Chapter 2
Sensor Coverage Model

Abstract  Sensor coverage models are used to reflect sensors’ sensing capability and quality. They are abstraction models trying to quantify how well sensors can sense physical phenomena at some locations, or in other words, how well sensors can cover such locations. In almost all cases, sensor coverage models can be mathematically formulated as a coverage function of distances and angles. The inputs of such a coverage function are the distances between a particular space point and sensors’ locations, and the output is a nonnegative real-valued number and is called coverage measure of this space point. In some cases, a space point is said to be covered if its coverage measure satisfies some predefined threshold. On the other hand, sensor types are diverse, and each sensor type has its own manner of sensing physical stimuli. Also application scenarios are various, and each application scenario has its own way of interpreting sensory data. As such, sensor coverage functions can be defined in different forms and are subject to different interpretations, depending on sensor types and application scenarios. This chapter introduces some common sensor coverage models, including their motivations, formulations, interpretations, and applications.

2.1 Motivations

A sensor converts physical stimuli into electrical or other recordable signals. These signals are further processed to output digital sensing data which are embedded with comprehensible information. An interesting and important question is: how well a sensor works? In order to address this question, many measurement mechanisms have been used to quantify and compare sensors. As introduced in the previous chapter, some sensor characteristics, such as transfer function, sensitivity, dynamic range, accuracy, etc., can be used to measure how well a sensor reacts to physical stimuli. In this chapter, we introduce sensor coverage model as another mechanism to measure sensors’ sensing capability and quality. In what follows, we first discuss some motivations of sensor coverage models with examples.
Example  Let us first consider an example of using an acoustic sensor to measure sound pressure. Figure 2.1 illustrates the radiation of sound waves from a sound source in the free space. Suppose that the sound source emits a pure tone characterized by a sine function $a \sin(2\pi f + \theta)$, where $a$ is the maximum amplitude, $f$ the frequency, and $\theta$ the initial phase. The root-mean-square (RMS) amplitude $a_{\text{rms}} = \frac{a}{\sqrt{2}}$ is used to measure the sound pressure

$$L_p = 20 \log_{10} a_{\text{rms}} + C \text{ (dB)},$$

where the constant $C$ is the reference sound pressure (an internationally agreed value of $C$ is 94 dB). A free space is a homogeneous medium, free from boundaries or reflecting surfaces. In such a free space, the sound waves radiated from a sound source will diffuse in all directions, and its amplitude (or energy in terms of $a_{\text{rms}}$) attenuates with distances. The sound pressure at a given point, at a distance $d$ (in meters) from the source, can be computed as [7, 9, 16]

$$L_p = L_{\text{ref}} - 20 \log_{10} \left( \frac{d}{d_{\text{ref}}} \right) \text{ (dB)},$$

where $L_{\text{ref}}$ is the sound pressure at a reference point (usually greater than 1 meter to avoid source near field effects), and $d_{\text{ref}}$ is the distance between the reference point and sound source. From this expression it can be seen that in the free space, the sound pressure decreases by 6 dB when the distance $d$ doubles.

All kinds of acoustic sensors have limitations on the measurable sound pressure level (in dB), beyond which an accurate measurement cannot be obtained. The lower measurement limit of a microphone is established by its cartridge thermal noise [27]. There are two sources of thermal noise, air damping and preamplifier circuitry. The air damping causes a white noise that is a property of the microphone. The preamplifier has low-frequency noise which is inversely proportional to frequency and white noise. The thermal noise determines the lower measurement limit of an acoustic sensor, which is the dB level that would be read by a measurement instrument connected to the microphone output when there is no acoustic pressure applied to the microphone. If the distance between a sound source and an acoustic sensor is too large, the sound pressure at the sensor location is too small and cannot be accurately measured by the sensor. This suggests that a sensor may only sense...
some object within a limited range. Or in other words, a sensor can cover some region with limited area.

**Example** Let us consider another example of using a thermometer sensor to measure environmental temperature. The thermocouple of a thermometer consists of two electrical conductors of dissimilar materials. One is called the measurement junction, and the other is called the reference junction with known reference temperature. When the temperature of the measurement junction is different from the reference junction, a current will flow in the circuitry with the intensity proportional to their temperature difference. Although the measurement is the temperature of the air around the thermometer, it may be extended to infer the temperature of some other space points not far away from the thermometer. Such inferences may not be very accurate; however, they are useful in practice. For example, if the measured temperature is 30°C, then we may infer that a space point with distance of 10 meters away from the thermometer has the same temperature. This suggests that the sensing data of a sensor can be applied to the space points not only around the sensor but also close to the sensor. Or in other words, a sensor can cover some space with limited area.

