2 Basic Laser Optics

Boswell: Then, Sir, what is poetry? Johnson: Why, Sir, it is much easier to say what it is not. We all know what light is; but it is not easy to tell what it is

_Boswell’s Life of Johnson_

Open the second shutter so that more light can come in

_Attributed as the dying words of Johann Wolfgang von Goethe (1749–1832)_

In this chapter the basic nature of light and its interaction with matter is described and the fundamentals of how such energy can be manipulated in direction and shape are presented.

2.1 The Nature of Electromagnetic Radiation

Electromagnetic radiation has been a puzzle ever since man first realised it was there. Pierre de Fermat (1608–1665) stated the principles of ray propagation: “The path taken by a light ray in going from one point to another through any set of media is such as to render its optical path equal, in the first approximation, to other paths closely adjacent to the actual one” (i.e., the path will be the one with the minimum time: a concept much in vogue at the time following in the traditions laid down by Euclid and Hero of Alexandria). This is a rather complicated statement from which the laws of reflection and refraction can be derived. Christian Huygens (1629–1695) [1] introduced the wave concept of light to explain refraction and reflection. This he did through the “Huygens principle” that _each point on a wavefront may be regarded as a new source of waves_. Sir Isaac Newton (1642–1727) in 1704 unravelled the puzzle of colour and introduced the concept of light consisting of a number of tiny particles moving through space and subject to mechanical forces, the “corpuscular theory” [2]. Albert Einstein (1879–1955) [3] in 1905 invented the concept of the photon to explain the photoelectric effect and gave birth to the quantum theory of radiation. In fact there is still some mystery left. For example, if light passes through two parallel slits and then falls on a screen, as in Thomas Young’s (1773–1829) famous double-slit experiment, a diffraction pattern is formed on
the screen. The phenomenon can be simply explained by assuming that the radiation passing through the slits expands as a wavefront from the slits and makes an interference pattern on the screen. It is difficult to explain the outcome of the experiment by assuming the light is a stream of particles. However, in the photoelectric effect light falling on a target will give off electrons of fixed energy, \( E \), from the target regardless of the intensity of the incident light. \( E \) is given by

\[
E = h\nu - p, \tag{2.1}
\]

where \( h \) is Planck's constant \((6.625 \times 10^{-34} \text{ J s})\), \( \nu \) is frequency \((c/\lambda\), where \( c \) is the velocity of light, \( \text{i.e.}, 2.99 \times 10^8 \text{ m s}^{-1} \), and \( \lambda \) is the wavelength of light in metres\) and \( p \) is a constant characteristic of the material.

In the wave theory, the radiation would be spread over the surface and would not all be available for one electron.

This dichotomy between waves and particles varies in significance with the wavelength or energy of the “photons”. Thus, at the long wavelengths from radio to blue light, the wave theory explains most phenomena observed for normal intensities. With X-rays and \( \gamma \)-rays, which are highly energetic photons of short wavelength, the particle theory explains most events.

The quantum theory, of which we are talking here, was initiated by Werner Karl Heisenberg (1901–1976) \[4\] and Erwin Schrödinger (1887–1961) \[5\] in 1926. It makes a link between these states through Neil Bohr’s analysis of Planck’s constant. He suggested that the constant is the product of two variables, one characteristic of the wave and the other of a particle. Thus, if the wave has a period, \( T \), a wavelength, \( \lambda \), particle energy, \( E \), and momentum, \( p \), Bohr suggested, on dimensional grounds amongst others, that \( h = ET = p\lambda \). Thus, if the particle aspects are strong, then the wave aspects will be weak. It just happens that the size of Planck’s constant is such that the electromagnetic spectrum takes us from strongly particle type radiation to strongly wave type radiation. Why Planck’s constant is of such a size is unknown and must be left as an exercise for the readers and their heirs and successors! However, this concept that \( \lambda = h/p \) suggests all matter with momentum has a wavelength. This was shown to be the case for electrons by Davisson and Germer in the USA and G.P. Thomson in the UK, but the size of \( h \) makes the wavelength very small. The wavelength of Earth, for example, would be calculated as follows. The mass of Earth \( m = 5.976 \times 10^{24} \text{ kg} \) and the velocity of Earth \( v = 3 \times 10^4 \text{ m s}^{-1} \); therefore, Earth's wavelength \( \lambda = 6.625 \times 10^{-34}/(5.976 \times 10^{24} \times 3 \times 10^4) = 3.7 \times 10^{-63} \text{ m} \), which is a bit difficult to measure!

The momentum of a photon can be found from Planck’s law \( E = h\nu \) (justified from the photoelectric effect and other phenomena, where \( \nu \) is the frequency) and Einstein’s equivalence of mass and energy \( E = mc^2 \) (justified by experiments on nuclear disintegration).

Together these give

\[
h\nu = hc/\lambda = mc^2, \tag{2.2}
\]

and since the momentum \( p = mc \) we have \( p = h/\lambda \), which is the same as Bohr’s relationship quoted earlier.
Table 2.1  Photon properties of different lasers

<table>
<thead>
<tr>
<th>Device</th>
<th>Source of laser energy</th>
<th>Wavelength, λ (μm)</th>
<th>Frequency, ν (Hz)</th>
<th>Energy, E&lt;sup&gt;a&lt;/sup&gt; (eV)</th>
<th>(J × 10&lt;sup&gt;−20&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclotron</td>
<td>Accelerator</td>
<td>0.1 (X-ray)</td>
<td>2.9 × 10&lt;sup&gt;15&lt;/sup&gt;</td>
<td>12.3</td>
<td>192</td>
</tr>
<tr>
<td>Free-electron laser</td>
<td>Magnetic wiggler</td>
<td>1 × 10&lt;sup&gt;3&lt;/sup&gt;–10&lt;sup&gt;6&lt;/sup&gt;</td>
<td>1 × 10&lt;sup&gt;8&lt;/sup&gt;–10&lt;sup&gt;11&lt;/sup&gt;</td>
<td>1 × 10&lt;sup&gt;−6&lt;/sup&gt;</td>
<td>1 × 10&lt;sup&gt;−2&lt;/sup&gt;–10&lt;sup&gt;−5&lt;/sup&gt;</td>
</tr>
<tr>
<td>Excimer laser</td>
<td>Atomic electron orbits</td>
<td>0.249 (UV)</td>
<td>1.2 × 10&lt;sup&gt;15&lt;/sup&gt;</td>
<td>4.9</td>
<td>79.4</td>
</tr>
<tr>
<td>Argon ion</td>
<td></td>
<td>0.488 (blue)</td>
<td>6.1 × 10&lt;sup&gt;14&lt;/sup&gt;</td>
<td>2.53</td>
<td>40.4</td>
</tr>
<tr>
<td>He–Ne laser</td>
<td></td>
<td>0.6328 (red)</td>
<td>4.7 × 10&lt;sup&gt;14&lt;/sup&gt;</td>
<td>1.95</td>
<td>31.1</td>
</tr>
<tr>
<td>Nd:YAG laser</td>
<td>Molecular vibration</td>
<td>1.06 (IR)</td>
<td>2.8 × 10&lt;sup&gt;14&lt;/sup&gt;</td>
<td>1.16</td>
<td>18.5</td>
</tr>
<tr>
<td>CO laser</td>
<td></td>
<td>5.4</td>
<td>5.5 × 10&lt;sup&gt;13&lt;/sup&gt;</td>
<td>0.23</td>
<td>3.64</td>
</tr>
<tr>
<td>CO&lt;sub&gt;2&lt;/sub&gt; laser</td>
<td></td>
<td>10.6</td>
<td>2.8 × 10&lt;sup&gt;13&lt;/sup&gt;</td>
<td>0.12</td>
<td>1.85</td>
</tr>
</tbody>
</table>

<sup>a</sup> Energy calculated from \( E = hν \); 1 eV = 1.6 × 10<sup>−19</sup> J.

Incidentally this suggests that the pressure, \( P \), on a mirror from a photon from a CO<sub>2</sub> laser incident normally is (see Section 12.2.3.1) \( 2p = P = 2 \times 6.625 \times 10^{-28}/10.6 \times 10^{-6} = 1.25 \times 10^{-28} \) N s per photon, not of any great significance until one considers the avalanche of photons possible with the laser.

The energy of a photon from a CO<sub>2</sub> laser is given as 1.85 × 10<sup>−20</sup> J in Table 2.1, where it is compared with the energy of photons from other optical generators. Thus, in a 1-kW CO<sub>2</sub> laser beam there will be a flux of 1,000/1.85 × 10<sup>−20</sup> = 5 × 10<sup>22</sup> photons per second and the overall force will be 6 × 10<sup>−6</sup> N – still not very exciting, but possibly measurable. However, over the focused spot from this laser, of, say, 0.1 mm diameter, the pressure would be \((4 × 6 × 10^{-6})/[\pi(0.1 \times 10^{-3})^2] = 760 \) N m<sup>−2</sup>. This is equivalent to a depression in molten steel of approximately 1 cm! This is very close to what is observed. One wonders whether we have missed something in ignoring photon pressure.

It is assumed that the velocity of a photon is always \( c \), the velocity of light in a vacuum or the limiting velocity of all objects with finite rest mass, and that this is a universal constant. Photons do not behave as normal particles, which can have a variable velocity. The early explanations of refraction, for example, in which the wave theory explains the process by suggesting that the velocity of light varies from one medium to another, has to be interpreted as follows: the photon travels at the speed \( c \) always, but in passing through a medium, the wavefront slows owing to the absorption/re-emission processes taking place as the photon interacts with the molecules of the medium through which it travels. The reason for this universal constant is related to the concept of time: it has an uncanny ring that we have more thinking to do to understand this subject.

## 2.2 Interaction of Electromagnetic Radiation with Matter

When electromagnetic radiation strikes a surface, the wave travels as shown in Figure 2.1. Some radiation is reflected, some absorbed and some transmitted. As it passes
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Incident ray:
\[ E_i = E_{i0} \cos \left( \omega t - \frac{\omega z}{c} \right) \]

Reflected ray:
\[ E_r = E_{i0} \cos \left( \omega t - \frac{\omega z}{c} \right) \]

Transmitted ray:
\[ E_t = E_{i0} e^{-\beta z} \cos \left( \omega t - \frac{\omega z}{c} \right) \]

Figure 2.1 Phase and amplitude, \( E \), of an electromagnetic ray of frequency \( \omega \) travelling in the \( z \) direction striking an air–solid interface and undergoing reflection and transmission

through the new medium, it will be absorbed according to some law such as the Beer-Lambert law, \( I = I_0 e^{-\beta z} \). The absorption coefficient, \( \beta \), depends on the medium, the wavelength of the radiation and the intensity (see Section 2.2.1). The manner in which this radiation is absorbed, reflected or transmitted is considered to be as follows. Electromagnetic radiation can be represented as an electric vector field and a magnetic vector field as illustrated in Figure 2.2. When this passes over a small charged particle, the particle will be set in motion by the electric force from the electric field, \( E \). Provided that the frequency of the radiation does not correspond to a natural resonance frequency of the particle, then fluorescence or absorption will not occur, but a forced vibration would be initiated. The force induced by the electric field, \( E \), is very small and is incapable of vibrating an atomic nucleus. We are therefore discussing photons interacting with electrons which are either free or bound. This process of photons being absorbed by electrons is known as the “inverse bremsstrahlung effect”. (The bremsstrahlung effect is the emission of photons from excited electrons.) As the electron vibrates so it

Figure 2.2 The electric and magnetic field vectors of electromagnetic radiation
will either re-radiate in all directions (the reflected and transmitted radiation) or be re-
strained by the lattice phonons (the bonding energy within a solid or liquid structure),
in which case the energy would be considered absorbed, since it no longer radiates. In
this latter case the phonons will cause the structure to vibrate and this vibration will
be transmitted through the structure by the normal diffusion-type processes due to the
linking of the molecules of the structure. We detect the vibrations in the structure as
heat. The flow of heat is described by Fourier’s laws on heat conduction – a flux equation
\( \frac{q}{A} = -k \frac{dT}{dx} \) (see Chapter 5). If sufficient energy is absorbed, then the vibration
becomes so intense that the molecular bonding is stretched so far that it is no longer
capable of exhibiting mechanical strength and the material is said to have melted. On
further heating, the bonding is further loosened owing to the strong molecular vibra-
tions and the material is said to have evaporated. The vapour is still capable of absorbing
the radiation but only slightly since it will only have bound electrons; with sufficient ab-
sorption the electrons are shaken free and the gas is then said to be a plasma.

