Chapter 2
Babbage's Engines

Although Babbage was a significant figure in English mathematics at the start of the nineteenth century, he is now principally remembered for his work on the design and development of a series of what he described as mechanical calculating engines. He devoted a large amount of time and money to these projects, but unfortunately none of them was ever fully completed.

The earliest machine, the Difference Engine, was intended to compute and print accurate mathematical and navigational tables. In 1822, Babbage completed a small prototype of this machine, which demonstrated the feasibility of his ideas. This was displayed and widely commented on, and the subsequent development of a full-scale machine was funded by the British Government. Progress was slower than expected, however, and was further hindered by Babbage's occasionally problematic relationship with the engineers he employed to work on the construction. By about 1830, work had come to a standstill, despite the investment of a considerable amount of Babbage's own money in the project. Government funding was not renewed, and the Difference Engine was never completed.

In the early 1830s, when he was no longer occupied with work on the Difference Engine, Babbage conceived of a more powerful and flexible machine, which became known as the Analytical Engine. Over a period of years, he produced a large number of drawings and associated specifications for this machine, but he never seriously considered undertaking its construction.

Late in life, he revisited his earlier ideas, developing the design for a new and simplified difference engine, using insights gained from his work on the Analytical Engine. This machine became known as the Difference Engine No. 2, though again, Babbage did not succeed in completing the construction of the engine.

Babbage's machines, and in particular the Analytical Engine, are often portrayed as isolated precursors of the modern computer, and Babbage as an anachronistic genius who had the misfortune to live at a time when technology was insufficiently developed to enable him to fully implement his vision. When viewed in context, however, a rather different picture emerges. Babbage engaged whole-heartedly with the scientific, industrial and even political life of his time, and this chapter describes how his calculating engines can be seen in a fuller historical context.

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2.1 The Division of Mental Labour

From the onset of the Industrial Revolution, observers were struck by the increases in productivity that were brought about by the application of machinery and the factory system to the manufacture of goods of all sorts. In *The Wealth of Nations*, published in 1776, the Scottish philosopher and economist Adam Smith attempted to explain the great discrepancies observable between the levels of production in different countries. The starting point of his analysis was the idea of the division of labour, or breaking down the overall task of manufacturing a product into a number of simple processes, and the allocation of these processes to different workers within a factory.

Famously, Smith took as his primary example of the division of labour the trade of pin-making, and he described how as many as 18 different processes were involved in the manufacture of pins. These ranged from fundamental operations, such as cutting the wire used to make the pins to the length and thickness required, to less obvious tasks such as inserting the completed pins into paper holders before they could be sold. He took as an example a 'small manufactory' of ten workers which produced 48,000 pins per day, and by estimating that a single worker carrying out all the required operations could make at most 20 pins in a day, concluded that the division of labour had increased the productivity of the workers in the pin-making trade by a factor of at least 240.

Smith attributed the effect of the division of labour to three major causes. Firstly, he observed that a workman specializing in one simple process would become more skilled and so quicker in carrying it out than someone for whom it was one of many different tasks to be carried out during a day's work. A further economy of time was obtained by having each workman perform the same task continually, thus saving the time lost in switching from one type of work to another. Finally, in an observation that is particularly relevant to the concerns of this chapter, Smith pointed out that once an activity had been broken down by the division of labour, it was often possible to develop machinery that would assist or replace human workers in the execution of the simple processes involved, increasing the ease and efficiency with which the work was carried out.

Babbage whole-heartedly agreed with Smith's view of the division of labour: he devoted two chapters to it in his book *The Economy of Machinery and Manufactures*, describing it as "perhaps the most important principle on which the economy of a manufacture depends". In addition to the factors listed by Smith, Babbage believed he had identified a further important principle to explain the cheapness of articles manufactured through the division of labour, namely that "the master manufacturer, by dividing the work to be executed into different processes, each requiring different degrees of skill or force, can purchase exactly that precise quantity of both which is necessary for each process".[1] He also gave a typically detailed account of the pin-making trade in England and France, with details of the productivity and wages of the workers in each of the processes into which it was divided.

[1]Babbage (1835b), pp. 175–176.

Babbage went beyond Smith, however, and considered the effect of applying the division of labour not only to manufacturing processes but also to activities which involved mental rather than physical labour, and in particular, to the calculation of mathematical tables. His principal example was the production of a large set of tables recently undertaken in France under the direction of the engineer Gaspard Riche de Prony.[2]

De Prony's tables were produced as part of the reform of weights and measures instigated in the 1790s by the French revolutionary government. The reform was intended to introduce the decimal system wherever possible, and one application of this principle was a proposal to divide a right angle into 100, rather than 90, degrees. This system had been used in the survey of the Paris meridian that led to the definition of the new unit of length, the metre, and in 1794 it was decided that new tables of trigonometrical functions and their logarithms were required which were adapted to this new angular measure. The calculation of these tables was entrusted to de Prony, then the director of the Bureau de Cadastre, an organization concerned with the production of accurate land surveys required for the purposes of taxation.

De Prony had been instructed to ensure that the tables "left nothing to desire with respect to exactitude" and would be "the vastest and most imposing monument to calculation that had ever been executed or even conceived". The tables required an unprecedented amount of calculation, and when complete would contain several hundred thousand figures, calculated to an accuracy of between 12 and 22 decimal places. Clearly, a project of this magnitude would require careful organization, and according to his own account, de Prony explicitly drew upon Smith's ideas as an inspiration for the organization of his workforce: "I came across the chapter where the author treats of the division of work … I conceived all of a sudden the idea of applying the same method to the immense work with which I had been burdened, and to manufacture logarithms as one manufacture pins".[3]

In order to apply the division of labour to computation, de Prony turned to a mathematical technique known as the method of differences. This method enabled complex logarithmic and trigonometric functions to be calculated by employing only the much simpler operations of addition and subtraction. Suppose, for example, that it is required to calculate the values of the formula $f(x) = x^2 + x + 41$. A few values of this formula are given in the second column of Table 2.1.

The *first differences*, shown in the column headed Δ_1, are found by calculating the difference between two adjacent values of the function. So the first figure in this column is found by computing $f(2) - f(1) = 47 - 43 = 4$. In a similar way, the *second differences*, shown in the column headed Δ_2 are found by computing the difference between adjacent values in the first difference column. It can be seen that in the case the second differences are all the same, a property that is always true for polynomial formulae in which the highest power is 2.

We can use this property to reverse the process and compute the next value of the formula using only addition. The fourth entry in the Δ_2 column will be 2; this

[2]Descriptions of this project have been given by Grattan-Guinness (1990) and Daston (1994).

[3]de Prony (1824), quoted in Grattan-Guinness (1990).

Table 2.1 Application of the method of differences to the function $x^2 + x + 41$

x	$f(x)$	Δ_1	Δ_2
1	43	4	2
2	47	6	2
3	53	8	2
4	61	10	
5	71		

can be added to the entry in the Δ_1 column to give the fifth Δ_1 value, which is 12; this can in turn be added to the last value in the $f(x)$ column to give the value 83. It can be checked by direct computation that the value of the formula for $x = 6$ is in fact 83, but the method of differences has allowed us to work this out without performing any multiplications. In a similar way, provided that the first number in each column is known, all the values of the function can be tabulated. Subtraction will be required as well as addition if negative numbers appear in the table.

The application of the method of differences is not always as straightforward as in the example above, but provided that some preliminary mathematical work is done, the method can be used to calculate the logarithmic and trigonometric functions that de Prony was concerned with to any required degree of accuracy. To bring this about, De Prony divided his workforce into three sections.

The first section consisted of mathematicians who derived the formulae that would be used to calculate the required functions. This work required a considerable degree of mathematical expertise; on the other hand, the number of workers required in the first section was relatively small, and they carried out no numerical work.