There are also some other types of sensors whose sensing capability and quality can be related to the distances between a space point and sensors. These motivate the use of sensor coverage model, as one of many other mechanisms, to model how well sensors can sense physical phenomena at some locations, or in other words, how well sensors can cover such locations. We will introduce some commonly used sensor coverage models in the next section. Before that, we make an important note here. As abstraction models, sensor coverage models only apply to some types of sensors. There exist some other types of sensors to which coverage considerations may not apply at all.

### 2.2 Sensor Coverage Models

Sensor coverage models measure the sensing capability and quality by capturing the geometric relation between a space point and sensors. In almost all cases, a sensor coverage model can be formulated as a function of the Euclidean distances (and the angles) between a space point and sensors. The inputs of such a coverage function are the distances (and angles) between a particular space point and sensors’ locations, and the output is called coverage measure of this space point, which is a nonnegative real number.

We introduce the concept of coverage function in the context of two-dimensional space. Let us consider a space point \( z \) and a set of sensors \( S = \{s_1, s_2, \ldots, s_n\} \). We use \( d(s, z) \) \( (d(s, z) \geq 0) \) to denote the Euclidean distance between a sensor \( s \) and a space point:

\[
d(s, z) = \sqrt{(s_x - z_x)^2 + (s_y - z_y)^2}
\]  

(2.1)
in the two-dimensional space, where \((s_x, s_y)\) and \((z_x, z_y)\) are the Cartesian co-ordinates of the sensor \(s\) and the space point \(z\), respectively. We use \(\phi(s, z)\) \((0 \leq \phi(s, z) < 2\pi)\) to denote the angle between them. For a sensor \(s\), we draw a horizontal line starting from the sensor and pointing to right. We connect the sensor \(s\) and the space point \(u\) with another line \(\overline{sz}\). Then \(\phi(s, z)\) is the anticlockwise angle between the two lines, starting from the horizontal line and ending at the line \(\overline{sz}\). Figure 2.1 illustrates an example of \(d\) and \(\phi\). We use \(d_n = (d(s_1, z), d(s_2, z), \ldots, d(s_n, z))\) to denote the vector of such distances and \(\phi_n = (\phi(s_1, z), \phi(s_2, z), \ldots, \phi(s_n, z))\) to denote the vector of such angles between the set of sensors and the space point. A sensor coverage model can be formulated as a coverage function \(f\) mapping \((d_n, \phi_n)\) to a nonnegative real number, that is,

\[
f : (d_n, \phi_n) \rightarrow \mathbb{R}^+,
\]

where \(\mathbb{R}^+\) stands for the set of nonnegative real numbers. We call \(f(d_n, \phi_n)\) the coverage measure of a space point with respect to the sensors \(s_1, s_2, \ldots, s_n\). Similar definition can also be applied in three-dimensional space yet with some simple modification of the definition of angles.

Many sensor coverage models have been proposed in the literature. In some models, the inputs of a coverage function are only the distance and angle between a space point and one sensor. In some other models, the inputs of a coverage function are the distances and angles between a space point and more than one sensor. In our view, two types of coverage functions can be classified: One type is a kind of Boolean coverage models, where the coverage measure is either 0 or 1 for one space point; and the other can be called general coverage models, where the coverage measure can take various nonnegative values. If the angle argument is not included in the coverage function, then such coverage models are called omnidirectional coverage models. On the other hand, if it is included, such coverage models are called directional coverage models. In what follows, we elaborate some commonly used coverage models in details.