Plasmas can be strongly absorbing if their free-electron density is high enough. The
electron density in a plasma is given by equations such as the Saha equation (2.3) \([6]\),
which assumes thermal equilibrium in the plasma so that standard free-energy changes
can be calculated using conventional thermodynamic principles, which is not necessarily true with short laser pulses:

\[
\ln \left( \frac{N_1}{N_0} \right)^2 = -5040 \left( \frac{V_1}{T} \right) + 1.5 \ln \left( T + 15.385 \right),
\]

where \( N_1 \) is the ionisation density, \( N_0 \) is the density of atoms, \( V_1 \) is the ionisation po-
tential (eV) and \( T \) is the absolute temperature (K).

This indicates that temperatures of the order of 10,000–30,000 °C are required for
significant absorption (Figure 2.3) \([7]\). This sequence in the stages of absorption is il-
lustrated in Figure 2.4.

It is interesting to note that the energy absorbed by an electron may be that of one
or more photons; however, it will only be in extreme cases, such as the Vulcan laser
operating at 1 PW or so that a sufficient number of photons would be simultaneously

![Figure 2.3 Degree of ionisation as a function of temperature](image)
absorbed to allow the emission of X-rays during laser processing. This is a strategic advantage for the laser over electron beam processes, which require shielding against this hazard.

At these very high photon fluxes the electric field is sufficient to strip electrons from the atoms, which become charged and then repel each other. With femtosecond pulses ($10^{-15}$ s) there is no time for conduction and so the material forms a solid-state plasma, similar no doubt to the interior of stars.

Incidentally, the mean free time between collisions of electrons in a conductor is calculated to be around $10^{-13}$ s. This means that only for extremely short laser pulses of around 1 ps ($10^{-12}$ s per pulse) is it possible that the material would contain two temperatures not at equilibrium – the electron temperature and the atomic temperature. Also, for very short pulses non-Fourier conduction has been postulated [8], in which a compression or heat wave forms; this may be related to the acoustic signals noted in Section 12.2.3.1 or shock hardening mentioned in Section 6.19.

### 2.2.1 Nonlinear Effects

Ordinarily, the optical effects we experience are linear effects. When light interacts with matter, the matter responds in a proportionate way. Thus, we have the linear effects of reflection, refraction, scattering and absorption, all of which occur at the same frequency; the frequency of the light is not altered by the process. However, in 1961 Peter
Franken and others at the University of Michigan focused a high-powered ruby laser (red light) onto a quartz crystal and generated ultraviolet light mixed with the transmitted light. This was the birth of the new subject of nonlinear optics.

Today many electro-optic devices of practical importance depend upon nonlinear optical effects. These effects include second-harmonic generation as observed by Franken and his colleagues and optical rectification, the Pockel's electrooptic effect, sum and difference frequency mixing, the Kerr electro-optical effect, third-harmonic generation, general four-wave mixing, the optical Kerr effect, stimulated Brillouin scattering, stimulated Raman scattering, phase conjugation, self-focusing, self-phase modulation and two-photon absorption, ionisation and emission.

This exciting new area of physics has been opened up by the laser since the focused beam can generate huge electric and magnetic fields affecting the atomic dipoles (Lorentzian dipoles). At normal levels of radiation, several watts per square metre, the dipoles respond in one-to-one correspondence with the driving force, in fact linearly; however, at high levels of irradiation, several megawatts per square metre, the dipoles no longer respond linearly but more in the style of an overdriven pendulum and they exhibit a variety of harmonic oscillations. Via such effects it is possible to mix the frequencies of light waves. This is quite remarkable and against all the principles of the superpositioning of waves which were used to explain so much of earlier light theory, such as Young’s experiment.

2.2.1.1 Fluorescence

If a solid or a liquid is strongly illuminated by a frequency of radiation that it is able to absorb, it will become excited. To lose this energy the structure may simply become hot, or re-radiate at the same frequency “resonance radiation” or at a lower frequency
“fluorescence”. The lower frequency is predicted by Stokes law (Sir George G. Stokes, 1819–1903, Lucasian Professor of Mathematics at Cambridge University, who worked on spectroscopy, diffraction, viscosity – another Stokes law – and vector analysis). The reason is illustrated in Figure 2.5.

Fluorescence lifetime is an important diagnostic tool in medical studies to determine chemical groups such as amino acids and their environment – the subject is known as “fluorimetry” (see Chapter 11). Some materials emit very slowly and can be seen to glow after exposure as in the case of phosphors on watches and some TV screens – phosphorescence.

Fluorescent radiation is usually of a lower frequency than the stimulating radiation but it may be at a higher frequency (anti-Stokes radiation) if some extra energy is provided by the material being hot or a multiphoton event occurring.

Fluorescence of some materials can be stopped by irradiating them with infrared radiation. This has the effect of removing the excess energy in the structure of the material as heat. There are some commercial fluorescent screens on the market which will fluoresce in ultraviolet light from a lamp; the glowing screen can be used to image an infrared laser beam falling on it. On the other hand, a change in frequency can in some cases stimulate the fluorescence.

2.2.1.2 Stimulated Raman Scattering

If low-intensity light is transmitted through a transparent material, a small fraction is converted into light at longer wavelengths, with the frequency shift (Stokes shift) corresponding to the optical phonon frequency in the material. This process is called Raman scattering; see Figure 2.6. At higher intensities Raman scattering becomes stimulated and from the spontaneous scattering a new light beam can be built up. Under favourable

![Figure 2.6](image)

**Figure 2.6** Raman scattering. Input radiation of $\nu_0$ is inelastically scattered. In Stokes Raman scattering an overall transition to a higher vibrational state occurs, giving less energetic radiation of frequency $\nu_S$. In anti-Stokes Raman scattering the radiant shift is from a higher vibrational state to a lower one, giving more energetic radiation. Thus, Raman spectroscopy gives data on vibrational levels of a molecule, from which it can sometimes be identified.
conditions, the new beam can become more intense than the remaining original beam. The amplification is equally high in the forward and the backward directions. This may lead to a situation where a large fraction of the radiation is redirected towards the light source rather than towards the target. This could be a problem with intense light being transmitted in fibres, but also forms the basis of certain detection techniques, such as LIDAR (see Section 1.4.7).

2.2.1.3 Stimulated Brillouin Scattering

The same process takes place with the acoustical phonons as opposed to the lattice vibrations. The corresponding frequency shift is much smaller. Acoustical phonons are sound waves and the frequency shift exists only for the wave in the backward direction. Again, at high intensities the Brillouin effect becomes a stimulated process and the Brillouin wave may become much more intense than the original beam. Almost the entire beam may be reflected towards the laser source.

2.2.1.4 Second-harmonic Generation

Light waves are not supposed to interact with one another, but in the case of nonlinear interactions the nonlinear radiation itself couples the energy from one beam to another. This would not be possible in a vacuum. One can imagine the overstimulated structure being distorted and so affecting the absorption of other beams. In second-harmonic generation the nonlinear polarisation wave moves through the structure at one velocity and the primary refracted wave moves at another. For them to interact constructively, the phase velocities of the two waves must match. This can be done by using birefringent crystals, such as lithium niobate (LiNbO₃), lithium borate (LiB₃O₅) and others as listed in Table 2.2, whose refractive index depends on the direction and polarisation of the propagating light. If a polarised light wave passes through a birefringent crystal at just the right angle, the phase velocities of the induced polarisation wave and the second-harmonic wave can be made equal. However, this does mean that the angle and the temperature of the crystal have to be very carefully maintained. Once done, though, the effect is near magic. Thus, for example, a beam from a Nd:YAG laser is shone into the LiNbO₃ crystal held in a temperature enclosure at the correct an-

<table>
<thead>
<tr>
<th>Table 2.2 Common electro-optic materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
</tr>
<tr>
<td>Ba₂NaNb₅O₁₅</td>
</tr>
<tr>
<td>LiNbO₃</td>
</tr>
<tr>
<td>BaTiO₄</td>
</tr>
<tr>
<td>NH₄H₂PO₄</td>
</tr>
<tr>
<td>KH₂PO₄</td>
</tr>
<tr>
<td>LiIO₃</td>
</tr>
<tr>
<td>CdSE</td>
</tr>
<tr>
<td>KD₂PO₄</td>
</tr>
</tbody>
</table>
gle and the invisible infrared beam of 1.06 μm emerges, with some 30% converted to green light at 0.53 μm. Frequency tripling can also be obtained from crystals of different structures.

### 2.2.1.5 The Kerr Effect

When light is reflected from a magnetised medium, its state of polarisation and even its amplitude are changed. This effect is known as the Kerr effect after John Kerr (1824–1907), a Scottish physicist who was one Lord Kelvin’s first research students. When light is reflected from a surface, the surface electrons are moved by the incoming radiation electric field. If there is a magnetic field, then the direction of movement of the electrons will be affected as by the normal laws of electromagnetism and their angle of movement will be altered and hence the angle of polarisation with which they are emitted will be altered owing to their change in direction. The effect depends on the direction and strength of the magnetic field relative to the radiation.

There is an “optical Kerr effect”, which is a third-order nonlinear polarisation effect which can cause a change in the refractive index of the material subject to high-intensity radiation (see Section 2.2.1.6).

One of the more bizarre effects using this optical Kerr effect is optical phase conjugation. In one form, called degenerate four-wave mixing, two beams converge in the material and set up a form of grating within the material; a third wave couples nonlinearly with the others to form a phase-conjugated wave. This principle is applied to phase-conjugated mirrors. Phase-conjugated mirrors return the light to the source; any distortions between the source and the phase-conjugated mirrors are automatically compensated because of the phase reversal. Phase-conjugated mirrors are finding their way into commercial lasers to mitigate beam distortions and applications in adaptive optics are under development [9]. Self-focusing fibres are also a possibility using this effect.

### 2.2.1.6 The Pockel Effect

When an electric field is applied to certain materials, the electrostatic forces can distort the locations of the molecules of the material and result in a redistribution of the internal charges, causing a change in refractive index for noncentrosymmetric crystals such as CdTe and GaAs and anisotropic materials such as LiNbO₃ and KDP. This is known as the linear electro-optic effect, or the Pockel’s effect. The effect is used in a Pockel cell to spoil the lasing oscillations in some solid-state lasers by deflecting the beam. This is one form of Q switch known as an electro-optic Q switch (see Section 1.3.2.1).

For materials that have inversion symmetry, such as silicon, germanium, diamond and liquids and gases in general, the Pockel effect vanishes and the second-order electro-optic effect becomes noticeable, known as the optical Kerr effect (see the previous section).
2.3 Reflection or Absorption

The value of the absorption coefficient will vary with the same effects that affect the reflectivity. For opaque materials,

\[
\text{Reflectivity} = 1 - \text{absorptivity}.
\]

For transparent materials,

\[
\text{Reflectivity} = 1 - (\text{transmissivity} + \text{absorptivity}).
\]

In metals the radiation is predominantly absorbed by free electrons in an “electron gas”. These free electrons are free to oscillate and re-radiate without disturbing the solid atomic structure. Thus, the reflectivity of metals is very high in the waveband from the visible to the DC, i.e., very long wavelengths; see Figure 2.7. As a wavefront arrives at a surface, then all the free electrons in the surface vibrate in phase, generating an electric field 180° out of phase with the incoming beam. The sum of this field will be a beam whose angle of reflection equals the angle of incidence. This “electron gas” within the metal structure means that the radiation is unable to penetrate metals to any significant depth, only one to two atomic diameters. Metals are thus opaque and they appear shiny.

The reflection coefficient for normal angles of incidence from a dielectric or metal surface in air \((n = 1)\) may be calculated from the refractive index, \(n\), and the extinction coefficient, \(k\) (or absorption coefficient as described above), for that material:

\[
R = \frac{\left[ (1 - n)^2 + k^2 \right]}{\left[ (1 + n)^2 + k^2 \right]}.
\] (2.4)

For an opaque material such as a metal, the absorptivity, \(A\), is

\[
A = 1 - R,
\]

\[
A = \frac{4n}{\left[ (n + 1)^2 + k^2 \right]}.
\] (2.5)

Some values of these constants are given in Tables 2.3 and 2.4. The value of the reflectivity, \(R\), shown in Table 2.3 is 1 for a perfectly flat clean surface – which is rarely the case.