From these formulae, the workers in the second section derived the more detailed information required to compute the results using the method of differences. Unlike the simple example given above, logarithmic and trigonometrical functions do not have a constant final difference. If a sufficient number of differences are computed, however, the final difference will be constant over significant ranges of values of x. The job of the second section was to work out what these ranges were and to compute, for each range, the values in the top row of the table, including the constant last difference.

The results were handed on to the workers in the third section in the form of sheets "ruled with fifty horizontal lines … and divided into a number of vertical columns according to the number of orders of differences which had to be written. The topmost line of each of these sheets reproduced the numbers determined by the calculators of the second section, and thus served as a point of departure".[4] The workers in the third section could then complete the sheets by performing only additions and subtractions, as explained above. Once completed, the sheets were passed back to the second section for checking; this could be done without repeating the detailed calculations.

[4]Lefort (1858), p. 127.

The workers in the third section were instructed in how to complete the sheets. This would have involved learning the order in which the intermediate results were to be calculated. The actual additions and subtractions were carried out on loose sheets which were discarded once the results had been transcribed onto the sheets prepared by the second section.[5] The third section, therefore, had little, if any, scope for the exercise of judgement or initiative.

Babbage later made an explicit comparison between the organization of de Prony's calculations and the typical organization of "a cotton or silk mill, or any similar establishment". Furthermore, he observed that the work of the third section might "almost be termed mechanical", and referred to a time "when the completion of a calculating engine shall have produced a substitute for the whole of the third section of computers".[6] As he makes clear in a footnote, this is a reference to his project to build a 'difference engine' which occupied much of his time, effort and money during the 1820s. This comment also provides a nice illustration of the way in which Babbage's work exemplifies Smith's third principle, according to which the introduction of new forms of machinery is a consequence of employing a suitable division of labour.

2.2 The Difference Engine

In Babbage's own accounts, the invention of the Difference Engine closely followed Smith's principle whereby the simple processes resulting from the division of labour became susceptible to mechanization. As early as 1813, Babbage and Herschel had made a case for the introduction of a suitable division of labour into computation:

> The ingenious analyst who has investigated the properties of some curious function, can feel little complaisance in calculating a table of its numerical values; nor is it for the interest of science, that he should *himself* be thus employed, though perfectly familiar with the method of operating on symbols; he may not perform extensive arithmetical operations with equal facility and accuracy; and even should this not be the case, his labours will at all events meet with little remuneration.[7]

In his autobiography, Babbage recounted an anecdote suggesting that the idea of mechanical calculation had occurred to him at this time.[8] However, it was not until several years later that he seriously entertained the idea of building a machine which would perform calculations. In 1821, he and Herschel were reflecting on the experience of overseeing a large computation in which "the calculations ... were distributed among several computers", thus sparing Babbage and Herschel from "that wearisomeness and disgust, which always attend the monotonous repetition of arithmetical operations". However, they still found the preliminary calculations

[5]Lefort (1858), p. 134.

[6]Babbage (1835b), p. 195.

[7]Babbage and Herschel (1813), p. viii.

[8]Babbage (1864).

and subsequent comparison and verification of the results "a considerable trial of the patience of those who superintend them". As a result of this frustration, "it was suggested by one of us, in a manner which certainly at the time was not altogether serious, that it would be extremely convenient if a steam-engine could be contrived to execute calculations for us; to which it was replied that such a thing was quite possible",[9] and in Babbage's account, it was this casual suggestion which sparked in him a serious consideration of the possibility of mechanical computation.

In the following year, Babbage gave a similar account of the origin of the machine in a letter to Humphry Davy, then the president of the Royal Society:

> The intolerable labour and fatiguing monotony of a continued repetition of similar arithmetical calculations, first excited the desire, and afterwards suggested the idea, of a machine, which, by the aid of gravity or any other moving power, should become a substitute for one of the lowest operations of human intellect.[10]

In the same letter, he gave a description of de Prony's project, and stated explicitly the intended scope of his machine: "If the persons composing the second section, instead of delivering the numbers they calculate to the computers of the third section, were to deliver them to the engine, the whole of the remaining operations would be executed by machinery".[11]

Babbage did not view the Difference Engine as simply a calculator, however, but rather as an attempt to mechanize a complete process, namely the production of mathematical tables. Tables were by far the most important mathematical aid in use at the time: those in use ranged from simple tables of products to those tabulating complex trigonometric functions and, as in the project entrusted to de Prony, were often produced in both natural and logarithmic forms. The areas in which tables were used also varied widely: a particularly significant market was the production of astronomical tables, which found a very practical application to the maritime navigation on which international trade depended.

As well as the saving of labour, Babbage repeatedly stressed a further benefit to be hoped for from the mechanization of calculation, namely the avoidance of errors. It was well known that all published tables contained errors, and the detection and correction of errors was an important aspect of the work of calculation. Errors could arise not only in the process of computation, but also in the subsequent printing process, and Babbage placed as much emphasis on the mechanization of typesetting as on that of calculation. His overall vision was of a machine divided into two parts, each dealing with one of these fundamental processes.

> In order to produce printed tables free from error I proposed the engine should be able to calculate any tables whatever, and that it should produce a stereotype plate of the computed results ... This view of the subject naturally divided the engine into two great branches: one part must make the calculations, another must produce the plates.[12]

[9] Babbage (1822c), p. 15.

[10] Babbage (1822b), p. 3.

[11] Babbage (1822b), p. 10.

[12] Babbage (1822c), p. 16.

Table 2.2 A schematic calculating sheet equivalent to the Difference Engine

x	$f(x)$	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6
1	7	...	...	...	...	...	...
...	...	...	...	...	...	...	...

In order to be able to produce "any tables whatever", Babbage needed a simple yet general mathematical method and, like de Prony and many table-makers before him, he turned to the method of differences. As well as its generality, this method recommended itself to Babbage because of "the great uniformity which it would necessarily introduce into all [the machine's] parts ... The whole difficulty was now reduced to that of forming a machine which should add or subtract, as might be required, any number of differences".[13]

The basic idea behind the calculating part of the engine was to provide a physical representation of one row of the calculating sheets used by de Prony. Babbage planned to build a machine which would calculate with six orders of differences and up to 20 digits in each number stored; if de Prony had organized his calculations on a similar scale, the sheets used would have looked something like Table 2.2.

Babbage presented the design of the Difference Engine as proceeding through a number of stages; at each stage he showed how a complex operation could be analyzed in terms of simpler ones, which could then be recombined to produce the desired overall effect.[14]

To complete such a table, a computer would start from the right, adding the last difference to the preceding column, then adding the resulting number to the column to its left, and so on until the next function value had been obtained. Babbage's first step of analysis, therefore, was to observe that if he could devise a mechanism for adding one number to another, this mechanism could be replicated as often as necessary to add the successive differences across a row in the sheet.

However, each number in the table was itself made up of a number of digits; Babbage further observed that a mechanism that could add one digit to another could simply be repeated as often as required to add together all the digits in a number. This plan was slightly complicated by the necessity of dealing with *carries*, when the sum of two digits was greater than 9 and required a further increment to the digit in the next place. Babbage's strategy at this point was to separate these two fundamental operations: the overall design of the engine is therefore based on the repetition of two basic mechanisms, one to add together two digits, and the other to deal with any resulting carry. Babbage saw an analogy between the structure of the machine and that of arithmetic itself:

> In fact, the parts of which it consists are few but frequently repeated, resembling in this respect the arithmetic to which it is applied, which, by the aid of a few digits often repeated, produces all the wide variety of number.[15]

[13]Babbage (1822c), p. 17.

[14]The following description is based on the account given in Babbage (1822c).

[15]Babbage (1822b), pp. 6–7.

It is not necessary to consider the mechanical details of the engine here, but the overall organization of the machine is of some interest. Individual digits were represented by wheels mounted on an axle. The numerals 0 to 9 were inscribed on the circumference of some of the wheels and positioned in such a way that the number stored on the axle could be read off at the front of the machine. Babbage later referred to the assemblage consisting of a figure wheel and its associated machinery as a 'cage'. A complete number was represented by a set of cages mounted in a column on a single axle.