### 2.2.1 Boolean Sector Coverage Models

The Boolean sector coverage model (sometimes called the sector model), which might be motivated from a directional camera, is a Boolean directional coverage model [12]. Figure 2.2(a) illustrates such a sector model, where \(\phi_s\) is called an orientational angle, \(\omega\) is called a visual angle of the sector model, and \(R_s\) is called a sensing range. The coverage function of the sector model is given by

\[
f(d(s, z), \phi(s, z)) = \begin{cases} 
1 & \text{if } d(s, z) \leq R_s \text{ and } \phi_s \leq \phi(s, z) \leq \phi_s + \omega, \\
0 & \text{otherwise},
\end{cases}
\]

where \(d(s, z)\) is the Euclidean distance between a sensor \(s\) and a space point \(z\), and \(\phi(s, z)\) is their angle. This coverage function defines a sector: All space points
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**Fig. 2.2** Illustration of (a) a directional Boolean sector coverage model; (b) a directional Boolean sector coverage model with adjustable orientational angle; and (c) a space point being 3-covered by three sectors

within such a sector have the coverage measure of 1 and are said to be *covered* by this sensor. All space points outside such a sector have the coverage measure of 0 and are said to be not covered by this sensor. In Fig. 2.2(a), the space point marked by a star has a coverage measure of 1 and is covered by the sensor.

The orientational angle of a directional sensor might be adjustable after a sensor has been deployed [2, 3, 6]. Obviously, the area that can be covered by such a sensor will be different when it takes different orientational angle. For example, in Fig. 2.2(b), if the sensor takes $\phi_s$ as its orientational angle, then the space point $z_1$ is covered, and $z_2$ is not. If it takes $\phi_s'$ as its orientational angle, then the space point $z_1$ is not covered, and $z_2$ is covered.

A space point may be covered by more than one sector [10]. With the Boolean sector coverage model, the coverage measure of a space point relative to a set of sensors can be the addition of the coverage measure of the point relative to each individual sensor. Formally, the coverage function can be defined as

$$ f(d_n, \phi_n) = \sum_{i=1}^{n} f_i(d(s_i, z), \phi(s_i, z)), \quad (2.4) $$

where $f_i$ is the coverage function of a sensor $s_i$ and is given by (2.3). If $f(d_n, \phi_n) = k$ ($k \geq 1$), then we say that the point is *k-covered*. Obviously, if a point is *k-covered*, it is also $(k - 1)$-covered. Figure 2.2(c) illustrates an example of space point being 3-covered, where the space point marked by the star is within the sensing sectors of sensors $s_2$, $s_4$ and $s_5$.

### 2.2.2 Boolean Disk Coverage Models

The Boolean disk coverage model (often simplified as the disk model) might be the most widely used sensor coverage model in the literature. The coverage function of
the disk model is given by

$$f(d(s, z)) = \begin{cases} 
1 & \text{if } d(s, z) \leq R_s, \\
0 & \text{otherwise}, 
\end{cases}$$

where $d(s, z)$ is the Euclidean distance between a sensor $s$ and a space point $z$, and the constant $R_s > 0$ is called sensing range. Indeed, this function defines a disk (often called a sensing disk) centered at the sensor with the radius of the sensing range. Figure 2.3(a) illustrates a disk coverage model. The disk coverage model is an omnidirectional coverage model. All space points within such a disk have the coverage measure of 1 and are said covered by this sensor. All space points outside such a disk have the coverage measure of 0 and are said not covered by this sensor.

The sensing range $R_s$ is used to characterize the sensing capability of a sensor. Normally, different sensor types are assumed to have different sensing ranges. Some researchers even argue that a single sensor unit may have different sensing ranges and can choose one sensing range as its working sensing range [4, 22, 33]. For example, Fig. 2.3(b) illustrates a sensor with two sensing ranges, $R_{s1}$ and $R_{s2}$. The space point marked by the star is not covered if the sensor uses $R_{s1}$ as sensing range; it is covered if the sensor uses $R_{s2}$ as sensing range. It is generally assumed that a sensor consumes more energy when it uses a larger sensing range.

A space point may be located within more than one sensing disks. Under the disk coverage model, the coverage measure of a space point relative to a set of sensors can be the addition of the coverage measure of the point relative to each individual sensor. Formally, the coverage function can be defined as

$$f(d_n) = \sum_{i=1}^{n} f_i(d(s_i, z)), \quad (2.6)$$

where $f_i(\cdot)$ is the coverage function of a sensor $s_i$ and is given by (2.5). If $f(d_n) = k$, then we say that the point is $k$-covered. Obviously, if a point is $k$-covered, it is also $(k - 1)$-covered. Figure 2.3(c) illustrates an example of space
point being 3-covered, where the space point marked by the star is within the sensing disks of sensors $s_2$, $s_4$, and $s_5$.

