades ............................... 462
  3.1  Fast Join Algorithm .................................................................... 462
  3.2  Computational Example ............................................................. 463
  3.3  Grade Stability of Fast Join Algorithm ...................................... 464
  3.4  Improved Fast Join Algorithm .................................................. 465
  3.5  Numerical Stability of the Fast Join Algorithms ....................... 465
4  Implementation ................................................................................ 466
  4.1  Code Generation ....................................................................... 467
  4.2  Implementation of the Fast Factorization Algorithm ................ 467
  4.3  Implementation of the Fast Join Algorithm ............................... 467
  4.4  Implementation of the Delta Product ........................................ 469
  4.5  Benchmarks ............................................................................... 469
5  Discussion ......................................................................................... 473
  5.1  Fast Factorization Algorithm .................................................. 473
  5.2  FastJoin Algorithms .................................................................. 474
  5.3  Simultaneous Computation of Meet and Join Costs More .......... 474
6  Conclusion ......................................................................................... 475
References ......................................................................................... 476

Gaalop—High Performance Parallel Computing Based on Conformal
Geometric Algebra ............................................................................ 477
Dietmar Hildenbrand, Joachim Pitt, and Andreas Koch
1  Introduction .................................................................................... 477
Some Applications of Gröbner Bases in Robotics and Engineering

Rafal Abłamowicz

1 Introduction .................................................. 495
2 Gröbner Basis Theory in Polynomial Rings ................. 495
   2.1 Examples of Using Gröbner Bases ..................... 498
3 Fermat Curves and Bézier Cubics .......................... 503
   3.1 Fermat Curves ........................................... 503
   3.2 Bézier Cubics .......................................... 504
4 Conclusions .................................................. 516
References ..................................................... 516

Index ............................................................ 519
Geometric Algebra Computing
in Engineering and Computer Science
Bayro Corrochano, E.; Scheuermann, G. (Eds.)
2010, XXII, 526 p., Hardcover