Adjacent columns represented the numbers stored in a row of a calculating sheet. Addition was performed by a rotation of the axis to which a number was fixed. By means of various contrivances, this would cause the digits represented on the moving axle to be added to those on its stationary neighbour, while at the same time the machine would record the various carries that were required. The operation of adding the carries to the resulting sum was performed in a separate step, once the rotation of the moving axle was completed.

The action of the various parts of the machine was to be coordinated in such a way that a number of additions could be performed simultaneously. In one step, the numbers stored on the columns representing the even differences would be added to the adjacent columns; then the columns would be geared differently, and the odd differences added to the even columns. In a description of the engine written after construction had halted, the journalist Dionysius Lardner described the functioning of the engine in poetical style:

> There are two systems of mechanical action continually flowing from bottom to top; and two streams of similar action constantly passing from the right to the left. The crests of the first system of adding waves fall upon the last difference, and upon every alternate one proceeding upwards whilst the crests of the other system touch upon the intermediate differences. The first stream of carrying action passes from right to left along the highest row and every alternate row, whilst the second stream passes along the intermediate rows.[16]

This nicely evokes the way in which the calculating part of the engine embodied the complexity of the method of differences, despite being built out of relatively simple components. By contrast, the design for the printing part of the engine was worked out in much less detail, and its details need not be considered here.

The difficulties that Babbage encountered, or created, in trying to construct a working difference engine have been described elsewhere, and it is not necessary to recount them in detail here. A prototype engine was built in 1822, computing only two rather than the proposed six orders of differences. Its application was therefore rather limited, but it served for Babbage as a demonstration of the feasibility of his ideas, and was displayed at his house. He used it to construct tables of square and triangular numbers, and tabulations of formulae such as $x^2 + x + 41$. With the support of the Royal Society and some financial assistance from the Government, construction of the full-size engine started in 1823 and occupied much of Babbage's time for several years. After a number of financial disagreements with his engineer, construction was suspended in 1833 and never restarted. Babbage made appeals to

[16] Lardner (1834), p. 298.

successive Prime Ministers for additional funding, but none was forthcoming, and in 1842 the Government finally decided to withdraw support from the machine. The completed portions of the machine ended up in the South Kensington Museum.

2.3 The Meanings of the Difference Engine

The Difference Engine was not, of course, the first mechanical calculator. However, earlier machines such as those of Pascal and Leibniz had only mechanized single arithmetical operations such as addition and multiplication, where the user would supply the numbers to be operated on and the machine would calculate the result.

By contrast, Babbage's engine was intended to automate several aspects of a complex process, namely the production of scientific and navigational tables by means of the method of differences. In contrast to the earlier machines, it would be *productive*: as it implemented a complete algorithm and not just a single operation, large numbers of results could be produced from a relatively small amount of input data. Furthermore, not only was it inspired by the division of labour apparent in the manual production of tables, but it also embodied a further division in the distinction between the calculating and printing parts of the machine.

The scale and novelty of Babbage's proposal generated a lot of interest in the Difference Engine; in the 1820s, Babbage had a small prototype set up in his house and demonstrated it to many interested visitors. The engine was widely perceived as having more significance than as a simple aid to calculation; rather, it was seen to be a novel application of the processes of mechanization that were clearly visible elsewhere in society.

The Mechanization of the Mental As discussed in Chap. 1, machines were used in many areas of manufacturing and industry by the 1820s, and their introduction had profound social and practical implications. Machine performance surpassed that of humans in many physical tasks, leading to great changes in the nature of work. Among the benefits claimed for mechanization were that it increased the power that a worker could apply to a task, and also enabled work to be carried out faster and more accurately than before.

Even more fundamental than these practical advantages, however, was the fact that the Difference Engine appeared to increase the scope of what could potentially be mechanized. In 1825, in an address given when Babbage received the gold medal of the Astronomical Society, Henry Colebrooke observed that hitherto "mechanical devices have substituted machines for simpler tools or for bodily labour", before commenting:

> But the invention, to which I am adverting, comes in place of mental exertion: it substitutes mechanical performance for an intellectual process: and that performance is effected with celerity and exactness unattainable in ordinary methods.[17]

[17] Colebrooke (1825), p. 509.

As Babbage later put it in a letter to the Prime Minister, the Duke of Wellington, the engine represented "the first conversion of mental into mechanical processes".[18] The fact that Babbage had disparaged calculation as "one of the lowest operations of the human intellect" does not alter the fact that this marks a significant extension of the reach of the mechanical.

Despite the novelty of the application area, however, Babbage and his associates appeared to align mental with physical labour, at least in terms of the benefits that mechanization could be expected to deliver.

Economy In common with other mechanical innovations, the Difference Engine was expected to deliver greater productivity at lower costs. The bulk of de Prony's workforce was made up of the calculators in the third section, and it was precisely this section that would be made redundant by Babbage's engine. Babbage also pointed out that the automatic printing facilities would result in "the whole work of the compositor being executed by the machine, and the total suppression of that most annoying of all literary labour, the correction of the errors of the press".[19]

A key factor in this projected saving was the fact that the operator would only have to supply a small amount of initial data, from which the engine would be able to calculate a significant portion of a table along with all its intermediate differences. For Colebrooke, this distinguished Babbage's proposal from the earlier machines of Pascal and Leibniz where the individual numbers to be operated on had to be set on the machine before each operation. The Difference Engine was not just a calculator, but encoded a complete mathematical process; furthermore, the constant repetition of the simple operations required to perform the calculation made the process a suitable one to be carried out by the unvarying behaviour of a machine.

The ability of the Difference Engine and related machines to automate an entire arithmetical process, rather than just single operations, also promised to remove an obstacle that was impeding the progress of mathematics in some areas. For example, Colebrooke referred to certain equations of Lagrange "which involve operations too tedious and intricate for use, and which must remain without efficacy, unless some mode be devised of abridging the labour or facilitating the means of its performance".[20]

Avoidance of Error Even more than its productivity, the capacity of the engine to deliver tables that were more reliable than those produced manually was stressed by Babbage who wrote that "[t]he quantity of errors from carelessness in correcting the press, even in tables of the greatest credit, will scarcely be believed".[21]

This theme was given great prominence by Lardner, who gave a compendious summary of the tables currently in use, but commented that their usefulness was

[18] Babbage (1834), p. 4.

[19] Babbage (1822b), p. 10.

[20] Colebrooke (1825), p. 512.

[21] Babbage (1822b), p. 5.

limited by their "want of numerical correctness", [22] reiterating Babbage's point that the engine was designed to remove not only errors of calculation, but also those introduced during the processes of typesetting and printing.

Mathematical Innovation The purpose of the difference engine was to express a familiar mathematical procedure in machinery. In a curious reversal of this process, Babbage was led to investigate the mathematical properties of some novel functions that had been suggested to him by a consideration of some modifications that could be made to the machine. In 1822, he explained to the Astronomical Society how he had been led to this discovery.[23]

The engine was designed to compute tables which had a constant last difference. Many important functions, particular trigonometrical functions, did not have this property, but could nevertheless be approximated over suitably chosen intervals by functions which did have a constant difference. A disadvantage of this procedure was that, at the boundaries between these intervals, the constant difference in use would have to be manually altered to the value appropriate for the next interval. Babbage considered that this could be a significant source of error, and therefore looked for ways to avoid this procedure.