The variation of the amplitude of the electric field, \(E\), with depth, \(d\), is given by the Beer–Lambert law for a wavelength \(\lambda\) in a vacuum as

\[
E = E_0 \exp(-2\pi kd/\lambda).
\]

The intensity is proportional to the square of the amplitude and hence the variation of intensity with depth is given by

\[
I = I_0 \exp(-4\pi kd/\lambda).
\] (2.6)

For example, iron has a value of the extinction coefficient, \(k\), of 4.49 (Table 2.4) for 1.06-\(\mu\)m radiation. Thus, the intensity would have fallen to \(1/e^2\) (i.e., 0.13 times the incident value) after a depth of 0.038\(\mu\)m; and for 10.6-\(\mu\)m radiation with \(k = 32.2\) (Table 2.4), this depth becomes 0.052\(\mu\)m.
Table 2.3  Complex refractive index and coefficient of reflection for some materials to 1.06-μm radiation [45]

<table>
<thead>
<tr>
<th>Material</th>
<th>$k$</th>
<th>$n$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>8.50</td>
<td>1.75</td>
<td>0.91</td>
</tr>
<tr>
<td>Cu</td>
<td>6.93</td>
<td>0.15</td>
<td>0.99</td>
</tr>
<tr>
<td>Fe</td>
<td>4.44</td>
<td>3.81</td>
<td>0.64</td>
</tr>
<tr>
<td>Mo</td>
<td>3.55</td>
<td>3.83</td>
<td>0.57</td>
</tr>
<tr>
<td>Ni</td>
<td>5.26</td>
<td>2.62</td>
<td>0.74</td>
</tr>
<tr>
<td>Pb</td>
<td>5.40</td>
<td>1.41</td>
<td>0.84</td>
</tr>
<tr>
<td>Sn</td>
<td>1.60</td>
<td>4.70</td>
<td>0.46</td>
</tr>
<tr>
<td>Ti</td>
<td>4.0</td>
<td>3.8</td>
<td>0.63</td>
</tr>
<tr>
<td>W</td>
<td>3.52</td>
<td>3.04</td>
<td>0.58</td>
</tr>
<tr>
<td>Zn</td>
<td>3.48</td>
<td>2.88</td>
<td>0.58</td>
</tr>
<tr>
<td>Glass</td>
<td>0</td>
<td>1.5</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 2.4  Refractive index and Brewster angles for various materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$\lambda$ (μm)</th>
<th>Refractive index</th>
<th>Brewster angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k$</td>
<td>$n$</td>
<td>$\theta_B$</td>
</tr>
<tr>
<td>Al</td>
<td>1.06</td>
<td>8.5</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>10.6</td>
<td>34.2</td>
<td>0.108</td>
</tr>
<tr>
<td>Fe</td>
<td>1.06</td>
<td>4.49</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>10.6</td>
<td>32.2</td>
<td>5.97</td>
</tr>
<tr>
<td>Ti</td>
<td>3.48</td>
<td>2.88</td>
<td>70.8</td>
</tr>
<tr>
<td>Glass</td>
<td>–</td>
<td>1.5</td>
<td>56.3</td>
</tr>
</tbody>
</table>

2.3.1 Effect of Wavelength

At shorter wavelengths, the more energetic photons can be absorbed by a greater number of bound electrons and so the reflectivity falls and the absorptivity of the surface is increased (Figure 2.7).
2.3.2 Effect of Temperature

As the temperature of the structure rises, there will be an increase in the phonon population, causing more phonon–electron energy exchanges. Thus, the electrons are more likely to interact with the structure rather than oscillate and re-radiate. There is thus a fall in the reflectivity and an increase in the absorptivity with a rise in temperature for some metals, as seen in Figure 2.8 [10].

2.3.3 Effect of Surface Films

The reflectivity is essentially a surface phenomenon and so surface films may have a large effect. Figure 2.9 shows that for interference coupling the film must have a thickness of around \[ \frac{(2n + 1)}{4} \lambda \] to have any effect, where \( n \) is any integer. The absorption variation for CO\(_2\) radiation by a surface oxide film is shown in Figure 2.10 [10,11]. One form of these surface films may be a plasma [12] provided that the plasma is in thermal contact with the surface.

Figure 2.8 Reflectivity as a function of temperature for 1.06-μm radiation

Figure 2.9 A surface film as an interference coupling, “antireflection” coating. If \( 2d/\cos \phi = [(2n + 1)/2] \lambda \), then there will be destructive interference of the reflected ray
2.3.4 Effect of Angle of Incidence

The full theoretical analysis of reflectivity was first done by Drude [13] from atomistic considerations of the electron flux in a radiant field, which he then applied to the Maxwell (1831–1879) equations. It is sometimes known as “Drude reflectivity”. It showed a variation in reflectivity with both the angle of incidence and the plane of polarisation. If the plane of polarisation is in the plane of incidence, the ray is said to be a “p” ray (parallel); if the ray has its plane of polarisation at right angles to the plane of incidence, it is said to be an “s” ray (Senkrecht meaning “perpendicular”). The reflectivities for these two rays reflected from perfectly flat surfaces are given by:

\[ R_p = \frac{[n - (1/\cos \phi)]^2 + \kappa^2}{[n + (1/\cos \phi)]^2 + \kappa^2} \]  
\[ R_s = \frac{[n - \cos \phi]^2 + \kappa^2}{[n + \cos \phi]^2 + \kappa^2}. \]  

The variation of the reflectivity with angle of incidence is shown in Figure 2.11. At certain angles the surface electrons may be constrained from vibrating since to do so would involve leaving the surface. This they would be unable to do without disturbing the matrix, i.e., absorbing the photon. Thus, if the electric vector is in the plane of incidence, the vibration of the electron is inclined to interfere with the surface at high angles of incidence and absorption is thus high; however, if the plane is at right angles to the plane of incidence, then the vibration can proceed without reference to the surface or angle of incidence and reflection is preferred. There is a particular angle – the “Brewster angle” – at which the angle of reflection is at right angles to the angle of refraction. When this occurs it is impossible for the electric vector in the plane of incidence to be reflected since there is no component at right angles to itself. Thus, the reflected ray
2.3 Reflection or Absorption

will have an electric vector mainly in the plane at right angles to the plane of incidence. This is the reason why Polaroid\textsuperscript{®}\textsuperscript{1} spectacles reduce the glare from puddles. At this angle the angle of refraction \( \theta = (90^\circ - \text{angle of incidence}) \) and hence by Snell's law (see Section 2.4) the refractive index, \( n = \tan(\text{Brewster angle}) \). Any beam which has only one or principally one plane for the electric vector is called a “polarised” beam. Some values of the refractive index and the Brewster angles for different materials are given in Table 2.4.

Most lasers produce beams which are polarised owing to the nature of the amplifying process within the cavity which will favour one plane. Any plane will be favoured in a random manner, unless the cavity has folding mirrors, in which case the electric vector, which is at right angles to the plane of incidence on the folded mirrors, will be favoured because that is the one suffering the least loss.

2.3.5 Effect of Materials and Surface Roughness

Roughness has a large effect on absorption owing to the multiple reflections in the undulations (see Table 6.1, page 299). There may also be some “stimulated absorption” due to beam interference with sideways-reflected beams [14]. Provided the roughness is less than the beam wavelength, the radiation will not suffer these events and hence will perceive the surface as flat. The reflected phase front from a rough surface, formed from the Huygens wavelets, will no longer be the same as the incident beam and will spread in all directions as a diffuse reflection. It is interesting to note that it should not be possible to see the point of incidence of a red He–Ne beam on a mirror surface if the mirror is perfect.

\textsuperscript{1} Polaroid\textsuperscript{®} is a registered trademark of the Polaroid Corporation 4350 Baker Road Minnetonka, MN 55343-8684, USA. www.polaroid.com
2.4 Refraction

On transmission the ray undergoes refraction described by Snell’s law (Willebrord Snell, 1591–1626, Professor of Mathematics at Leiden University, Holland): “The refracted ray lies in the plane of incidence, and the sine of the angle of refraction bears a constant ratio to the sine of the angle of incidence”:

\[ \frac{\sin \phi}{\sin \psi} = \frac{n_1}{n_2}, \]

(2.9)

where \( n \) is the refractive index, \( \phi \) is the angle of incidence, \( \psi \) is the angle of refraction, \( v_1 \) is the apparent speed of propagation in medium 1 and \( v_2 \) is the apparent speed of propagation in medium 2.

The apparent change in the velocity of light as it passes through a medium is the result of scattering by the individual molecules. The scattered rays interfere with the primary beam, causing a retardation in the phase. Consider a plane wave striking a very thin, transparent sheet whose thickness is less than the wavelength of the incident light [15], as shown in Figure 2.12. Let the electric vector have a unit amplitude and then it can be represented at a particular time as \( E = \sin(2\pi x/\lambda) \). If the scattered intensity is small, then the intensity reaching some point, \( P \), will be essentially the intensity of original wave plus a small contribution from all the light scattered from all the atoms of the sheet. Now the energy scattered by one atom will be proportional to its scattering cross-section, \( \sigma \), which is that part of the area of the atom presented to the oncoming radiation. Thus, the scattered amplitude is proportional to \( \sqrt{\sigma} \). If there are \( N \) atoms per cubic centimetre, the total scattered amplitude per square centimetre would be proportional to \( Nt\sqrt{\sigma} \); where \( t \) is the thickness. Since it is assumed that \( t \sim \lambda \), the waves leaving the sheet will all be in phase. At point \( P \), however, their phases will differ by the different distances travelled, \( R \). We can calculate the net effect by summing the scattered amplitudes of all the atoms over the surface, \( E_s \) – allowing for the amplitude being proportional to \( 1/R \):

\[ E + E_s = \sin \left( \frac{2\pi x}{\lambda} \right) + \sqrt{\sigma} Nt \int_0^\infty \frac{2\pi r dr}{R} \sin \left( \frac{2\pi R}{\lambda} \right). \]

Figure 2.12  Radiation passing through a thin transparent layer
Since \( x^2 + r^2 = R^2 \) and \( x \) is constant, we have \( rd\,r = Rd\,R \), and the integral may be written as

\[
\int_0^\infty \frac{2\pi}{R} \sin \left( \frac{2\pi R}{\lambda} \right) r\,dr = 2\pi \int_x^\infty \sin \left( \frac{2\pi R}{\lambda} \right) \,dR = \frac{2\pi\lambda}{2\pi} \left[ -\cos \left( \frac{2\pi R}{\lambda} \right) \right]_{R=x}^{R=\infty}.
\]

(The integral limits are 0 to \( \infty \) for \( r \) and \( x \) to \( \infty \) for \( R \).)

At \( R = \infty \), the quantity in brackets is equal to zero and so we have

\[
E + E_s = \sin \left( \frac{2\pi x}{\lambda} \right) + \sqrt{\sigma} Nt\lambda \cos \left( \frac{2\pi x}{\lambda} \right).
\]

This is of the form \( \sin A + B\cos A \), where \( B \) is assumed to be very small. Under these conditions we may write

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B \approx \sin A + B\cos A.
\]

Therefore,

\[
E + E_s = \sin \left( \frac{2\pi x}{\lambda} + \sqrt{\sigma} Nt\lambda \right),
\]

which shows that the phase of the wave at point \( P \) has been altered by the amount \( Nt\lambda\sqrt{\sigma} \). However, we know that the presence of a sheet of refractive index \( n \) and thickness \( t \) would have retarded the phase by

\[
2\pi(n - 1) t/\lambda;
\]

hence

\[
\sqrt{\sigma} Nt\lambda = \frac{2\pi}{\lambda} (n - 1) t
\]

and so

\[
n - 1 = \frac{1}{2\pi} N\lambda^2 \sqrt{\sigma}.
\]

This derivation is not precise (it has not allowed for absorption) but it has shown the nature of the refraction process and how the material properties affect the refractive index. For example, introduce a strain and the value of \( N \) may vary, and so on. It does not show how \( n \) varies with \( \lambda \) since the scattered intensity does not just depend upon \( \sigma \) but also depends on \( 1/\lambda^4 \) -- the Rayleigh scattering law. The normal form of a dispersion curve (refractive index versus wavelength) is known as a Cauchy equation,

\[
n = A + B/\lambda^3 + C/\lambda^4,
\]

a semiempirical equation which is useful away from absorption bands.
2.4.1 Scattering

So far we have assumed that the medium through which the light is passing is uniform, but if it consists of numerous inhomogeneities acting as re-radiating centres the phenomenon of scattering is observed in which light may appear to no longer travel in straight lines: the back glare of car headlights in fog is an example. The extent of the scattering depends on the particle size and density. It comes in various forms.

2.4.1.1 Rayleigh Scattering

Particles much smaller than the wavelength of the incident light (for example, molecular clusters or imperfections in the silica lattice of a fibre) will scatter the radiation in the form of a spherical wave. The extent of this power loss depends on the number of particles and the wavelength. It has been found that this effect is proportional to $1/\lambda^4$. This is the reason the sky is blue, but it can also be a limiting factor in the design of fibres and some optics. For example, the attenuation of a laser beam, $E_{\text{attenuation}}$, passing through a plasma cloud, as in laser welding, could be described by the equation [16]

$$E_{\text{attenuation}} = P\left[1 - e^{-(Q_{\text{sca}} + Q_{\text{abs}})\pi r^2 N z}\right],$$

where $P$ is the laser beam power (W), $r$ is the average radius of the particles (m), $N$ is the number of particles per cubic metre and $z$ is the beam path length (m).