### 2.2.3 Attenuated Disk Coverage Models

Some researchers argue that the sensing quality of a sensor reduces with the increase of the distance away from the sensor [13, 19]. An attenuated disk coverage model is used to capture such attenuated sensing qualities. An example of an attenuated disk coverage model is given by

$$ f(d(s, z)) = \frac{C}{d^\alpha(s, z)}, \quad (2.7) $$

where $\alpha$ is the path attenuation exponent, and $C$ a constant. Since it is a nonnegative function, a single sensor enforces its coverage measure to any point in the space. Figure 2.4(a) illustrates such an attenuated disk coverage model. The coverage measure of $z_1$ is larger than that of $z_2$, as it is closer to the sensor.

There may be more than one sensor in a sensor field. Under the attenuated disk coverage model, the coverage measure of a space point relative to a set of sensors is the addition of the coverage measure of the point relative to each individual sensor. Formally, the coverage function is modified as

$$ f(d_n) = \sum_{i=1}^{n} \frac{C}{d^\alpha(s_i, z)}. \quad (2.8) $$

In some cases, only the sensors close to a space point are included in the computation of the above equation for simplification.

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**Fig. 2.4** Illustration of (a) an attenuated disk coverage model; (b) a truncated attenuated disk coverage model; (c) a truncated multilevel attenuated disk coverage model
2.2.4 Truncated Attenuated Disk Models

In the attenuated disk coverage model, the coverage measure becomes very small when the distance between a space point and a sensor becomes very large. In such cases, the coverage measure might be neglected, and some approximations can be made by truncating the coverage measure for larger values of distance. For example, Zou and Chakrabarty [32] propose the following truncated attenuated coverage function:

\[
f(d(s,z)) = \begin{cases} 
    Ce^{-\alpha d(s,z)} & \text{if } d(s,z) \leq R_s, \\
    0 & \text{otherwise},
\end{cases}
\]

where \(\alpha\) is a parameter representing the physical characteristics of the sensor unit, and \(R_s\) the sensing range. Figure 2.4(b) illustrates such a coverage model.

Another truncated attenuated disk model [31] is defined as follows:

\[
f(d(s,z)) = \begin{cases} 
    1 & \text{if } d(s,z) \leq R_s - R_u, \\
    e^{-\alpha(d(s,z)-(R_s-R_u)^{\beta})} & \text{if } R_s - R_u < d(s,z) \leq R_s, \\
    0 & \text{if } R_s < d(s,z),
\end{cases}
\]

where \(R_s\) is the sensing range, \(R_u\) is called the uncertain range, and \(\alpha\) and \(\beta\) are constants. The use of \(R_u\) is to capture the reducing but not yet vanishing of the sensing quality when the distance between a sensor and a space point increases. Figure 2.4(c) illustrates such a coverage model.

Figure 2.5 illustrates the relation between the coverage measure and sensor–point distance for the aforementioned disk coverage models.

**Fig. 2.5** Illustration of the relation between the coverage measure and sensor–point distance for (a) Boolean disk coverage model; (b) attenuated disk coverage model; and (c) and (d) truncated attenuated disk coverage model (for the color version, see Color Plates on p. 207)
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2.2.5 Detection Coverage Models

An important application of sensor networks is to detect some event occurred at some location. In the context of detection application, the sensing quality of a sensor can be represented by its detection probability. The detection probability of a space point by a single sensor is also related to, among other factors, the distance between them. However, the detection probability of a space point relative to a set of sensors is no longer simply computed as the addition of the detection probability of the point relative to each individual sensor (otherwise, it might be larger than one). Instead, a value fusion or decision fusion can be used to derive the detection probability. Based on different event scenarios and detection techniques, many detection coverage models have been proposed in the literature [1, 5, 8, 15, 17, 18, 26, 28–30].

Let us consider a general signal propagation model where the signal parameter $\theta$ (e.g., the sound pressure of a sound source) attenuates along with the signal propagation. Depending on the hypothesis of whether the target is present ($H_1$) or not ($H_0$), the readings at the sensor $s_k$ are given by

$$H_0 : x_k = n_k,$$
$$H_1 : x_k = \frac{\theta}{d_\alpha^\alpha} + n_k,$$

where $\alpha$ is the attenuation exponent, $d_\alpha^\alpha = d_\alpha^\alpha(s_k, z)$ is the Euclidean distance between the sensor $s_k$ and the space point $z$, and $n_k$ is the measurement noise (e.g., circuitry thermal noise). It is often assumed that the noise follows a Gaussian distribution with zero mean and variance $\sigma_k^2$, denoted by $\mathcal{N}(0, \sigma_k^2)$.