For some functions, he found that there was a mathematical relationship between the value to be calculated and one of the later differences. For example, it can be shown by simple algebraic manipulation that

$$\Delta_2 \sin(x) = K \sin(x+1),$$

for a constant value K. In other words, the value of $\sin(x+1)$ can be calculated from the second difference of the preceding value of the function. Although it seems rather circular, this in fact means that the sine function can be tabulated if an initial value $\sin_1$ and first difference $\Delta_1 \sin_1$ are given, even though it has no column of constant differences. The calculations proceed as follows (where for clarity $\sin_1$ is written in place of sin(1), and so on):

$$\begin{aligned} \sin_2 &= \sin_1 + \Delta_1 \sin_1, \\ \Delta_2 \sin_1 &= K \sin_2, \\ \Delta_1 \sin_2 &= \Delta_1 \sin_1 + \Delta_2 \sin_1, \\ \sin_3 &= \sin_2 + \Delta_1 \sin_2, \\ &\ldots \end{aligned}$$

Babbage observed that if this idea were to be implemented, one requirement would be the ability to pass values between columns, in this case from the result column to the second difference column. He considered ways in which this could be done, and in fact the portion of the engine that was actually assembled in 1832 included mechanism which provided the ability to transfer a single digit between these two columns.[24]

[22] Lardner (1834), p. 283.

[23] Babbage (1822a).

[24] Collier (1970), pp. 111–112.

Babbage then noticed that connecting the columns of the engine in this way would enable it to "produce tables of a new species altogether different from any with which I was acquainted".[25] As his first example of such a table, he considered one where the second difference was equal to the units figure of a number already computed. This is, of course, a table that could be easily have been computed by the machinery assembled in 1832.

Babbage proceeded to analyze these new functions, and succeeded, with some difficulty, in deriving formulae that would generate the same numerical sequences. He commented later that this episode provided the first ever "example of analytical enquiries, suggested and rendered necessary by the progress of machinery adapted to numerical computation".[26] He was further surprised to discover that one of the functions he investigated was related to enquiries that he had made years previously in the course of an investigation into the problem of describing knight's tours on a chessboard.

A further example of a table that Babbage claimed the engine would be able to compute was a series of cube numbers, "subject to this condition, that whenever the number 2 occurred in the tens' place, that and all the succeeding cubes should be increased by ten".[27] This example appears to introduce a new requirement, namely the ability for the machine to detect the occurrence of a 2 and to react accordingly. Unfortunately, Babbage did not give a detailed explanation of how the engine might have performed this task.

Babbage attached great significance to these discoveries,[28] even giving a detailed example of the phenomenon in *The Economy of Machinery and Manufactures*, an unexpected detail in a book intended for the general reader.[29] He was struck in particular by the contrast between the ease with which such new tables could be mechanically computed and the difficulty of finding an analytical solution. This led him to reflect that the ability to perform mechanical calculation could produce useful results even when no theoretical account was given, and also that "one of the first effects of machinery adapted to numbers, has been to lead us to surmount new difficulties in analysis".[30]

2.4 The Mechanical Notation

In the course of his early work on the design of the Difference Engine, Babbage reflected on his working practices, and formulated a explicit account of the process

[25] Babbage (1822a), p. 312.

[26] Babbage (1826b), p. 217.

[27] Babbage (1823), p. 125.

[28] Baily (1823), p. 419, describes Babbage as considering that the "mechanical contrivances" embodied in the engine were of "a secondary kind" compared to these theoretical results.

[29] Babbage (1835b), p. 198.

[30] Babbage (1823), p. 127.

of "mechanical invention", a process which he broke down into three main stages. Given a description of what the proposed machine was intended to do, and taking advantage of any natural divisions such as that between calculation and printing in the Difference Engine, Babbage recommended that the inventor start with what appeared to be the most difficult part. Then,

> [a] kind of analysis of it must be made, and it will be subdivided into a number of different movements, some of which must be executed simultaneously, others in succession; some actions must take place at regular, others at irregular, periods.[31]

Some means to implement each of these individual movements should then be found, and they should all be assembled into a complete design without regard to elegance or mechanical efficiency. At this stage, the only requirement was "that supposing them all executed with perfect accuracy and supposing no flexure nor friction in the materials the machine would do its work". The inventor could then begin the second stage of the process, which "ought to consist almost entirely of simplification", and Babbage gave a number of detailed heuristics which could be applied to help spot ways of simplifying complex mechanical designs. Once the design had been simplified as much as possible, the third stage of Babbage's method involved selecting, or if necessary developing, the mechanisms to implement the new, simplified design.

To indicate the level of simplification that could be hoped for at the second stage, Babbage stated that his initial design for the prototype engine had consisted of 96 wheels and 24 axes, but that after simplification he was able to reduce this to 18 wheels and three axes. Lardner gave a similar account some years later of an instance where Babbage had managed dramatically to reduce the number of revolutions of a particular axis required to perform a specific task, indicating that simplification could be applied to the functioning as well as to the structure of the mechanism.

In the course of this work, Babbage found that the traditional method of using drawings to describe machinery was inadequate. A drawing could only represent the state of a machine at one instant, and so provided little assistance in understanding the sequences of movements involved in a complex mechanism or in working out the appropriate timing of the movements of its interacting parts. Babbage rejected as impractical the idea of producing a series of drawings of successive states of the machine, and believed that natural language was too verbose and ambiguous to be used. Instead, "being convinced from experience of the vast power which analysis derives from the great condensation of meaning in the language it employs", he decided "to have recourse to the language of signs".[32]

Unlike existing symbolic languages such as that of algebra, the notation Babbage developed was partly graphical: the machine to be analyzed was represented by a two-dimensional diagram containing various textual annotations. A diagram began with an enumeration of the moving parts of the machine in question, which were labelled to enable easy cross-reference with the drawings of the machine. No attempt

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[31] Babbage (1822c), p. 24.

[32] Babbage (1826a), p. 251.

was made in the notation to describe the actual form of the parts, although a number of their properties could be listed, such as their velocity. The names of the parts were listed in a row across the top of the diagram, and each part had a vertical 'indicating line' below it.

Arrows were drawn between the indicating lines to show a second major feature of mechanisms, namely the connections between parts by means of which motion was communicated from one to another. Different forms of arrow showed alternative types of connection, indicating whether motion was communicated by attaching one part to another, by friction, by a ratchet-driven mechanism, or a variety of other means.

Most importantly, the notation showed the "succession of the movements which take place in the working of the machine". Babbage assumed that a machine's action was periodic, and that after a certain period of time it would return to its initial state and the sequence of movements would be repeated. The movements of each part were shown by means of various symbols written next to the indicating lines. Thus reading down the indicating line for a part would show its motion through one cycle of machine operation, whereas reading horizontally across all the indicating lines would show the motions of all the parts at a given instant. As Lardner later put it, "it exhibits in a map, as it were, that which every part of the machine is doing at each moment of time".[33]

Lardner devoted several pages of his article to the notation, giving examples of its utility and power. He described it as an aid to "invention and discovery", a claim that had also been made when symbolic notation was introduced into algebra two centuries earlier. Lardner himself drew attention to this comparison, writing that "[w]hat algebra is to arithmetic, the notation we now allude to is to mechanism".[34] He stressed its applicability to the second, simplifying stage of Babbage's method: if the complexity of the whole machine was represented in easily manipulable signs, it would help the inventor to identify possible modifications and simplifications that would otherwise have remained obscure. Finally, in an observation which throws an interesting light on contemporary understanding of the relationships between machines and society, he claimed that the notation could be applied to describe the organization of "an extensive factory, or any great public institution, in which a vast number of individuals are employed, and their duties regulated (as they generally are or ought to be) by a consistent and well-digested system".[35]

Babbage continued to use and further develop the notation while working on the Analytical Engine, but he wrote little else about it and it never became widely known or applied. Nevertheless, it is of interest as a novel attempt to apply a symbolic approach to the design and description of complex processes.

[33] Lardner (1834), p. 313.

[34] Lardner (1834), p. 315.

[35] Lardner (1834), p. 319.

2.5 The Analytical Engine

It is misleading to talk about Babbage's 'Analytical Engine' as if it was a single, definite artefact: the term refers to a projected machine which Babbage described in a series of drawings and other documents over a period of many years, during which time his plans naturally changed considerably. Babbage published no description of the machine until the appearance of his autobiography, and the machine was never built, though a model of certain parts of it was constructed late in Babbage's life.