The Rayleigh scattering efficiency is given by

$$Q_{\text{sca}} = \frac{8}{3} \left(\frac{2\pi r}{\lambda}\right)^4 \left(\frac{m^2 - 1}{m^2 + 2}\right),$$

with the complex refractive index $m = (n + ik)$ and $\lambda$ the wavelength.

2.4.1.2 Mie Scattering

When the diameter of the particles is approximately the size of the incident wavelength, the scattering is less dependent on the wavelength. This is known as Mie scattering [17]. It is possibly very relevant to laser material processing as Hansen and Duley [18] reported. Within the keyhole or interaction zone, when there is some form of boiling or ablation, there is almost certainly an aerosol which will cause scattering of the incident beam, thus affecting the focus and processing conditions. Some interesting results were recorded by Akhter [19] when laser welding with a powder feed in which the absorption was enhanced by the presence of the powder. The calculations of Hansen and Duley [18] showed, for particles of radius $r$, that for $2\pi r/\lambda \gg 1$ there was strong forward scattering, a form of refocusing of the beam. This is a subject area which will merit further study in the years to come (see also Sections 2.2.1.2, 2.2.1.3).
2.4.1.3 Bulk Scattering

For particles much greater than the wavelength of incident radiation the scattered intensity is almost independent of the wavelength. This is the reason why snow and fog are white. Some of this form of radiation transfer must be present in blown powder laser cladding processes.

2.5 Interference

Light waves are electromagnetic disturbances that travel through space. A vibrating electric charge sets up changing electric and magnetic fields around it which spread through space at the speed of light in the form of spherical waves oscillating transverse to the direction of travel. Enough “spherical” wavelets integrate to make a wavefront of any given shape. The description of the relationship between these electric and magnetic fields is given in Maxwell’s famous set of four equations, from which all electromagnetic phenomena can be deduced – although that requires some effort! From them it is possible to show that the velocity of light \( c = \frac{1}{\sqrt{\left(\mu_0 \varepsilon_0\right)}} \), where \( \mu_0 \) is the magnetic permeability of space and \( \varepsilon_0 \) is the electric permeability of space representing the storing of energy in inductive or capacitative form – which is the basis of the oscillation.

Since they have a transverse wave form, for normal energies these waves can be linearly superimposed (see Section 2.2.1). Thus, for two waves travelling in opposite directions a standing wave may form, as in the laser cavity. Two waves of similar frequency but of slightly different direction travelling in the same direction gives rise to a standing transverse wave form – an interference pattern used in the laser Doppler anemometer and Michelson interferometer (Section 1.4) or mode structures as observed coming from a laser. If several beams of slightly differing frequency are collinear, this could create almost any wave form. If they are all sinusoidal wave forms, they can be separated analytically into their constituent waves by Fourier analysis. Two waves travelling with the same frequency and direction but with different planes of polarisation will give rise to elliptical or circularly polarised beams. The addition or subtraction of waves is known as interference.

2.6 Diffraction

On striking a sharp edge, the electromagnetic waves will spread and not remain as a collimated stream. One can imagine waves on water striking an edge, such as a harbour wall, after which they will expand into the harbour. The divergence angle of the wave stream is a function of the wavelength, the longer ones spreading more than the shorter ones. Thus, the roar from a distant road will have a lower note than that from a road roar nearer the source. This diffraction phenomenon was first noted by Francesco Grimaldi (1618–1663) and was demonstrated elegantly in Young’s double-slit experiment. Diffraction often leads to interference as two beams overlap. If the beams have
a plane front (far field), then the phenomenon will be described as Fraunhofer diffraction (after Joseph von Fraunhofer, 1787–1826) and if they have curved front (near field), then it will be described as Fresnel diffraction (after Augustin-Jean Fresnel, 1788–1827, who did the first analytical analysis of diffraction). The calculation of diffraction from a slit is given in Section 2.8.

2.7 Laser Beam Characteristics

The energy from a laser is in the form of a beam of electromagnetic radiation. Apart from power, it has the properties of wavelength, coherence, power distribution or mode, diameter and polarisation. These are now discussed in the following sections.

2.7.1 Wavelength

Since the invention of the laser in 1960, many hundreds of lasing systems have been developed but only a few of commercial significance in material processing. Some of the wavelengths of the important material processing lasers are shown in Table 0.1. The wavelength depends on the transitions taking place by stimulated emission. The wavelength may be broadened by Doppler effects due to the motion of the emitting molecules or by related transitions from higher quantised states as with the CO laser. On the whole, the radiation from a laser is amongst the purist spectral forms of radiation available. Very high spectral purity can be achieved by using a frequency-selecting grating as the rear mirror of the laser optical cavity, but this is rarely worth the effort for material processing. In consequence, if one wishes to achieve a very short pulse of light, for example, of 1 fs (a beam of light around 0.3 μm long!), it is not possible without first making a laser with a broader waveband, as is required by the Fourier series, which defines such a short pulse wavefront. But that is a problem for others who are not so involved in material processing.

2.7.2 Coherence

The stimulated emission phenomenon means that the radiation is generating itself and in consequence a continuous waveform is possible with low-order mode beams. The length of the continuous wavetrain may be many metres long. The comparison of laser light with standard random light is illustrated in Figure 2.13. This long coherence length allows some extraordinary interference effects with laser light, as noted in Chapter 1, such as length gauging, speckle interferometry, holography and Doppler velocity measurement. This property has not yet been used in material processing. In years to come it may be that someone will be able to use it as a penetration meter or to carry out subtle experiments with interference-banded heat sources.
2.7 Laser Beam Characteristics

2.7.3 Mode and Beam Diameter

A laser cavity is an optical oscillator. When it is oscillating there will be standing electromagnetic waves set up within the cavity and defined by the cavity geometry. It is possible to calculate the wave pattern for such a situation and it is found that there are a number of longitudinal standing waves at slightly varying angles. The number of such off-axis standing waves is related to the Fresnel number \( (a^2/\lambda L) \) (see Section 1.2.1.2). These standing waves interfere with each other giving a transverse standing wave which emerges from the cavity as the mode structure of the beam. For a nonamplifying, cylindrical cavity the amplitude of the transverse standing wave pattern, \( E(r, \phi) \), is given by a Laguerre–Gaussian distribution function of the form

\[
E(r, \phi) = E_0 \left( \frac{\sqrt{2} r}{w(z)} \right)^n L^n_p \left( \frac{2r^2}{w^2(z)} \right) \exp \left( -\frac{r^2}{w^2(z)} \right) \left( \frac{\sin}{\cos} n\phi \right),
\]

where \( E(r, \phi) \) is the amplitude at point \( r, \phi \), \( w(z) \) is the beam radius at point \( z \) along beam path, \( r \) is the radial position, \( \phi \) is the angular position, \( n \) is an integer and

\[
L^n_p(x) = e^x x^{n-4} \frac{d^p}{dx^p} \left( e^{-x} x^p \right),
\]

which is the generalised Laguerre polynomial (Edmond Laguerre 1834–1886). Some low-order polynomials are

\[
\begin{align*}
L^0_0(x) &= 1, \\
L^n_1(x) &= n + 1 - x, \\
L^2_2(x) &= 1/2(n + 1)(n + 2) - (n + 2)x + 1/2x^2.
\end{align*}
\]

The intensity distribution is found from the square of the amplitude:

\[
P(r, \phi) = E^2(r, \phi).
\]

These are the classical mode distributions for a circular beam. The distributions for a square beam are similar, but with Hermite polynomials. A plot of the amplitude and
spatial intensity distributions which this expression represents for various orders of mode is shown in Figures 2.14 and 2.15. Typical mode patterns that would be made from such beams are shown in Figure 2.16.

The classification of these transverse electromagnetic mode patterns is by \( \text{TEM}_{p \, l \, q} \) where \( p \) is the number of radial zero fields, \( l \) is the number of angular zero fields and \( q \) is the number of longitudinal zero fields.

Most slow flow lasers operate with a near perfect TEM00 or TEM01\(^*\) mode. The TEM01\(^*\) mode is made from an oscillation between two orthogonal TEM01 modes as illustrated in Figure 2.16.

Most fast axial flow lasers also give a beam with a low-order mode since they have long, narrow tubes – low Fresnel number \( (a^2/\lambda L) \) – (see Section 1.2.1.2). The modes from these lasers may be slightly distorted owing to plasma density variations.

Transverse flow lasers usually have multimode beams of indeterminate ranking. They are either quasi-Gaussian – in that they are a single lump of power – or asymmetric owing to the transverse amplification being different across the cavity owing to the heating of the gas as it traverses. To reduce this effect some cavities are ring-shaped – see Section 1.2.1.3.3.

The higher the order of the mode, the more difficult it is to focus the beam to a fine spot, since the beam is no longer coming from a virtual point.
A question arises in material processing as to what is the beam diameter. For example, the data in Figures 2.14 and 2.15 were calculated with the mathematical radius, \( w(z) \), the same. This is obviously not related to the diameter which affects heating processes. Sharp et al. [20] argue that the beam diameter should be defined as that distance within which \( 1/e^2 \) of the total power exists. (See Chapter 12 for methods of measuring the beam diameter.)

### 2.7.4 Polarisation

The stimulated emission phenomenon not only produces long trains of waves but these waves will also have their electric vectors all lined up. The beam is thus polarised. Many of the early lasers and some of the more modern ones which do not have a fold in the cavity will produce randomly polarised beams. In this case the plane of polarisation of the beam changes with time – and the cut quality may show it! To avoid this it is necessary to introduce into the cavity a fold mirror of some form. Outside the cavity such a fold would make no noticeable difference. Inside the cavity it is a different matter since the cavity is an amplifier and hence the least-loss route is the one being amplified in preference to the others – in fact almost to their total exclusion. Polarised beams have a directional effect in certain processes, for example, cutting, owing to the reflectivity effects on the sloping cut front shown in Figure 2.11 and discussed in Section 2.3.4. Hence, material processing lasers are usually engineered to give a polarised beam which is then fitted with a circular polariser – see Section 2.9.2.
Polarisation plays a role in the reflection and scattering of all light. If the electric vector is all aligned in one direction, then the beam is “linearly polarised”. If it has two vector directions at right angles to each other of equal intensity, it is said to be “circularly polarised” – if the field rotates clockwise to an observer looking into the beam then it is said to be “right-circular polarised” as opposed to “left-circular polarised”. With one vector stronger than the other it is “elliptically polarised”. The “extinction ratio” is the ratio between the maximum and minimum intensities of the beam after passing through a polarisation filter. Birefringent crystals have fast and slow indices of refraction for different states of polarisation. Certain molecules, notably quartz and sugars, can rotate the plane of polarisation of transmitted beams. Known forms of life are overwhelming composed of amino acids with left-handed optical activity and use sugars that are right-handed – unlike laboratory-prepared sugars and amino acids. A meteorite discovered in Australia in 1969 contained a surprising quantity of amino acids with this same bias towards left-handedness, thus posing some interesting questions. Bees are considered to navigate by the polarisation of the sunlight scattered from the atmosphere [21].

2.8 Focusing with a Single Lens

To manipulate the beam, to guide it to the workplace and shape it, there are many devices which have so far been invented. These devices are now discussed together with the basic theory of their design. In nearly all of them the simple laws of geometric optics listed in Table 2.5 are sufficient to understand how they work, but to calculate the precise spot size and depth of focus one needs to refer to Gaussian optics and diffraction theory.