Given the threshold $A$, a sensor makes its detection decision of whether a target is present by

$$x_k \underbrace{\gtrless}_{H_1} A.$$

That is, if the measurement is larger than $A$, it decides that a target is present, and if the measurement is less than $A$, it decides that a target is not present. When a target is present at the space point $z$, the detection probability $P_k^d$ of the sensor $s_k$ is given by

$$P_k^d = \mathbb{P} \left( \theta d_\alpha^\alpha + n_k \geq A \right) = Q \left( \frac{A - \frac{\theta}{d_\alpha^\alpha}}{\sigma_k} \right),$$

where $Q(\cdot)$ is the $Q$-function defined by

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$$

Since $Q$-function is a decreasing function, the detection probability $P_k^d$ decreases when the distance $d_k$ increases. Indeed, (2.14) defines an attenuated disk coverage
model. Furthermore, if we define a threshold for detection probability, $P_{\text{th}}^d$, and only those points with detection probability equal to or larger than such a threshold, i.e., $P_k^d \geq P_{\text{th}}^d$, are considered as covered by this sensor, then we actually define a truncated attenuated disk coverage model. If we do not discriminate the points with detection probability not less than the detection threshold and simply call these points being covered by the sensor, then finally we get a Boolean disk coverage model. In such a case, the points with the detection probability equal to the threshold consist of a circle, and their distances to the sensor are also equal and often regarded as the sensing range $R_s$. That is,

$$Q\left(\frac{A - \theta \frac{R_s}{\sigma_k}}{\sigma_k}\right) = P_{\text{th}}^d \implies R_s = \left(\frac{\theta}{A - \sigma_k Q^{-1}(P_{\text{th}}^k)}\right)^{\frac{1}{\alpha}}, \quad (2.16)$$

where $Q^{-1}(\cdot)$ denotes the inverse function of $Q(\cdot)$.

When $K$ sensors are used to cooperatively detect an event, the value fusion technique can be used to compute the detection probability of a space point by these sensors. Let $x_k, k = 1, 2, \ldots, K$, denote the readings of the $k$th sensor. With the value fusion, we compare the sum of $x_k$ and a threshold to make a decision whether or not a target is present. We assume that all the noises $n_k (k = 1, 2, \ldots, K)$ are independent Gaussian noises with zero mean and variance $\sigma^2$. When a target is present at the space point $z$, the detection probability by these sensors is given by

$$P_{K}^d = \Pr \left[ \sum_{k=1}^{K} \left( \frac{\theta}{d_k^\alpha} + n_k \right) \geq \sqrt{K}A \right] = Q\left(\frac{\sqrt{K}A - \sum_{k=1}^{K} \frac{\theta}{d_k^\alpha}}{\sqrt{K}\sigma}\right), \quad (2.17)$$

where $\sqrt{K}A$ is the value fusion threshold. Again, we can use the threshold of detection probability $P_{\text{th}}^d$, and the points with detection probability not less than the detection threshold are called covered by these sensors. In such a case, the covered points by $K$ sensors satisfy the following distance inequality:

$$\sum_{k=1}^{K} \frac{1}{d_k^\alpha} \geq \frac{\sqrt{K}}{R_s^\alpha}, \quad (2.18)$$

where $d_k$ is the distance between a point and a sensor $s_k$, and $R_s$ given by (2.16). Indeed, (2.18) defines a Boolean detection model for $K$ sensors, that is,

$$f(d_K) = \begin{cases} 1 & \text{if } \sum_{k=1}^{K} \frac{1}{d_k^\alpha} \geq \frac{\sqrt{K}}{R_s^\alpha}, \\ 0 & \text{otherwise.} \end{cases} \quad (2.19)$$

Figure 2.6 marks out the space points that are considered as being covered when using (2.19) ($\alpha = 1.0$) as the coverage model. The points within a disk are considered as being covered when only one sensor is used. They are also considered as being covered when more than one sensor is used. Furthermore, those points colored by yellow (and outside the disks) are not covered by only a single sensor but are considered as being covered by more than one sensor. These additionally covered space
points can be regarded as a kind of cooperation gain by using more than one sensor for a same sensing task.