The Origins of the Engine Babbage referred to a new engine in 1834, in a letter to the Prime Minister, the Duke of Wellington. Babbage explained that difficulties he had encountered in the construction of the Difference Engine had led to him having no access to the engineering drawings of that machine for two years. When he regained possession of the drawings, Babbage said, he "immediately began a re-examination and criticism of every part. The result of this, and of my increased knowledge, has been the contrivance of a totally new engine possessing much more extensive powers, and capable of calculations of a nature far more complicated".[36] However, he gave no specific details of the new machine, and in fact seemed rather pessimistic about his chances of success in persuading the British Government to support its construction.

This re-examination began with a reconsideration of the mechanism necessary to transfer numbers from one column to another. Babbage had examined this ten years earlier, when considering how transcendental functions could be computed on the difference engine, but had put the matter aside while concentrating on the practical problems of building the machine. He now referred to this technique as enabling the machine to "eat its own tail", and settled on a scheme where the columns would be arranged around a large central wheel, enabling numbers to be transferred between them easily.[37]

Computation of functions by this new method would also require the ability to handle negative as well as positive numbers, and to perform multiplications. Both of these requirements went beyond the capabilities of the difference engine, and so Babbage turned his attention to the design of machinery to carry out multiplication and division. This led to two significant developments. Firstly, in order to minimize the time taken, Babbage came up with more efficient methods for adding numbers, using in particular a technique known as *anticipatory carrying*. The mechanism for implementing this was rather extensive, however, and rather than making multiple copies of it for each number column, Babbage was led to the idea of separating the parts of the machine that carried out arithmetical operations from those which simply stored numbers.

Secondly, he came up with what he later called a "tentative" process for carrying out division, by repeatedly subtracting multiples of the divisor from the dividend.

[36] Babbage (1834), p. 6.

[37] This account is largely taken from the account of Babbage's notebook entries summarized by Collier (1970), pp. 116–140.

This process should come to an end when a subtraction caused the remaining value of the dividend to become negative. Babbage designed a mechanism which would enable the engine to detect when this had happened, undo the last subtraction, and proceed to the next part of the calculation.

At this point, towards the middle of 1835, Babbage mentioned the new engine in a letter to the Belgian statistician Quetelet. He described it as being "capable of having 100 variables (or numbers susceptible of change) placed upon it" and being able to perform a range of arithmetical operations on any of these variables. The operations Babbage lists are addition, subtraction, multiplication and division of two numbers, extracting the square root of a number, and reducing a number to zero. By these means, Babbage claimed, "if $f(v_1, v_2, \ldots, v_n)$, $n < 120$, be any given formula which can be formed by addition, multiplication, subtraction, division, or extraction of square root the engine will calculate the numerical value of f",[38] and he went on to give examples of some situations where this capability could be used.

This suggests that Babbage viewed the primary purpose of the new engine as being the evaluation of algebraic formulae. However, he had at this stage no easy way of controlling the great variety of operations that would be required in the evaluation of any significant computation. In 1836, however, he came up with the idea of using punched cards to control the progress of the machine, as Jacquard had done earlier in the century in the development of his automated loom.

This turned out to be last major innovation in the design of the new engine, and in 1837 Babbage described its structure and functioning in a lengthy but incomplete manuscript entitled *On the mathematical powers of the calculating engine*.[39] As the title suggests, Babbage was more concerned to describe the capabilities of the proposed machine than the details of its construction, but nevertheless he did not give a clear statement of its purpose. However, the final section of the manuscript, on the use of the machine, is headed "Of computing the numerical value of algebraic formulae", and the discussion and examples given strongly support the idea that at this period Babbage saw this as the primary application for the machine.

The Structure of the Analytical Engine Despite inevitable later revisions, most of the features described in the manuscript of 1837 remained constant during the years that Babbage worked on the Analytical Engine.[40] Like the Difference Engine, at the top level it was divided into two major components.

> The calculating part of the engine may be divided into two portions:
> First. The *mill* in which all operations are performed.

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[38] Babbage (1835a), pp. 12, 13. The apparent discrepancy between the number of variables stored in the machine and the number of function arguments is in Babbage's text.

[39] Babbage (1837b).

[40] According to Allan Bromley, who carried out extensive research on Babbage's unpublished design notations for the engine, the design that emerged around 1838 was the basis for Babbage's refinements and elaborations in the subsequent ten years (Bromley 1982). Bromley's paper gives a detailed physical description of the engine, including the engaging observation that the completed machine would "have been about the size and weight of s small railway locomotive".

> Secondly. The *store* in which all the numbers are originally placed and to which the numbers computed by the engine are returned.[41]

At one level, the top-level structure of the engine and the terminology used by Babbage to describe it can be seen as reflecting a division of labour similar to that employed in the cotton industry, where the raw material was worked on in the mill and the resulting fabrics were removed from and replaced in a storehouse. However, a deeper motivation comes from Babbage's philosophical views on the notation of mathematics, which took as fundamental the distinction between operations and variables, and saw the evaluation of a formula as a question of applying the required operations to the numbers provided. Babbage's distinction between the mill and the store, on this view, simply makes concrete this theoretical distinction. As Lovelace put it some years later,

> [i]n studying the action of the Analytical Engine, we find that the peculiar and independent nature of the considerations which in all mathematical analysis belong to *operations*, as distinguished from *the objects operated upon* and from the *results* of the operations performed upon those objects, is very strikingly defined and separated.[42]

The store consisted of a number of *figure axes*, also known as *variables*, each of which was capable of storing one number. Each axis consisted of 40 *figure wheels* mounted vertically on a single axle. Each wheel was marked with the digits 0 to 9, and could be moved to the desired position by hand, thus allowing manual entry of numbers into the store. A additional wheel recorded the sign of the number stored on the axis; this wheel also had ten positions, of which the even positions were marked with a plus sign and the odd positions with a minus sign. Each axis was identified by a fixed label or variable number, such as V_1, which was displayed above the axis. In addition, below the figure wheels each axis had

> a small square frame [...] in which may be inserted a card to be changed according to the nature of the calculation directed. On this card is written that particular variable or constant of the formula to be computed whose numerical coefficient and sign are expressed on the wheels above it.[43]

If, for example, it was necessary to calculate the value of the formula $\sqrt{a^2+x^2}$ from the values of a and x, it might be decided to place the values of a and x on the variable axes V_2 and V_3, respectively, and the final value on V_6. In this case, the symbols a, x and $\sqrt{a^2+x^2}$, using the mathematical notation relevant to the problem at hand, would be written on cards and placed beneath the variables V_1, V_2 and V_6. Clearly, this technique had no effect on the operation of the machine, and was purely intended as an aid to its human users.

Numbers could be transferred from the store to the mill, where they would be operated upon, and the results of these operations could then be transferred back to the store. The engine was designed to perform arithmetical operations, and in particular addition, subtraction, multiplication and division. Babbage occasionally

[41] Babbage (1837b), p. 15.

[42] Lovelace (1843), p. 692.

[43] Babbage (1837b), p. 23.

referred to other possible operations, such as the extraction of roots, but described only the four basic operations in detail.

The detailed progress of an operation such as the addition of two numbers was controlled by cylinders known as *barrels*. Studs were attached to the barrels in a number of vertical columns, and as the barrels rotated, the studs caused different parts of the mechanism of the mill to be brought into action, in a manner similar to that employed in a music box or barrel organ. The barrels were capable of more than simply controlling a sequence of actions, however, and various mechanisms enabled the mill to detect and respond to situations that might arise in the course of carrying out an operation.