2.8.1 Focused Spot Size

2.8.1.1 Diffraction-limited Spot Size

A beam of finite diameter is focused by a thin lens onto a plate as shown on Figure 2.17. The individual parts of the beam striking the lens can be imagined to be point radiators of a new wavefront. The lens will draw the rays together at the focal plane and constructive and destructive interference will take place there. When two rays arrive at the screen and they are half a wavelength out of phase, then they will destructively interfere and the light intensity will fall; the converse will occur when they arrive in phase. Thus, if ray AB (Figure 2.17) is $\lambda/2$ longer than ray CB, point B will represent the first dark ring of what is known as a “Fraunhofer diffraction pattern” (assuming the wavefronts are planar). The central maximum will contain approximately 86% of all the power in the beam. The diameter of this central maximum will be the focused beam diameter, usually measured between the points where the intensity has fallen to $1/e^2$ of the central value.
Table 2.5 Gaussian optical properties

<table>
<thead>
<tr>
<th>Terminology</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_2$</td>
</tr>
<tr>
<td>$h_1$</td>
</tr>
<tr>
<td>$R_1$</td>
</tr>
<tr>
<td>$R_2$</td>
</tr>
<tr>
<td>$f$</td>
</tr>
<tr>
<td>$u$</td>
</tr>
<tr>
<td>$v$</td>
</tr>
</tbody>
</table>

$m = h_2/h_1 = \text{magnification}; R_1, R_2 = \text{radii of curvature}; n_1, n_2 = \text{refractive index of two medium.}$

<table>
<thead>
<tr>
<th>Spherical surface</th>
<th>Plane surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$</td>
<td>$f = -\infty$</td>
</tr>
<tr>
<td>$f = -\frac{R}{2}$</td>
<td></td>
</tr>
<tr>
<td>$m = -\frac{v}{u}$</td>
<td>$m = +1$</td>
</tr>
<tr>
<td>Concave: $f &gt; 0, R &lt; 0$</td>
<td>Convex: $f &lt; 0, R &gt; 0$</td>
</tr>
<tr>
<td>Refraction at single surface</td>
<td></td>
</tr>
<tr>
<td>$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$</td>
<td>$v = -\frac{n_2 u}{n_1}$</td>
</tr>
<tr>
<td>$m = -\frac{n_1 v}{n_2 u}$</td>
<td>$m = +1$</td>
</tr>
<tr>
<td>Concave: $R &lt; 0$</td>
<td>Convex: $R &gt; 0$</td>
</tr>
<tr>
<td>Refraction at a thin lens</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$</td>
<td></td>
</tr>
<tr>
<td>$m = -\frac{v}{u}$</td>
<td></td>
</tr>
<tr>
<td>Concave: $f &lt; 0$</td>
<td>Convex: $f &gt; 0$</td>
</tr>
</tbody>
</table>

Figure 2.17 The diffraction-limited spot size
For a rectangular beam with a plane wavefront, the first dark fringe will occur when the beam path difference between the centre and the edge rays, \( d \), is \( \lambda/2 \)

\[
d = \lambda/2 = (D/2) \sin \varphi.
\]

That is, when \( \lambda = D \sin \varphi \), or for other fringes when \( m\lambda = D \sin \varphi \).

From geometry, \( 2y = 2f \tan \varphi \), and for small angles \( \tan \varphi = \sin \varphi = \lambda/D \).

Therefore, \( 2y = d_{\text{min}} = 2f\lambda/D \).

For plane front circular beams there is a correction of 1.22 and so the equation becomes

\[
d_{\text{min}} = 2.44f\lambda/D.
\] (2.11)

For Gaussian beams there is sometimes a further small correction. The focal spot size for a multimode beam will be larger because the beam is coming from a cavity having several off-axis modes of vibration and therefore not all coming from an apparent point source. This correction for a TEM_{plq} beam is

\[
d_{\text{min}} = 2.44\left(\frac{f\lambda}{D}\right)(2p + l + 1).
\] (2.12)

Radial nulls, \( p \), are more damaging to the focal spot size than angular nulls, \( l \). For example, the expected spot size for a CO_2 laser beam 22 mm in diameter with a TEM01 mode focused by a 125 mm focal length lens would be expected to be \( d_{\text{min}} = 2.44\left[\left(\frac{125 \times 10.6 \times 10^{-3}}{22}\right)\times 2\right] = 0.29\text{ mm}, \) whereas a TEM10 beam would be expected to focus to 0.44 mm.

2.8.1.2 \( M^2 \) Concept of Beam Quality

An unmodified laser beam diverges by diffraction from its initial waist value of \( D_0 \) at an increasing rate as shown in Figure 2.18, and reaches a maximum value only at infinity. This maximum value is the far-field divergence, \( \Theta_{0\infty} \). If a lens focuses the beam, it forms a new waist, \( D_1 \). The beam converges towards and diverges away from this new waist with a far-field divergence of \( \Theta_{1\infty} \) where

\[
D_0 \Theta_{0\infty} = D_1 \Theta_{1\infty} = \text{constant}.
\]

Figure 2.18  Variation of radius of curvature of the phase field with distance. A small value of \( R \) is known as the “near field”, whereas a large value is known as the “far field”
This constancy of $\theta$ values through the system with aberration-free optics allows the calculation of spot size, depth of focus, Rayleigh length and curvature of phase fronts.

To be able to use this property, we need to define a quality factor comparing the actual beam divergence, $\Theta_{\text{act}}$, with the divergence from a Gaussian laser beam with the same initial waist size, $\Theta_r$. Consider a laser cavity giving an actual beam divergence of $\Theta_{\text{act}}$ and having a beam waist radius $W_0$. A Gaussian beam originating from the same virtual origin as the actual beam would have a divergence $\Theta_{\text{Gauss}}$ and a beam waist radius $w_0$ defined by the Gaussian beam propagation equation for a diffraction-limited Gaussian beam (TEM00) (see Figure 2.18):

$$w^2(z) = w_0^2 \left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]$$

(2.13)

where $w(z)$ is the beam radius at a distance $z$ from the waist position of radius $w_0$ for a beam of wavelength $\lambda$.

In the far field, $z$ becomes large; hence,

$$\left(\frac{\lambda z}{\pi w_0^2}\right)^2 \gg 1$$

and hence $\Theta_{\text{Gauss}}$, which is equal to $w(z)/z = \lambda/\pi w_0$ from Equation 2.13.

It can be seen that $\Theta_{\text{Gauss}} w_0 = \lambda/\pi = \text{constant}$ for all Gaussian beams as noted above.

Using the same propagation equation, the divergence, $\Theta_r$, of a Gaussian beam with the same waist radius as the actual beam, $W_0$, is

$$\Theta_r = \frac{\lambda}{\pi W_0}.$$ 

If we define the ratio $M = \Theta_{\text{act}}/\Theta_{\text{Gauss}}$, this equals $W_0/w_0$ since the Gaussian comparator beam and the actual beams have the same virtual origin at a point at a distance $l$ from the waist. Thus, $\Theta_{\text{act}} = W_0/l$ and $\Theta_{\text{Gauss}} = w_0/l$, making

$$\Theta_{\text{act}}/\Theta_{\text{Gauss}} = W_0/w_0 = M$$

and

$$w_0 = W_0/M.$$ 

Then

$$\Theta_{\text{Gauss}} = \frac{\lambda}{\pi \left( W_0/M \right)}$$

therefore,

$$\Theta_{\text{act}} = M \left( \frac{\lambda M}{\pi W_0} \right).$$
But

\[ \Theta_r = \lambda / (\pi W_0) \]

and thus

\[ M^2 = \Theta_{act} / \Theta_r. \] (2.14)

This is the comparator which we sought. It is sometimes expressed as \( Q = M^2 \), which avoids the rather tedious argument just presented [22]. In a recent International Organization for Standardization (ISO) standard it is also described as \( 1/K \), where \( K \) is yet another measure of quality. All are based on the same comparison with Gaussian beams.

Applying this to a lens, we have

\[ \Theta_{act} = \frac{D_L}{2f} \text{ and } \Theta_r = \frac{2\lambda}{\pi d_{\text{min}}}; \]

therefore,

\[ d_{\text{min}} = \frac{4M^2 f \lambda}{\pi D_L}. \]

It can be seen that this quality factor, \( M^2 \) or \( Q \), allows real beams of higher-order mode than the basic Gaussian TEM00 to be treated as Gaussian by using a modified wavelength, \( M^2 \lambda \).

Thus, knowing \( M^2 \), one can calculate various beam characteristics:

1. The beam diameter, \( D \), at any distance along the beam path, \( z \), from the beam waist is given from the basic propagation equation:

\[ D_z = D_0 \left[ 1 + \left( \frac{4M^2 \lambda z}{\pi D_0^2} \right)^2 \right]^{\frac{1}{2}} \] (2.15)

2. The wavefront radius, \( R_z \), at any distance, \( z \), from the beam waist is given by

\[ R_z = z \left[ 1 + \left( \frac{\pi D_0^2}{4M^2 \lambda z} \right)^2 \right]. \] (2.16)

3. The Rayleigh range, \( R \), which is the distance from the beam waist of diameter \( D_0 \) to the position where it is \( \sqrt{2}D_0 \), is

\[ R = \left( \frac{\pi D_0^2}{4M^2 \lambda} \right). \] (2.17)
2.8 Focusing with a Single Lens

The Rayleigh range is the multiplier in the equations for $D_z$ and $R_z$:

\[
D_z = D_0 \left[ 1 + \left( \frac{z}{R} \right)^2 \right]^{\frac{1}{2}} \quad \text{and} \quad R_z = z \left[ 1 + \left( \frac{R}{z} \right)^2 \right].
\]

4. The depth of focus is the distance either side of the beam waist, $D_0$, over which the beam diameter grows by 5% (see also Section 2.8.2):

\[
DOF = \pm 0.08 \pi \frac{D_0^2}{M^2 \lambda}.
\] (2.18)

5. Focused spot size. Since $D_0 \Theta_\infty = D_1 \Theta_\infty$ for all aberration-free optical systems, then $D_1 = D_0 \Theta_0 / \Theta_\infty$ for a focusing lens placed at the beam waist, the preferred place since the wavefront is plane at that location.

\[
\Theta_\infty = D_0 / 2f
\]

\[
\Theta_\infty = 2M^2 \lambda / (\pi D_0).
\] (2.19)

Therefore,

\[
d_{\text{min}} = f \Theta_\infty = 4fM^2 \lambda / (\pi D_0).
\]

For a focusing lens placed $z$ millimetres from the beam waist,

\[
d_{\text{min}} = f \Theta_\infty \left( \frac{D_0}{D_z} \right) = \frac{4fM^2 \lambda}{\pi D_z}.
\]

At this point it is interesting to note that $d_{\text{min}} = f \Theta_\infty$ is independent of wavelength if $\Theta_\text{act}$ is mainly decided by the cavity optics. This is a result of $M^2$ being inversely proportional to $\lambda$. Thus, there is no particular focusing advantage in using shorter-wavelength lasers for a given cavity, for example, either CO or CO$_2$ lasers using the same cavity.

The usefulness of $M^2$ is apparent from the above equations. However, like all good things in life there is a snag – how to measure $\Theta_\text{act}$?

The beam expands as described in Equation 2.13 and shown in Figure 2.18. However, unless one measures the beam expansion at infinity, one is likely to measure something other than $\Theta_\text{act}$ such as the trigonometric divergence, $\Theta_T$, or the local divergence, $\Theta_L$ [22]. Figure 2.19 shows these three values. They can be calculated approximately from the wave propagation equation (2.13):

\[
D_z = D_0 \left[ 1 + \left( \frac{z}{R} \right)^2 \right]^{\frac{1}{2}}
\] (2.20)

where the Rayleigh range $R = \frac{\pi D_0^2}{4M^2 \lambda}$. 
Figure 2.19 The various angles of divergence discussed in establishing the value of the beam quality factor $M^2$.

Now $dD_z/dz \rightarrow \Theta_\infty$ as $z \rightarrow \infty$; thus, by differentiating, we get

$$\frac{dD_z}{dz} = \frac{D_0}{R^2} \left[ \frac{z}{1 + \left( \frac{z^2}{R^2} \right)^{1/2}} \right].$$

As $z \rightarrow \infty$

$$\left( 1 + \frac{z^2}{R^2} \right)^{1/2} \rightarrow \left( \frac{z^2}{R^2} \right)^{1/2} = \frac{z}{R};$$

hence,

$$\Theta_\infty = \frac{dD_z}{dz} \bigg|_{z=\infty} = \frac{D_0}{R^2} \left[ \frac{z}{z/R} \right] = \frac{D_0}{R}.$$

Also

$$\Theta_T = \left( \frac{D_z - D_0}{z} \right) = \frac{D_0 \left[ 1 + \left( \frac{z}{R} \right)^2 \right]^{1/2} - 1}{z},$$

and

$$\Theta_L = \frac{D_1 - D_2}{L}.$$

This divergence can be corrected to infinity if $\Theta_T$ is multiplied by $\left[ \left( \frac{v + 1}{v - 1} \right)^{1/2} \right]$, where $v = D_z/D_0$, and noting from Equation 2.20 that

$$v = \left[ 1 + \left( \frac{z}{R} \right)^2 \right]^{1/2},$$
then

\[ \Theta_{T, \text{corrected}} = \frac{D_0}{z} \left[ \frac{(v - 1)(v + 1) \frac{1}{2}}{(v - 1) \frac{1}{2}} \right] = \frac{D_0}{z} \left( v^2 - 1 \right) \frac{1}{2}. \]

But

\[ v = \left[ 1 + \left( \frac{z}{R} \right)^2 \right]^{\frac{1}{2}}. \]

Therefore,

\[ \Theta_{T, \text{corrected}} = \frac{D_0}{R} = \Theta_\infty. \]

Similarly, if \( \Theta_L \) is multiplied by \( \left[ v/(v^2 - 1) \right]^{\frac{1}{2}} \), the value is corrected.