There are also many decision fusion techniques that can be used to derive the detection probability by a set of sensors. For example, the following decision fusion computes the overall detection probability by a set of sensors:

\[ P_{d}^{K} = 1 - \prod_{k=1}^{K} (1 - P_{d}^{k}), \]  

(2.20)

where \( P_{d}^{k} \) is given by (2.14). Note that \( P_{d}^{k} \) depends on the distance between a sensor and the space point. Equation (2.20) can also be used to define a coverage model (It has been called as probabilistic coverage in some papers [1, 8, 17].) Again, we can set a threshold and define that a point is covered by \( K \) sensors \((s_{1}, \ldots, s_{K})\) if its overall detection probability is not less than such a threshold.

Another commonly used decision fusion technique is the majority voting. Suppose that there are \( K \) sensors, each independently making a local binary decision \( \delta_{k} \). If \( x_{k} \geq A \), then a sensor decides that a target is present, and \( \delta_{k} = 1 \); otherwise, a sensor decides that a target is present, and \( \delta_{k} = 0 \). Note that \( \delta_{k} \) is dependent on the distance between a sensor and a space point. The consensus decision rule is given by

\[ \sum_{k=1}^{K} \delta_{k} \geq H_{1} \left\lceil \frac{K}{2} \right\rceil. \]  

(2.21)

That is, if more than a half of sensors decide a target being present, then the overall decision fusion result is that a target is present; otherwise, the final result is that a target is not present. The consensus detection probability hence is given by

\[ P_{d}^{K} = \Pr \left[ \sum_{k=1}^{K} \delta_{k}^{d} \geq \left\lceil \frac{K}{2} \right\rceil \right] = \sum_{j=\lceil \frac{K}{2} \rceil}^{K} \prod_{k=1}^{j} P_{d}^{k} \prod_{k=j+1}^{K} \left( 1 - P_{d}^{k} \right), \]  

(2.22)
where $P^d_k$ is given by (2.14). The second sum term is to add up the product of detection probabilities and missing probabilities over all possible permutations over $k$. Equation (2.22) can also be used to define a coverage model. Also, we can set a threshold and define that a point is covered by $K$ sensors ($s_1, \ldots, s_K$) if its overall detection probability is not less than such a threshold.

### 2.2.6 Estimation Coverage Models

Another important application of sensor networks is estimating signal parameters. In the context of estimation application, the sensing quality of a sensor can be represented by its estimation errors. The estimation error of a space point by a single sensor is also related to, among other factors, the distance between them. However, when multiple sensors are used in estimation, the estimation error of a parameter of some signal at a space point is no longer simply computed as the addition of the estimation error of the point relative to each individual sensor. Instead, different estimation techniques can be used, and their estimation errors are also different. Based on different event scenarios and estimation techniques, some estimation coverage models have been proposed [11, 20, 21, 23, 25].

We use a simple signal estimation scenario to illustrate an estimation coverage model. We assume that a signal occurs at some space point $z$ and that its signal parameter $\theta$ attenuates along with the signal propagation. For example, $\theta$ can be the acoustic amplitude due to a motor engine or due to a leakage of gas barrel. For magnetic wave such as acoustic wave, its amplitude is attenuated when propagating. The measurement of the signal parameter by a sensor $s_k$ is given by

$$x_k = \frac{\theta}{d_{k}^\alpha} + n_k,$$

(2.23)

where $\alpha$ is the attenuation exponent, $d_{k}^\alpha = d^\alpha(s_k, z)$ is the Euclidean distance between the sensor $s_k$ and the space point $z$, and $n_k$ is the measurement noise (e.g., circuity thermal noise). It is often assumed that the noise follows a Gaussian distribution with zero mean and variance $\sigma_k^2$, denoted by $\mathcal{N}(0, \sigma_k^2)$. We note that this measurement model is the same as the one in (2.12) when the target is present.