The most important such situation was when a larger number was subtracted from a smaller, giving rise to a negative result. This particular event was known as *running up*; it was, in effect, a carry that, because of the numerical representation used, ran off the end of the figure axes, hence the name. The machine contained "a lever on which the *running up* warning acts and this lever governs many parts of the engine according as the circumstances demand".[44] This feature gave the engine the capacity to detect and react to certain events that took place during the course of a computation by affecting the way in which barrel selected the next column of studs to be applied to the mechanism.[45]

The course of a computation was determined by the sequence of operations that was to be performed. Operations were selected by means of *operation cards*, which would be presented in order to the mill. These cards were to be perforated pieces of pasteboard or thin metal similar to those used in Jacquard's automated loom. A card would be pressed against an array of levers; the unperforated positions in the card would engage a particular set of levers, which would then control the progress of the specified operation. As Babbage commented, "by arranging a string of cards with properly prepared holes any series of orders however arbitrary and however extensive may be given through the intervention of these levers".[46]

The process of reading an operation card and carrying out the specified operation was carried out under the control of the barrels. When an operation was requested, the numbers to be operated on would be fetched from the store and placed on two figure axes in the mill; on completion of the operation, the result would be returned to the store. In the same way as operation cards selected the operations that were to be performed, *variable cards* selected the quantities that would be operated on. At the start of an operation, the mill would request numbers from the store; the numbers selected would be those on the figure axes specified by the next variable cards to be presented to the store. At the conclusion of an operation, the result would be placed on a figure axis specified by the next variable card.

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[44] Babbage (1837b), p. 17.

[45] Allen Bromley describes in detail the conditional operations implemented within the mill, and gives Babbage the credit for originating "the whole concept of a conditional sequence of operations in a machine, and in particular of a conditional dependence on the outcome of previous actions of the machine" (Bromley 1982).

[46] Babbage (1837b), p. 20.

The independence of the operation and variable cards is a notable feature of the design of the Analytical Engine. The cards controlled the operations of separate parts of the machine, the mill and the store, respectively. Because the control levers differed, the two types of card were to be of different sizes and to have different patterns of perforation. An operation card could be used to operate on different data at different times, as the numbers used on each occasion would be determined solely by the variable cards that were presented to the store.

Babbage saw that particular operations and sequences of operations might need to be carried out repeatedly in the course of a computation. He therefore proposed that special cards, known as *combinatorial cards*, could be inserted in the sequence of operation cards. The function of the combinatorial cards was:

> To govern the *repeating apparatus* of the operation and of the variable cards and thus to direct at certain intervals the return of those cards to given places and to direct the number and nature of the repetitions which are to be made by those cards.[47]

Subsequent entries in Babbage's notebooks described a set of special counting wheels which would record the number of times operations should be repeated. When a combinatorial card was encountered, the number on these wheels would be reduced by one and the operation cards backed up as far as required. When the number on the wheels reached zero, the combinatorial card would be ignored and the computation would proceed with the next operation card. Babbage's first idea involved backing up the operation cards to the beginning, but he later proposed that the cards could be backed up to a predetermined spot in the sequence, or by a certain number of cards.[48]

A different further method of repeating operations, suggested in the early 1840s, was for each operation card to contain an index number which would specify how many times the operation on that card should be carried out before the next operation card was read.[49] The sole purpose of this suggestion seems to have been to minimize the number of operation cards that needed to be prepared for a calculation.

A further type of cards were the *number cards*; these were perforated in such a way as to make it possible to transfer a number from the card to a figure axis. An advantage of a number card as opposed to the manual entry of a number on an axis was the possibility of reusing the card whenever necessary. Babbage also considered that the cards could be used in cases where a calculation exceeded the number of variables provided in the store, and the use of number cards in a calculation was therefore to be controlled by the variable cards.[50]

The mechanical description of the engine concluded with a number of proposed auxiliary devices, such as a card punch which would enable numbers to be punched onto the number cards in the first place. Other planned output devices included an apparatus for printing results on paper or copperplate, and Babbage also mentioned

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[47] Babbage (1837b), p. 21.

[48] The relevant notebook entries have been summarized by Collier (1970), pp. 196–200.

[49] See the references listed by Collier (1970), p. 199.

[50] Babbage (1837b), p. 27.

the possibility of producing a graphical representation of results by means of a curve drawing apparatus.

The Mathematical View In the 1837 manuscript, Babbage gave few details of how the engine would be used. He seems to have envisaged a loose division of labour between the "employer of the engine", who would specify the operation and variable cards to be used for a computation along with any required constants, and a number of "attendants" and a "superintendent" who would actually operate the engine.[51] As well as simple sequences of operations, it was proposed that users could order the repetition of groups of operations using the combinatorial cards, but Babbage does not seem to have envisaged at this point that users could specify alternative sequences. The running up mechanism provided conditional execution at certain points in the *internal* implementation of operations, in particular division, but this capability was not made available to the user.

In 1840, Babbage travelled to Turin, where he presented his ideas to a number of Italian scientists. Following this visit, the mathematician L.F. Menabrea wrote an account of the engine, which was published in 1842. A translation of this article, which was written in French, was published by Ada Lovelace in the following year. Lovelace added some extensive notes of her own to the translation. Both Menabrea and Lovelace had discussed the engine at length with Babbage, and their writings form an important source of information about Babbage's ideas. In particular, they included a number of detailed descriptions of the operations that would be required to perform various computational tasks, derived from examples that Babbage had developed between 1837 and 1840, but never published. These examples included the tabulation of polynomials and iterative formulae, and the solution of simultaneous equations using Gaussian elimination.[52]

Unlike Babbage, Menabrea and Lovelace paid little attention to the mechanical details of the Engine, focusing instead on those aspects of it that would be important to someone wanting to use the completed machine to perform calculations. Lovelace characterized this situation by drawing a distinction between the "mechanical" and "mathematical" views of the engine. The mathematical view was that of someone "taking for granted that mechanism is able to perform certain processes, but without attempting to explain *how*", and describing instead "the manner in which analytical laws can be so arranged and combined as to bring every branch of that vast subject within the grasp of the assumed powers of mechanism".[53] She emphasized that the two views were complementary, and commented that "the same mind might not be likely to prove equally profound or successful in both".

Menabrea and Lovelace gave just enough mechanical detail about the engine to enable the reader to understand how it might be used to perform calculations. Rather than going into the details of how division was performed, for example, Menabrea wrote that "we must limit ourselves to admitting that the first four operations of

[51] Babbage (1837b), p. 52.

[52] Bromley (1982), p. 215.

[53] Lovelace (1843), p. 700.

arithmetic ... can be performed in a direct manner through the intervention of the machine". The general pattern for the execution of an arithmetical operation was the following:

> When two numbers have been thus written on two distinct columns, we may propose to combine them arithmetically with each other, and to obtain the result on a third column.[54]

To carry out an operation it was necessary first to configure the mill to perform that operation, and then to transfer the required numbers from the store to the mill; when the operation was completed, the answer should be transferred back to the store. Menabrea explained how operation and variable cards were used to carry out these tasks automatically. However, when giving detailed examples of calculations, Menabrea followed the example of Babbage's unpublished work and rather than describing the actual cards required, he listed the operations carried out, in a "three address" notation of a kind that would become familiar a century later.

It is worth noting, however, that this notation simplifies the actual behaviour of the engine. In order to deal with the complexities of adding and subtracting signed numbers, Babbage distinguished what he called the 'algebraic' sign of a number from its 'accidental' sign. In the formula $P - Q$, for example, the algebraic signs of P and Q are $+$ and $-$, respectively; however, their accidental signs could be $+$ or $-$, depending on whether the corresponding numbers in the store were positive or negative. Depending on the combination of these signs, the actual operation to be performed might differ from that specified on an operation card: in the formula above, if the accidental sign of Q was negative, for example, its absolute value would be added to that of P rather than subtracted from it.

Whereas the tabular notation suggests that addition and subtraction operations were specified by a single card, Babbage had actually stated that:

> For the processes of addition and subtraction two operation cards only are necessary. One of these is required for each quantity, and assigns to it the algebraic sign of the quantity.[55]

This approach allowed formulae such as $-P - Q$ to be handled in a consistent way, and also allowed repeated sums to be formed without moving partial answers back and forth between the mill and the store, making a potential saving in computation time.