Thus, to calculate \( M^2 \) for a given beam:

(a) find the beam waist from the cavity optics, e.g., output diameter for a flat output window is \( D_0 \);
(b) find the beam diameter, \( D_z \), at a known distance from the beam waist, \( z \) (two or three readings at different distances would help to confirm each other);
(c) calculate \( \Theta_T = (D_z - D_0)/z \);
(d) multiply by the correction factor with \( v = D_z/D_0 \) to obtain

\[ \Theta_\infty = \Theta_T \left[ \frac{(v + 1)}{(v - 1)} \right]^{\frac{1}{2}}; \text{ and} \]

(e) from Equation 2.19 derive

\[ M^2 = \left( \frac{D_0 \Theta_\infty \pi}{4\lambda} \right). \]

All other beam calculations follow.

However, notice in Figure 2.19 the significant understatement of \( \Theta_\infty \) defining \( M^2 \) which is often used in laser specifications.

For example, consider a CO\(_2\) laser with a 15-mm beam diameter from the flat output window, whose beam has expanded to 30 mm after an 8 m beam path:

\[ \Theta_T = (30 - 15)/8000 = 1.87 \text{ mrad}, \]
\[ v = 30/15 = 2. \]

The corrected value

\[ \Theta_\infty = \Theta_T \left( \frac{2 + 1}{2 - 1} \right)^{\frac{1}{2}} = 1.73 \Theta_T = 3.23 \text{ mrad}. \]
There are two values of $M^2$, one based on $\Theta_T$ and one on the correct value of $\Theta_\infty$:

$$M^2_{\Theta_T} = \frac{15 \times 1.87 \times 10^{-3} \times \pi}{4 \times 10.6 \times 10^{-3}} = 2.07,$$

and

$$M^2_{\Theta_\infty} = \frac{15 \times 3.23 \times 10^{-3} \times \pi}{4 \times 10.6 \times 10^{-3}} = 3.57.$$  

The Rayleigh range

$$R = \frac{\pi D_0^2}{4M^2 \lambda} = \frac{\pi \times 15^2}{4 \times M^2 \times 10.6 \times 10^{-3}};$$

therefore,

$$R = 16.7 \times 10^3/M^2,$$

$$R_{\Theta_T} = 8.05 \text{ m},$$

$$R_{\Theta_\infty} = 5.17 \text{ m}.$$  

The focal spot size for this laser is $f \Theta_\infty$ so for a 125-mm focal length lens

$$d_{\min \Theta_T} = 125 \times 1.87 \times 10^{-3} = 0.234 \text{ mm}$$

$$d_{\min \Theta_\infty} = 125 \times 3.23 \times 10^{-3} = 0.403 \text{ mm}$$

which gives a 72% error based on $d_{\min \Theta_T}$! Thus, corrected values of $M^2$ must be used in optical calculations.

### 2.8.1.3 Spherical Aberration

There are two reasons why a lens will not focus to a theoretical point; one is the diffraction-limited problem discussed earlier and the other is the fact that a spherical lens does not have a perfect shape. Most lenses are made with a spherical shape since this can be accurately manufactured without too much cost and the alignment of the beam is not so critical as with a perfect aspherical shape. The net result is that the outer ray entering the lens is brought to a shorter axial focal point than the rays nearer the centre of the lens, as shown in Figure 2.20. This leaves a blur in the focal point location. The plane of best geometric focus (the minimum spot size) is a little short of the plane of the planar wavefront – the paraxial point. The minimum spot size, $d_a$, is given by

$$d_a = K(n; q; p) \left( \frac{D_L}{f} \right)^3 S_2 = 2\Theta_a S_2,$$

where $\Theta_a$ is the angular fault (half-angle), $S_2$ is the distance from the lens, $D_L$ is the diameter of top hat beam mode on the lens, $f$ is the focal length of the lens and $K(n; q; p)$
is a factor dependent on the refractive index, $n$, the lens shape, $q$, and the lens position, $p$:

$$K(n; q; p) = \pm \frac{1}{128n(n - 1)} \left[ \frac{n + 2}{n - 1} q^2 + 4(n + 1)pq + (3n + 2)(n - 1)p^2 + \frac{n^3}{n - 1} \right],$$

where $q$ is the lens shape factor $(r_2 + r_1)/(r_2 - r_1)$, $r_2$ and $r_1$ are radii of curvature of the two faces of the lens and $p$ is the position factor $1 - 2f/S_2$.

Figure 2.21 shows the variation of spherical aberration with lens shape. The optimum shape is when the refraction angles at both faces of the lens are approximately equal. Note that there is a huge difference between a planoconvex lens mounted one way rather than the opposite way around.

Other lens faults are:

1. mechanical and optical axis are not correctly aligned – leading to coma effects; and
2. lens surface is not correctly spherical – leading to astigmatism if it has a cylindrical element.

Figure 2.21 Spherical aberration of a ray 1 cm off the optic axis passing through a lens of focal length 10 cm, diameter 2 cm and refractive index 1.517. (After Jenkins and White [15])
2.8.1.4 Thermal Lensing Effects

In optical elements which transmit or reflect high-power radiation there will be some heating of the component which will alter its refractive index and shape. As the power changes or the absorption changes, so will the focal point and the spot size. The two main elements usually concerned are the output coupler on the laser and the focusing lens, although the beam guidance mirrors could also be involved if adequate water cooling is not supplied. Transmissive optics can only be cooled from the edge or by blowing filtered, dry air onto the lens surface. Transmissive optics have a thickness chosen according to the pressure differential across them. Rarely is much thought given to thermal lensing, yet there is an optimum thickness to balance cooling with distortion [23].

Thermal lensing is mainly due to the rise in temperature of the optic causing variations in the refractive index ($dn/dT$) and only slightly in the expansion of the optics ($dl/dT$). The physical and optical constants for the principal infrared materials are given in Table 2.6. The focal length shift for a thin lens is given approximately by the quasi-statistical formula [24]

$$-\Delta f = \left( \frac{2APf^2}{\pi kD_L^2} \right) \frac{dn}{dT} \times 100,$$

where $\Delta f$ is the change in focal length (%), $A$ is the absorptivity of the lens material (m$^{-1}$), $P$ is the power incident on the lens (W), $n$ is the refractive index of the lens material (dimensionless), $k$ is the thermal conductivity of the lens (W m$^{-1}$ K$^{-1}$), $D_L$ is the incident beam diameter on the lens (m) and $T$ is the temperature (K).

Using this equation with $f/D_L = 10$ (i.e., an F10 optic) and $P = 2$ kW with a thin uncooled lens, the change in focal length due to thermal distortion for various materials is 0.02% for ZnSe to 2.6% for germanium.

By comparison, because of the geometric effects of thermal distortion on a 4-mm-thick lens, 38 mm in diameter made of ZnSe, which has a temperature difference of 14 $^\circ$C between the centre and the edge owing to the passing of a laser beam of 1,500 W [25], a change of focal length of approximately 4 $\mu$m would be expected owing to the change in the shape of the lens. This can be calculated from geometrical considerations and the simple lens formula for the focal length:

$$f = R/(n-1),$$

where $R$ is the radius of curvature of a planoconvex lens.

The expansion of the middle of the lens is expected to be

$$\beta l \Delta T = 7.57 \times 10^{-6} \times 0.04 \times 14 = 4.2 \mu m.$$

The effect this has on the lens focal length is

$$\Delta f = \delta R/(n-1) \sim -4.2/1.04 = 4 \mu m.$$
Table 2.6  Thermal and optical constants for principle infrared materials for 10.6 μm radiation

<table>
<thead>
<tr>
<th>Material</th>
<th>Absorptivity ((m^{-1} \times 10^{-6}))</th>
<th>Refractive index ((n))</th>
<th>(dn/dT) (\times 10^{-6} , ^\circ\text{C}^{-1})</th>
<th>Thermal conductivity ((W , m^{-1} , K^{-1}))</th>
<th>Specific heat ((J , kg^{-1} , ^\circ\text{C}^{-1}))</th>
<th>Thermal coefficient expansion ((\times 10^{-6} , ^\circ\text{C}^{-1}))</th>
<th>Density ((kg , m^{-3}))</th>
<th>Thermal diffusivity ((m^2 , s^{-1} \times 10^{-6}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZnSe</td>
<td>0.05</td>
<td>2.403</td>
<td>64</td>
<td>18</td>
<td>356</td>
<td>7.57</td>
<td>5270</td>
<td>9.6</td>
</tr>
<tr>
<td>CdTe</td>
<td>0.18</td>
<td>2.674</td>
<td>107</td>
<td>6.2</td>
<td>210</td>
<td>5.9</td>
<td>5850</td>
<td>5.05</td>
</tr>
<tr>
<td>GaAs</td>
<td>1</td>
<td>3.275</td>
<td>149</td>
<td>48</td>
<td>325</td>
<td>5.7</td>
<td>5370</td>
<td>27.5</td>
</tr>
<tr>
<td>Ge</td>
<td>3</td>
<td>4.003</td>
<td>408</td>
<td>59</td>
<td>310</td>
<td>5.7</td>
<td>5320</td>
<td>35.7</td>
</tr>
<tr>
<td>Si</td>
<td>150</td>
<td>3.418</td>
<td>160</td>
<td>156</td>
<td>716</td>
<td>2.56</td>
<td>2330</td>
<td>9.3</td>
</tr>
<tr>
<td>KCl</td>
<td>0.014</td>
<td>1.455</td>
<td>-33</td>
<td>6.5</td>
<td>683</td>
<td>36</td>
<td>1980</td>
<td>4.8</td>
</tr>
<tr>
<td>Quartz IR grade</td>
<td>1.45</td>
<td>1.0</td>
<td>1.4</td>
<td>745</td>
<td>0.55</td>
<td>2200</td>
<td>0.85</td>
<td></td>
</tr>
</tbody>
</table>

Data from II–VI handbook 1991.
There is a further problem since much of the absorption is on the surface. This creates a temperature gradient in the depth direction. The thicker the optic, the more bowed will be the internal isotherms. Such aberrations will affect the $M^2$ value of the beam.

In considering these issues, it is best to choose the material that will absorb less heat and show the least affect from being heated, e.g., ZnSe. The lens for high-power work should be cooled on the edge and by surface blowing if possible. It should also be as thin as the pressure differential will allow.

### 2.8.1.5 Beam Flight Tubes

An unexpected aspect of thermal lensing is to be found in the design of beam flight tubes which are used to pass the beam safely in open air if it cannot be transferred by a fibre, as with CO$_2$ radiation. For long flight tubes a mirage effect may be set up within the tube owing to thermal gradients caused by heating of the tube from sunshine or radiators, etc. Self heating of the gas in the tube by the absorption of the beam may distort or bend the beam. In both cases this would upset the alignment of a large gantry system.

To overcome this problem, flight tubes are often purged with dry nitrogen or helium gases which do not show self-heating problems for reasonable levels of power transmission; for ultrahigh powers a vacuum is recommended. A 10% increase in divergence has been found when the tube is filled with air as opposed to nitrogen or helium.

### 2.8.2 Depth of Focus

The depth of focus is the distance over which the focused beam has approximately the same intensity. It is defined as the distance over which the focal spot size changes by ±5%.

Considering the focusing beam to converge with an angle whose tangent is $D/(2f)$, by similar triangles we get

$$f/z_f = D/1.05d_{\text{min}} = D/1.05(2.44f\lambda/D)$$

$$z_f = \pm 2.56F^2\lambda,$$

where the $F$ number equals $f/D$. Allowing for multimode beams,

$$z_f = \pm 2.56F^2M^2\lambda.$$  \hspace{1cm} (2.21)

Table 2.7 shows some figures for the focal spot size and the depth of focus given by different lenses with beams of different mode structures.
2.9 Optical Components

2.9.1 Lens Doublets

We have so far discussed the single simple lens. A doublet is an alternative to an aspheric lens for overcoming the effects of spherical aberration. We have just noted that spherical aberration becomes the main issue for short focal length lenses of less than $F/5$. If such a short focus is needed, then the doublet is a cheaper option than an aspherical lens. Table 2.8 shows a comparison of lens types.

The effect of a doublet compared to a singlet is illustrated in Figure 2.22 [26]. This figure illustrates the advantages to be found for doublets at low $F$ numbers.