A parameter estimator can be used to estimate $\theta$ based on the measurements $x_k$, $k = 1, 2, \ldots, K$. Let $\hat{\theta}$ and $\tilde{\theta} = \hat{\theta} - \theta$ denote the estimate and the estimation error, respectively. If the estimation error is small, the estimate of the signal parameter is obtained with high confidence level. We can use the probability that the absolute value of the estimation is less than or equal to a predefined constant $A$, i.e., $\Pr[|\tilde{\theta}_K| \leq A]$, to measure how well a point is monitored (Pr[|\tilde{\theta}_K| \leq A] is called the information exposure in [24]). Some standard estimators can be used to perform the estimation. For example, if the best linear unbiased estimator (BLUE) estimator [14] is used, the estimate $\hat{\theta}_K$ is given by

$$\hat{\theta}_K = \frac{\sum_{k=1}^{K} d_k^{-\alpha} \sigma_k^{-2} x_k}{\sum_{k=1}^{K} d_k^{-2\alpha} \sigma_k^{-2}},$$

(2.24)
and the estimation error $\tilde{\theta}_K$ is given by

$$
\tilde{\theta}_K = \frac{\sum_{k=1}^{K} d_k^{-\alpha} \sigma_k^{-2} n_k}{\sum_{k=1}^{K} d_k^{-2\alpha} \sigma_k^{-2}}.
$$

(2.25)

If we further assume that all noises have the same variances, i.e., $\sigma_k^2 = \sigma^2$ for all $k = 1, 2, \ldots, K$, then we have

$$
\Pr[|\tilde{\theta}_K| \leq A] = 1 - 2 Q\left(\frac{A}{\sigma} \left(\sum_{k=1}^{K} d_k^{-2\alpha}\right)^{\frac{1}{2}}\right),
$$

(2.26)

where $Q$-function is defined in (2.15).

We can see that (2.26) dependence on the distances between the $K$ sensors and the space point and can be used as a coverage function to define an estimation coverage model. Now let us consider that only one sensor is used in estimation. In such a case, (2.26) is given by

$$
\Pr[|\tilde{\theta}_k| \leq A] = 1 - 2 Q\left(\frac{A}{\sigma} d_k^{-\alpha}\right).
$$

(2.27)

Since $Q(\cdot)$ is a decreasing function, $\Pr[|\tilde{\theta}_k| \leq A]$ decreases as the distance $d_k$ increases. Indeed, (2.27) defines an attenuated disk coverage model. Furthermore, if we define a threshold $\epsilon$ ($0 \leq \epsilon \leq 1$) and if only the points with $\Pr[|\tilde{\theta}| \leq A]$ equal to or larger than such a threshold, i.e., $\Pr[|\tilde{\theta}_k| \leq A] \geq \epsilon$, are considered as covered by this sensor, then we actually define a truncated attenuated disk coverage model. If we do not discriminate the points within such a disk, then finally we get a Boolean disk coverage model. In such a case, the points with $\Pr[|\tilde{\theta}_k| \leq A] = \epsilon$ consist of a circle, and their distances to the sensor are also equal and can be regarded as the sensing range $R_s$. That is,

$$
1 - 2 Q\left(\frac{A}{\sigma R_s^\alpha}\right) = \epsilon \implies R_s = \left(\frac{A}{\sigma Q^{-1}\left(\frac{1}{2} - \epsilon\right)}\right)^{\frac{1}{\alpha}},
$$

(2.28)

where $Q^{-1}(\cdot)$ denotes the inverse function of $Q(\cdot)$.

We can also define a Boolean estimation coverage model of $K$ sensors by comparing $\Pr[|\tilde{\theta}_K| \leq A]$ with the threshold $\epsilon$. In such a case, the covered points by $K$ sensors satisfy the following distance inequality:

$$
\sum_{k=1}^{K} \frac{1}{d_k^{2\alpha}} \geq \frac{1}{R_s^{2\alpha}},
$$

(2.29)

where $d_k$ is the distance between a point and a sensor $s_k$, and $R_s$ given by (2.28). Indeed, (2.29) defines a Boolean estimation coverage model for $K$ sensors (which
is called the information coverage model in [23]), that is,

$$f(d_K) = \begin{cases} 1 & \text{if } \sum_{k=1}^{K} \frac{1}{d_k^\alpha} \geq \frac{1}{R_{\alpha}^2}, \\ 0 & \text{otherwise.} \end{cases}$$

(2.30)

Figure 2.7 marks out the space points that are considered as being covered when using (2.30) ($\alpha = 1.0$) as the coverage model. It is also seen that when using more than one sensor for the same sensing task (estimation in this case), the covered space points are more than those by only using one single sensor. Again, the increased coverage area can be seen as a kind of cooperation gain.

References

Coverage Control in Sensor Networks
Wang, B.
2010, XV, 210 p. 79 illus., Hardcover
ISBN: 978-1-84996-058-8