For his first example, Menabrea described the operations required to calculate the value of x in the following pair of simultaneous equations:

$$mx + ny = d,$$
$$m'x + n'y = d'.$$

The series of operations specified by the cards together with the results of each operation were given in the tabular format shown in Table 2.3; the formula used to calculate x is displayed in the bottom right-hand cell of the table. Each line in this table represents a single operation and shows the two variables holding the numbers

[54] Menabrea (1842), p. 676.

[55] Babbage (1837b), p. 49.

Table 2.3 A computation on the Analytical Engine

Number of the operations	Operation cards: Symbol of operation	Variable cards: Columns on which operations are performed	Variable cards: Columns which receive results	Progress of the operations
1	$\times$	$V_2 \times V_4 =$	$V_8 \ldots$	$= dn'$
2	$\times$	$V_5 \times V_1 =$	$V_9 \ldots$	$= d'n$
3	$\times$	$V_4 \times V_0 =$	$V_{10} \ldots$	$= n'm$
4	$\times$	$V_1 \times V_3 =$	$V_{11} \ldots$	$= nm'$
5	$-$	$V_8 - V_9 =$	$V_{12} \ldots$	$= dn' - d'n$
6	$-$	$V_{10} - V_{11} =$	$V_{13} \ldots$	$= n'm - nm'$
7	$\div$	$\frac{V_{12}}{V_{13}} =$	$V_{14} \ldots$	$= x = \frac{dn'-d'n}{n'm-nm'}$

to be operated on as well as the variable that would receive the calculated result. The final column shows the mathematical meaning of the result variable, using the notation of the original problem. These are the formulae that would be written on the cards beneath the columns V_8 to V_{14}.

To execute this calculation on the Analytical Engine, a set of operation cards would have to be derived from the list of operations given in the second column of the table and a set of variable cards from the entries in the third and fourth columns. There was no need, however, to reconfigure the mill when the same operation was repeated, and therefore no need for a sequence of operation cards requesting the same operation. Menabrea observed that the number of operation cards required could be reduced if the machine included "an apparatus which shall, after the first multiplication, for instance, retain the card which relates to this operation, and not allow it to advance so as to be replaced by another one, until after this same operation shall have been four times repeated".[56] To reflect this, he used a revised tabular format, which included an additional column numbering the operation cards that were physically required.[57] He did not give any details of the postulated counting mechanism, however, nor do the tables include an explicit statement of the number of times any particular operation card is to be used.

A second modification to the notation concerned the reuse of numbers stored on the variable columns. The normal behaviour of the engine was to erase a variable when the number on it was transferred to the store. After operation 1 in Table 2.3, for example, the number stored on variable V_4 would be erased and would no longer be available for use by operation 3. It was possible to use an alternative form of variable card, however, which specified that when a number was transferred to the mill it would also be preserved in the store. Menabrea therefore extended the notation with an additional column headed "indication of the new columns on which the variables

[56]Menabrea (1842), p. 679.

[57]Menabrea (1842), p. 681.

are written"; for the first operation, the entry in this column would have read "V_2 on V_2, V_4 on V_4", indicating that both columns V_2 and V_4 kept the number stored on them after it was called into the mill for the multiplication.[58]

Menabrea gave further examples of calculations in which the variable columns were taken to represent, not simply the numbers in an algebraic formula, but the coefficients of the terms in a series such as $\Sigma_i \cos^i x$. This use of the engine, although it required a different approach to the planning of a calculation, did not require any additional features of the engine or notation.

As well as the ability to repeat cycles of operations, the need for the engine to be able to perform different sequences of operations in different circumstances was recognized. Menabrea introduced this requirement by considering "certain functions which necessarily change in nature when they pass through zero or infinity", and he described how the machine might stop and ring a bell to summon an operator when this happened. However:

> If this process has been foreseen, then the machine, instead of ringing, will so dispose itself as to present to the new cards which have relation to the operation that is to succeed the passage through zero and infinity. These new cards may follow the first, but may only come into play contingently upon one or other of the two circumstances just mentioned coming into place.[59]

To illustrate this capability, Menabrea considered the evaluation of terms of the form ab^n, where the number of multiplications to be performed depends on the value of the exponent n. He described how the value of n could be placed on a certain "registering-apparatus" and reduced by one each time a multiplication took place. The engine would detect when this value reached zero, and "pursuing its course of operations, will order the product of b^n by a".[60]

In a second example, he considered a calculation where it was required to tell when the two expressions $m + q$ and $n + p$ were equal. Menabrea wrote that:

> For this purpose, the cards may order $m + q$ and $n + p$ to be transferred to the mill, and there subtracted one from the other; if the remainder is nothing [...] the mill will order other cards to bring to it the coefficients Ab and Ba, that it may add them together.

However, no details were given of the mechanisms that would enable the engine to perform this kind of conditional execution of operations, nor of the details of the cards that would be required to specify them. However, Menabrea did draw the following general conclusion about the scope and power of this procedure:

> This example illustrates how the cards are able to reproduce all the operations which intellect performs in order to attain a determinate result, if these operations are themselves capable of being precisely defined.

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[58] In her translation, Lovelace slightly altered the heading of this new column and the notation used to express the preservation of the variables.

[59] Menabrea (1842), p. 685.

[60] Menabrea (1842), p. 686.

Translating Analysis As discussed at the beginning of this section, it appears that Babbage originally saw the Analytical Engine as a device to evaluate algebraic formulae. However, over the years a more expansive interpretation seems to have evolved, articulated around the notion that the engine was, in a very literal sense, a translation of the language of analysis into machinery. This idea was developed on two levels. On the material level, as pointed out above, the physical structure of the engine reflected Babbage's view of the structure of the mathematical universe, with the mill and the store corresponding to the basic categories of operations and variables, respectively.

The various operations that the engine could perform were carried out by largely independent pieces of machinery in the mill. To execute a particular operation, the perforations in an operation card would cause the machinery for that operation to be engaged. So in a very immediate way, mathematical operations were represented by machinery, a situation described by Lovelace as one in which "matter has been enabled to become the working agent of abstract mental operations".

However, as the engine was only planned to implement four basic arithmetical operations, it could be asked, as Lovelace did, whether the "executive faculties of this engine" were "*really* even able to *follow* analysis in its whole extent?" She answered this question in the affirmative, by listing the analytical procedures that the engine could carry out.

On a more abstract level, however, an equivalence was proposed between the engine and the notion of a function itself. Lovelace wrote that

> the engine may be described as being the material expression of any indefinite function of any degree of generality and complexity,

while receiving by means of the cards "the impress of whatever *special* function we may desire to develop or to tabulate".[61]

The cards themselves were described as literal translations of the corresponding algebraic formulae. Menabrea expressed this point of view very clearly, writing that "the cards are merely a translation of algebraical formulae, or, to express it better, another form of analytical notation".[62] Thus, to take a very simple example, the translation of the formula $x \times y$ would require one operation card corresponding to the multiplication symbol, and two variable cards denoting the variables in the store holding the values of x and y.

Another notion of generality was brought out in Babbage's comment that "the operation cards partake of that generality which belongs to the algebraic signs they represent", and Menabrea stated similarly that "the cards will themselves possess all the generality of analysis, of which they are merely a translation". One aspect of this generality had to do with the distinction between variables and numbers. Menabrea pointed out that just as "a formula simply indicates the number and order of the operations requisite for arriving at a certain definite result", so the corresponding

[61] Lovelace (1843), p. 691.

[62] Menabrea (1842), p. 688.

set of operation and variable cards "will serve for all questions whose sameness of nature is such as to require nothing altered excepting the numerical data".[63]

The combination of the claim that the cards translated formulae and the fact that the engine only implemented a limited range of operations raised certain problems, however. For example, consider the case of formulae of the form b^n, where one number is raised to an integral power. If the idea of translation is to be preserved, this formula cannot be interpreted as containing an exponential operation, as the engine contained no such operation. Rather, it should be interpreted as containing a reference to a multiplication operation, with the exponent n indicating how often the multiplication should be carried out. As Menabrea put it, as the cards were to be a translation of the analytical formula, the number of operation cards required to evaluate such terms should be the same whatever the value of n, even though differing values of n indicated that the multiplication operation must be performed a different number of times.