---

**Table 2.7** Effects of $F$ number on the focal length and depth of focus for different mode structures and wavelengths

<table>
<thead>
<tr>
<th>Wavelength ($\mu$m)</th>
<th>$F$ number</th>
<th>Mode</th>
<th>$d_{min}$</th>
<th>$\alpha$</th>
<th>$d_{min}$ focus, $z_f$</th>
<th>Bi-convex</th>
<th>Plano-convex</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>($\mu$m)</td>
<td>(f/D)</td>
<td>($\mu$m)</td>
<td>(mm)</td>
<td>(mm)</td>
</tr>
<tr>
<td>10.6</td>
<td>2</td>
<td>Top hat</td>
<td>0.26</td>
<td>0.08</td>
<td>0.5</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM00</td>
<td>0.13</td>
<td>0.5</td>
<td>0.08</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM01*</td>
<td>0.26</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM20</td>
<td>0.65</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>TEM00</td>
<td>0.26</td>
<td>0.5</td>
<td>0.02</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>10.6</td>
<td>2</td>
<td>Top hat</td>
<td>0.026</td>
<td>0.008</td>
<td>0.5</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM00b</td>
<td>0.013</td>
<td>0.05</td>
<td>0.08</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM01b</td>
<td>0.026</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>TEM20b</td>
<td>0.065</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>TEM00</td>
<td>-0.4</td>
<td>-2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Table 2.8** Comparison of basic lens types [15]

<table>
<thead>
<tr>
<th>Type</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singlet</td>
<td>Low cost</td>
<td>High SA at low $F$ number, no colour correction</td>
</tr>
<tr>
<td>Air space aplanat (doublet)</td>
<td>Excellent TWD</td>
<td>Cost, no colour correction</td>
</tr>
<tr>
<td>Cemented achromatic doublet</td>
<td>Better SA than singlet</td>
<td>Low power only fair TWD for low $F$ number</td>
</tr>
<tr>
<td>Air space achromat (triplet)</td>
<td>Colour correction OK, low SA at low $F$ number</td>
<td>Cost</td>
</tr>
</tbody>
</table>

---

The colour correction is not relevant for single-frequency lasers. SA spherical aberration, TWD transmitted wavefront distortion.
2.9.2 Depolarisers

When a polarised beam strikes a mirror surface at 45° to the plane of polarisation, the beam takes up two new planes: the p plane, parallel to the plane of incidence, and the s plane, perpendicular to the plane of incidence. When this reflected beam strikes a mirror having a surface coating which is $\lambda/4$ thick in the direction of propagation of the beam, then the p-polarised beam (parallel to the plane of incidence) will penetrate the film, whereas the s-polarised beam (perpendicular to the plane of incidence) will be reflected. The p-polarised beam will be reflected from beneath the film at the metal surface and so rejoin the main beam but then it will be phase-shifted by $2(\lambda/4)$. Thus, the final beam will be one in which the plane of polarisation alternates between two states at right angles with every beat of the wave form. This gives the impression to a viewer from the end of the beam that the plane of polarisation is rotating. The beam is said to be “circularly” polarised. Some care has to be taken with these carefully designed coatings, which are usually of MgF$_2$, because they are slightly hygroscopic and cannot be safely wiped clean. Nevertheless depolarisers are now fitted to nearly all commercial cutting machines.

An alternative to circularly polarised beams is radial polarisation (see Figure 2.23a). An intracavity conical prism has recently been introduced to make the beam from a fibre laser radially polarised; that is, when the polarisation axis is always radial from the centre of the beam. One such device is a double cone where the faces of the cone meet the beam at the Brewster angle, thus ensuring that only the radially polarised component of the incident beam enters the second collimating cone [27]. An alternative is a subwavelength circular grating in which normally reflected radiation will be polarised perpendicular to the grating rulings, i.e., radially [28]. The system is of potential use for optical tweezers and high-resolution microscopy, but it also shows considerable advantages for material processing, where it has been reported to increase the cutting speed by a factor of 10–50% compared with a linearly polarised beam [29]. The circular grating radial polariser can be mounted as the fully reflecting mirror inside the laser cavity, in which case it would have a convex GaAs lens on its surface to reduce diffraction losses (Figure 2.23b). The radius of curvature of the convex surface should...
2.9 Optical Components

**Figure 2.23**  a Different types of polarisation, and b an arrangement for mounting a radial polariser within the laser cavity to give a radially polarised beam [28]

be \( r' = (n-1)R \), where \( R \) is the original radius of curvature of the fully reflecting mirror prior to installing the radial element in its place.

### 2.9.3 Collimators

A collimator or beam expander is often used in installations where the beam path is long or the laser produces such a small beam diameter that it is difficult to focus without having the lens very close to the work piece and therefore vulnerable to spatter. A transmissive beam expander is illustrated in Figure 2.24 for the Galilean and Keplerian designs. The general principle is that the new beam size will be \( D_2 = D_1 f_2 / f_1 \). For long beam path work the beam divergence is one of the main criteria [30].
Most CO₂ laser cutting tables, gantries or robot systems have long beam paths and therefore need an optic to make the beam as parallel as possible. Failure to do so would mean the wavefront curvature and beam size would vary with position on the table and hence the focus would vary over the processing area. The objective of a beam expander or collimator is to locate the beam waist in the middle of the range of movement of the focusing optic to minimise this variation.

Beam expanders are usually marked on their barrels with the magnification, \( m \), and the focus setting, \( G \). \( G \) is not the beam expander focal length but is equal to \( m \) times the focal length.

For a simple lens where there is no change in beam size either side of the lens

\[
\frac{1}{R'} = \frac{1}{f} - \frac{1}{R},
\]

where \( R \) is the wavefront radius of curvature before the beam expander, \( R' \) is the wavefront radius of curvature after the beam expander and \( f \) is the focal length of the lens.

For a beam expander this becomes

\[
\frac{1}{R'} = \frac{1}{G} - \frac{1}{(m^2 R)},
\]

where in this case \( D_2 = mD_1 \) and the beam waist, for a beam of quality \( M^2 \), is located at [31]

\[
z_{\text{waist}} = \left( \frac{m^2 uG}{m^2 u + G} \right) \left\{ 1 + \left[ \frac{4M^2 \lambda}{\pi D^2} \frac{uG}{(m^2 u + G)} \right] \right\}^{-1},
\]

where \( u \) is the distance of the object from the first lens of the collimator.
The important aspect to note here is that there is a limiting value of $G$ beyond which normal diffraction dominates. When $G = \infty$, the beam waist is at the beam expander optic $z = 0$.

### 2.9.4 Metal Optics

#### 2.9.4.1 Plane Mirrors

The reflectivity of a mirror is a function of the material; therefore, most mirrors are made of a good conductor (good reflector) coated with gold for infrared radiation. The gold may be further coated with rhodium to allow gentle cleaning. New optics based on coated silicon are also used. In the case of some lasers they may be sufficiently thin to allow gentle flexing, giving some control over beam mode structure, if mounted within the cavity. The reason for having good conductivity mirror substrates, apart from reflectivity, is the need for good cooling. This is usually achieved by water but may be by air blast. Above 1 kW, mirrors must be cooled to avoid distortion. Mirror materials can be ranked against a figure of merit:

$$FOM = \frac{k}{A\beta},$$  \hspace{1cm} (2.22)

where $k$ is the thermal conductivity (W m$^{-1}$ K$^{-1}$), $A$ is the absorptivity (1 – reflectivity) and $\beta$ is the linear coefficient of expansion (K$^{-1}$).

The figure of merit for a number of mirror materials is shown in Table 2.9.

The flatness of mirrors is achieved by careful machining. The most popular technique is single-point diamond machining. The flatness is measured on an interferometer and recorded as $\lambda/\times$, for example, $\lambda/5$ means that there is a variation in flatness of one fifth of a wavelength over the mirror surface. The mirror must be mounted very carefully to avoid any mechanical distortion of this order. It must also be hard and tough, take a good polish and be cleanable.

Cleaning mirrors is done by placing a soft lens tissue on the mirror and allowing a drop of methanol or isopropyl alcohol to fall on it. The tissue is then drawn over the face of the mirror until it is dry. This will prevent scratching and also drying stains.

<table>
<thead>
<tr>
<th>Table 2.9 Properties of metal optic materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
</tr>
<tr>
<td>Thermal conductivity, $k$ (W m$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>Coefficient of expansion $\beta$ ($\times 10^{-6}$ oC$^{-1}$)</td>
</tr>
<tr>
<td>Density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>Hardness (Mohs)</td>
</tr>
<tr>
<td>Young's modulus ($\times 10^6$ MPa)</td>
</tr>
<tr>
<td>Specific heat (J kg$^{-1}$ oC$^{-1}$)</td>
</tr>
<tr>
<td>Reflectivity 0$^\circ$ AOI uncoated</td>
</tr>
<tr>
<td>Figure of merit, $k/A\beta$ ($\times 10^9$ W m$^{-1}$)</td>
</tr>
</tbody>
</table>

$AOI$ angle of incidence
Never rub a mirror surface. If a mirror becomes tarnished or damaged in any way, it is usually best to regrind and recoat it. If possible, mirrors should always be mounted so that they avoid dust falling on them.

2.9.4.2 Metal Focusing Optics (Parabolic Mirrors)

With the growing use of very high powered lasers with average powers over 5 kW, transmissive optics are near the limit of their thermal stress resistance. Most operators of such equipment prefer to use metal optics for focusing, collimating and guiding. One focusing element which uses the least number of mirrors is an off-axis parabolic mirror. They are very good if they are properly aligned, but they are very sensitive to alignment. Various arrangements are illustrated in Figure 2.25.

2.9.5 Diffractive Optical Elements – Holographic Lenses

2.9.5.1 Diffractive Optical Elements

Reflecting or transmissive plates finely etched or micromachined to two, three or 16 levels can be made in the form of a hologram and can thus reflect an image of any required shape. The early versions, known as “kinoforms” [32], had a reflectivity of

---

**Figure 2.25** Various ways of focusing using mirrors: a beam on mirror axis, b beam off axis, c Cassegranian lens, and d parabolic mirror
around 30% and there was some noise on the image at the edges. Modern versions are made of reflective material and are considerably more efficient [33].

The number of applications of diffractive optical elements is growing. At present they are used in processing for:

- multipoint soldering;
- beam shaping for uniform heating; and
- marking.

The optical applications include:

- Fresnel lenses;
- antireflection structures;
- achromatic lenses (the dispersion of a glass prism is in the opposite sense to a grating structure);
- coherent laser addition systems; and
- polarisation beam splitters.

### 2.9.5.2 Phase Plates

A variation on the etched diffractive optical elements is to insert thin surfaces into the beam path to change the phase from of a multimode beam [34].

### 2.9.6 Laser Scanning Systems

There are many occasions when a line beam is required. This can be achieved by a cylindrical lens or a scanning system [35]. These scanning systems can be based on oscillating aluminium mirrors as shown in Figure 2.26a. These systems have the weakness of giving a nonuniform power distribution owing to the turn point at the end of each oscillation. To avoid this, a rotating polygon is often used as shown in Figure 2.26b and c. This device has the problem of a varying velocity over the scan owing to the varying angle of incidence. Zheng [36] developed a double-polygon system which overcame that problem. There is some considerable geometry involved in designing these systems [35]. Computer control of the mirror oscillation allows the scanning of any pattern and hence laser marking, engraving, etc.

### 2.9.7 Fibre Delivery Systems

There are a variety of fibres being considered for delivering power beams for material processing [37]. The advantages appear obvious by analogy with electricity. There are, however, some difficulties which need to be faced when delivering power down a fibre. The first is the problem with the insertion into the fibre. The fibres are often a fraction of a millimetre in diameter, and thus when the focused beam is directed at the entry point into the fibre any dirt will cause catastrophic absorption. Once in the fibre the intensities are, of course, very high – for example, a 2-kW beam in a 0.5 mm-diameter
fibre would have an intensity of around $10^6 \text{ W cm}^{-2}$. Compare this value with the published values of damage thresholds for fibres shown in Table 2.10 [37] and the problem becomes apparent. If the fibre is made larger to reduce this value, then the focusability is reduced and a major property of the laser beam is lost. The finest focus of a beam from a multimode fibre is an image of the end of the fibre. This will never be anything like the fineness possible with a straight laser beam. The usual limit is a magnification of a half. A further problem is that of high loss due to nonlinear events such as Raman, Brillouin and Rayleigh scattering noted earlier. There is thus a limit on the power transmission of high-quality, high-powered beams in fibres; however, this limit is very high. Currently 5 kW is being routinely delivered down 0.4-mm-diameter fibres. One alternative is a multiplicity of fibres as shown in Figure 1.26 of three beams being focused through one lens.

There is a growing market for fibre optic delivery systems for Nd:YAG lasers [38]. Many high-powered Nd:YAG lasers of greater than 1 kW are now sold with only a fibre optic delivery option. This is partly due to an appreciation that the multimode output from such lasers is not seriously affected by passing the beam down a 400-μm fibre and partly by the greater freedom which fibre delivery gives the operator. For example, the laser can be in its own room some distance away and can be used to serve several workstations all in separate enclosures, which could be separated by up to 1 km or so.
Table 2.10 Published values of the damage thresholds in various fibres [37]

<table>
<thead>
<tr>
<th>Pulse duration</th>
<th>Wavelength (μm)</th>
<th>Material</th>
<th>Transmission loss (dB km⁻¹)</th>
<th>Core diameter (μm)</th>
<th>Breakdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 fs</td>
<td>0.620 SiO₂</td>
<td>~ 7</td>
<td>3</td>
<td>1.5 × 10⁵</td>
<td>&gt; 2 × 10¹²</td>
</tr>
<tr>
<td>5 ps</td>
<td>0.615 SiO₂</td>
<td>20</td>
<td>3</td>
<td>1.5 × 10⁴</td>
<td>2 × 10¹⁰</td>
</tr>
<tr>
<td>18 ns</td>
<td>0.248 SiO₂</td>
<td>2,000</td>
<td>1,000</td>
<td>1.6 × 10⁷</td>
<td>&gt; 2 × 10⁹</td>
</tr>
<tr>
<td>100 μs</td>
<td>1.06 SiO₂</td>
<td>1</td>
<td>10</td>
<td>~ 500</td>
<td>&gt; 5 × 10⁶</td>
</tr>
<tr>
<td>CW</td>
<td>1.06 SiO₂</td>
<td>~ 10</td>
<td>100</td>
<td>5 × 10⁶</td>
<td></td>
</tr>
<tr>
<td>500 ns</td>
<td>2.94 ZrF₄</td>
<td>12</td>
<td>100</td>
<td>~ 800</td>
<td>&gt; 1 × 10⁷</td>
</tr>
<tr>
<td>CW</td>
<td>5.2 As₂S₃</td>
<td>900</td>
<td>700</td>
<td>2.6 × 10⁴</td>
<td></td>
</tr>
<tr>
<td>CW</td>
<td>10.6 KRS-5</td>
<td>200–1,000</td>
<td>250</td>
<td>20</td>
<td>&gt; 4 × 10⁴</td>
</tr>
<tr>
<td>CW</td>
<td>10.6 Hollow</td>
<td>1,000</td>
<td>3,000</td>
<td>800</td>
<td></td>
</tr>
</tbody>
</table>

The fibres are made from extremely pure silica, often prepared from silane gas to avoid any impurities due to transition metal ions such as copper, iron and cobalt and hydroxyl ions. Such impurities are kept in the range of 1 ppb. The structure of the fibre consists of a core (the inner part of the fibre, Figure 2.27), the surrounding cladding of lower refractive index and an outer plastic protective coating; beyond that there is usually some form of metal sheathing. This metal sheathing may have within it thermal detectors to warn of damage. The light is confined to the core by total internal reflection at the core–cladding interface which occurs owing to the lower refractive index of the cladding. There are two main types of fibre: step-index fibre and graded-index fibre, as illustrated in Figure 2.27.

In step-index fibres the light rays take a zigzag path down the fibre until the rays homogeneously fill the core. The output beam has the diameter of the fibre core with an intensity pattern which is essentially flat-topped, although this will vary as the fibre

Figure 2.27 The structure of step-index and graded-index fibres
is bent. In the graded-index fibre the quantity of dopants affecting the refractive index varies across the fibre diameter, usually having a parabolical variation in refractive index. The rays propagate in an undulating manner. With a parabolical refractive index profile the path lengths of all rays are nearly equal for every angle of propagation. This is the condition for conserving the beam–parameter product: waist diameter $d_{\text{waist}} \times \text{angle of divergence, } \Theta_\infty$ (see Section 2.8.1.2 on the $M^2$ concept of beam quality). The output power profile from a homogeneously filled graded-index fibre yields an intensity distribution similar to the refractive index profile. In real fibres the beam parameters increase because of the finite size of the fibre, imperfect fibre geometry, inhomogeneities or impurities of the silica material, bending and imperfect fibre coupling.

2.9.7.1 Fibre Coupling

The laser beam will propagate along the fibre with low loss when it is coupled into the fibre within the maximum angle of acceptance ($\Theta_{\text{max}}$) determined by the numerical aperture of the fibre [39]:

$$NA = \sin(\Theta_{\text{max}}/2),$$

where $NA$ is the numerical aperture and equals the square root of the differences between the refractive indices of the core axis, $n^0_{\text{core}}$, and the cladding, $n_{\text{clad}}$:

$$NA = \left[\left(n^0_{\text{core}}\right)^2 - \left(n_{\text{clad}}\right)^2\right]^{1/2}.$$  

Typical values of the numerical aperture for fused silica range from 0.17 to 0.25 (i.e., acceptance angles up to 28°). Any higher values require increased dopant concentration, running the risk of disturbances in the refractive index. The basic coupling requirements are (Figure 2.28)

$$d_{\text{in}} < d_{\text{core}} \text{ and } \sin(\Theta_{\text{in}}/2) < NA.$$
There are more complex problems when the beam is elliptical or suffers from astigmatism. There is also the problem of thermal lensing discussed in Section 2.8.1.4 which could alter the value of \( d_{in} \) as a function of time. In practice, the core diameter and the numerical aperture are chosen so that they exceed the beam parameters \( d_{in} \) and \( \Theta_{in} \) by factors of 1.5–3 to guarantee safe operation. Fibre coupling optics are usually based on a telescope system.

Preparation of the end face of the fibre which is to receive the focused laser beam is critical. It is usually prepared by cleaving or very careful polishing.

### 2.9.7.2 Fibres for Lasers Other than the Neodymium-doped Yttrium Aluminium Garnet Laser

Table 2.10 lists some of the fibre material available for other wavelengths. In addition to these, CO radiation at 5.4 μm wavelength can be passed down metal halide fibres such as fibres of CaF\(_2\) and zinc halides. CO\(_2\) radiation at 10.6 μm can be transmitted down special thallium-based fibres with heavy loss, giving a power limit currently of approximately 100 W. An alternative is hollow waveguides, such as thin-bored sapphire tubes (0.5–1-mm internal diameter) coated internally with a dielectric coating of lower refractive index than that of air. These devices have passed several kilowatts of 10.6-μm radiation over distances of several metres. The losses, \( \alpha \), of such waveguides have been calculated by Miyagi and Karasawa \[40\] to be

\[
\alpha \propto I / D_{core}^3 \quad \text{and} \quad \alpha \propto 1/R,
\]

where \( D_{core} \) is the tube internal diameter and \( R \) is the bend radius. Thus, very fine waveguides would suffer serious loss, but large-diameter waveguides would be difficult to focus.

In general, engineers still think like electricians and would prefer fibre delivery of power, regardless of the fact that by so doing they are discarding one of the significant characteristics of optical energy – that it is one of the few forms of energy which can be transmitted through air or space without the need for a conductor.

### 2.9.8 Liquid Lenses

Liquid optics have a certain appeal in that they should be unbreakable, highly flexible, easily cleaned, easily cooled and possibly cheap. Various versions of liquid optics have been attempted or are being developed; but remember that you are currently reading this with a form of liquid optics – your eye!

1. **Gas jets of different refractive index such as cold nitrogen.** This works as a weak cylindrical lens to a beam passing at right angles to the jet.
2. **Water stream as a waveguide.** The water waveguide has been commercialised by Synova \[41\]. The laser beam from a Nd:YAG laser at 1,064 nm or a Yb:YAG laser at 1,070 nm or a frequency-doubled Nd:YAG laser at 532 nm is focused through a clean, deionised water pool into a fine orifice (25–150-μm diameter) from which
it emerges with the water jet flowing under a pressure of 2–50 MPa at approximately 1 l/h. The beam is homogenised and waveguided down the water jet by internal reflections. The working distance can be anywhere up to 1,000 times the nozzle diameter. Some very fine cutting has been demonstrated, producing parallel-sided cuts with a greatly reduced heat-affected zone (HAZ). The cut edges are clean and ablation products are removed in the water. There are no noxious gases, low mechanical pressures and no focal position problems. The applications to date have been in dicing SiC chips, cutting organic LED (OLED) masks, cutting medical stents and cutting hard materials such as diamond and cubic boron nitride.

3. Enclosed liquid surface whose shape can be controlled. A liquid optic is now being used in mobile phones developed by Varioptic [42]. In these devices a small droplet of oil is held in a tiny water chamber – it has to be small so that surface tension forces overcome waves and gravity effects. The design is illustrated in Figure 2.29. By applying an electric charge, one can make the oil surface curve in a controlled manner, since the electric charge affects the wettability of the oil on the walls of the chamber. The advantages are significant: no moving parts, little power to drive it (less than 15 mW), small size (8 mm diameter × 2 mm thickness) and focal range from F5 to infinity. The applications expected are as autofocus units for mobile

![Figure 2.29](image1.png)  The Varioptic liquid lens based on electrowettability. On application of a voltage, the meniscus shape changes owing to surface tension effects. This changes the power of the lens. The changes are both rapid and reversible

![Figure 2.30](image2.png)  Example of a liquid lens: an expanding hydrogel ring creates a meniscus at a water–oil boundary
phones, cameras, webcams, bar code readers, biometric readers for face, iris and fingerprint recognition and medical endoscopes, fibre scopes and dental cameras. An alternative is a small pool of liquid whose volume can be changed by pressure either pushing more fluid into the chamber or by the chamber changing size. The chamber changing size is an interesting concept illustrated in Figure 2.27. It has been developed by Jiang and Dong [43] of University of Wisconsin. The chamber holding water is made of a hydrogel that can respond to a stimulus. The chamber is covered with a water-repellent sheet with a hole in it. As the hydrogel expands, the water is pushed upwards but is pinned at the edges of the hole, thus forming a well-defined meniscus. This design is getting close to the way our eyes work.

4. Enclosed liquid or polymer whose refractive index can be controlled [43]. A nematic liquid crystal (a substance in which the molecules are oriented in parallel but not arranged in well-defined planes) can change refractive index depending on the orientation of the molecules within it. If a layer of such material is placed between two transparent electrodes, one possibly being hemispherical, when a voltage is applied, the electric field varies symmetrically about the centre of the electrodes – being highest at the edge and lowest at the centre. The orientation of the molecules varies with the electric field, causing a corresponding change in the refractive index; i.e., a lens is formed of variable focal length depending on the electric field. There are problems with this type of lens, namely, astigmatism, distortion, light scattering and from the laser material processing point of view an inability to transmit large powers. Developments are taking place on the types of liquid crystals (they must not rotate in the electric field) and the mixture in which they are contained.

2.9.9 Graded-index Lenses

Graded-index (GRIN) lenses may be radially or axially graded. They are usually shaped optics with the index gradient acting as an aspherical corrector.

Radial graded-index lenses are used in the input scanner section of photocopiers or fax machines. Axial graded-index lenses are often used as objective lenses in CD players and laser diode collimators. They have a potential future in camcorders and military optical systems, since they allow colour-corrected low F numbers and a reduced number of optical components in a zoom system [44].

2.10 Conclusions

Radiant energy is one of the most adaptable forms of energy available today. Not only can it be shaped by the devices just discussed, it can also have properties in polarisation, wavelength and power. It can interact with itself to give interference and diffraction effects or even change to double its frequency. This makes the subject of optical engineering one of the stronger subjects in the future of engineering.
Questions

1. a. Derive a formula for the relationship between the focal length of a lens and the resulting minimum spot size.
   b. Using this formula, show how the focal length is related to the depth of focus.
   c. List the occasions on which this formula may not work.
   d. A 2 kW CO₂ laser beam of 19 mm diameter and \( M^2 = 2 \) is to be expanded to form a wider parallel beam. This beam is then used to cut cloth using a galvanometer-driven focusing concave mirror. The concave mirror is mounted 3 m above the cutting table. The required spot size for successful cutting has to be less than 400 μm. How big should the beam be when it is incident in the concave mirror? What is the radius of curvature of the mirror?

2. a. A spot size of 150 microns or μm is required from a CO₂ laser whose raw beam from the laser cavity is 19 mm in diameter and \( M^2 = 2.5 \). What lens is required to achieve this?
   b. Using this lens from (a), how could one change the spot size to 180 or 120 μm?
   c. For \( F \) numbers less than 5, spherical aberration may be a problem. What spot size could be achieved with an \( F \) number of 5?
   d. What could be done to achieve a 100 μm spot size?

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“Now you know the difference between a moon beam and a laser beam!”
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2010, XVIII, 558 p. 374 illus., Softcover
ISBN: 978-1-84996-061-8