While this interpretation preserves the analogy between operation symbols and operation cards, however, it is difficult to square with the idea that variables simply store quantities that are used in calculations. In the formula b^n, n is not a straightforward symbol of quantity: the value of n is not used directly in calculation, but is rather used to *control* the progress of the calculation, specifying how often the quantity represented by b should be multiplied by itself. Lovelace blamed this on the deficiencies of current analytical notation, writing that "figures, the symbols of *numerical magnitude*, are frequently also the *symbols of operations*, as when they are the indices of powers",[64] and she went on to describe how numbers representing operations and those representing quantities are kept separate in the store, although it appears that this distinction was a matter of conventional usage, rather than being enforced mechanically.

2.6 The Science of Operations

The distinction between operations and the objects operated on was fundamental to the design of the Analytical Engine. This reflected current philosophical thinking about mathematics, as Lovelace pointed out in the first of the notes she added to her translation of Menabrea's account. She went on to give a general account of this new outlook, broadening its scope away from the purely mathematical.

She first offered an abstract definition of what an operation was, as "*any process which alters the mutual relation of two or more things*". Although it was inspired by mathematics, this definition was meant to be applicable to "all subjects in the universe", and Lovelace saw the study of operations as having a completely general scope:

[63] Menabrea (1842), pp. 685, 688.

[64] Lovelace (1843), p. 693.

> the science of operations, as derived from mathematics more especially, is a science of itself, and has its own abstract truth and value; just as logic has its own peculiar truth and value, independently of the subjects to which we may apply its reasonings and processes.[65]

Lovelace did not give details of other applications of the science of operations, although she did speculate that if "the fundamental relations of pitched sounds in the science of harmony" could be expressed as operations of the requisite sort and adapted to the mechanism of the engine, then it might be able to "compose elaborate and scientific pieces of music of any complexity or extent".[66] Later, she referred to "*symbolical results*" being generated by the machine, but without giving a clear example of what was meant by this.[67]

For Lovelace, therefore, the Analytical Engine was not simply a machine which evaluated formulae. In her view, it had a more general significance as "an *embodying of the science of operations*, constructed with particular reference to abstract number as the subject of those operations", in contrast with the Difference Engine, which was "the embodying of *one particular and very limited set of operations*".[68]

By "the science of operations", Lovelace appears to have meant some kind of general study of the way in which the operations required in calculations could be specified. The engine was designed primarily to execute a sequence of operations, and given the assumption that at each step in the calculation "the *same operation* would be performed on different *subjects of operation*" she found it natural to use a notation which represented the sequence of operations, but not the variables required at each step. For a simple example, she notes that she required

> In all, seven multiplications to complete the whole process. We may thus represent them:
>
> $$(\times, \times, \times, \times, \times, \times, \times), \quad \text{or} \quad 7(\times).$$

A more complex formula, which involved finding the quotient of two terms, was represented as follows:

$$\{7(\times), 2(\times), \div\}, \quad \text{or} \quad \{9(\times), \div\},$$

suggesting that Lovelace was not concerned whether the sequence of operations manifested the structure of the original formula or not.

In a later note, Lovelace gave a more complex example, in which she expanded and commented on this notation. The example was to multiply two trigonometrical series with coefficients A_n and B_n by calculating the coefficients C_n of the terms

[65] Lovelace (1843), p. 693; emphasis in original.

[66] Lovelace (1843), p. 694.

[67] Lovelace (1843), p. 695; emphasis in original.

[68] Lovelace (1843), p. 694; emphasis in original.

of the resulting series. The required coefficients C_n could be calculated using the following formulae:[69]

$$C = BA + \frac{1}{2}B_1A_1,$$
$$C_1 = BA_1 + B_1A + \frac{1}{2}B_1A_2,$$
$$C_n = BA_n + \frac{1}{2}B_1 \cdot (A_{n-1} + A_{n+2}).$$

Lovelace then gave the following expression, representing "the successive sets of operations for computing the coefficients of $n + 2$ terms":

$$(\times, \times, \div, +), (\times, \times, \times, \div, +, +), n(\times, +, \times, \div, +),$$

observing that "the brackets ... point out the relation in which the operations may be *grouped*, while the comma marks *succession*". The operations in the first set of brackets compute the term C, those in the second the term C_1 and those in the third any subsequent term; it is not altogether straightforward to check the equivalence between the sequences of symbols and the operations that would be performed to evaluate the formulae.

This expression contains what Lovelace termed a *recurring group* of operations, or a *cycle*, and she observed that "[w]herever a general term exists, there will be a *recurring group* of operations". Further, "[i]n many cases of analysis there is a *recurring group* of one or more *cycles*; that is a *cycle of a cycle*, or a *cycle of cycles*". For example, if it was required to form the quotient of two polynomials of order p, the operations required to form the coefficient of one term of the quotient are

$$\{(\div), p(\times, -)\},$$

and so the operations required to calculate n terms of the quotient could be represented as

$$n\{(\div), p(\times, -)\}.$$

Lovelace then took a further step, adapting "some of the notation of the integral calculus" to her evolving notation. She rewrote the previous expression in what was intended to be an equivalent form as:

$$\Sigma(+1)^n\{(\div), \Sigma(+1)^p(\times, -)\}.$$

Lovelace explains this notation by writing that "p stands for the variable; $(+1)^p$ for the function of the variable, that is for ϕp; and the limits are from 1 to p, or from 0 to $p - 1$, each increment being equal to unity".[70]

This notation is further applied to the case of *varying cycles*, where "each cycle contains the same group of operations, but in which the number of repetitions of the group varies according to a fixed rate, with every cycle". What Lovelace appears to have in mind here is the case of a cycle within a cycle, where the inner cycle is

[69] Lovelace (1843), pp. 715–716; A, B and C can here be read as A_0, B_0 and C_0.

[70] Lovelace (1843), p. 719.

repeated a different number of times at each repetition of the outer cycle. Without the new notation, the outer cycle must be written out in full, as in the following example:[71]

$$p(1, 2, \ldots, m), (p-1)(1, 2, \ldots, m), (p-2)(1, 2, \ldots, m), \ldots, (p-n)(1, 2, \ldots, m).$$

Lovelace claimed that this could be equivalently written as

$$\Sigma p(1, 2, \ldots m), \text{the limits of } p \text{ being from} p-n \text{ to } p,$$

but this idea was not explored or explained further, and it was left unclear how the variable n of the outer cycle could be related to the inner cycle, and how the informal description of the limits might have been symbolized.

Lovelace's notes, then, contain some suggestions for a potentially sophisticated notation for expressing complex sequences of operations, and in particular those containing recurring groups of operations. These ideas were not integrated into the more comprehensive notation that she used for planning calculations, however. Her final example, a "diagram for the computation by the Engine of the Numbers of Bernoulli", used a tabular format similar to that used by Menabrea, but with a greater number of annotations describing the mathematical progress of the calculation. On this diagram, cycles and cycles of cycles were indicated by means of brackets in the margins of the table indicating the recurring groups of operations, but the notation gave no indication of how often a particular cycle would be repeated.

Also, the relationship between the notation and those aspects of the machine that would control the execution of cycles was left rather vague. Lovelace stated that the backing mechanism was to be used to execute the operations in a cycle, but her explanation of how the extent of the cycle or the number of required recurrences were communicated to the engine was restricted to the following suggestion:

> Σ, in reality, here indicates that when a certain number of cards have acted in succession, the prism over which they revolve must *rotate backwards*, so as to bring those cards into their former position; and the limits 1 to n, 1 to p, etc., regulate how often this backward rotation is to be repeated.[72]

2.7 The Meanings of the Analytical Engine

Mechanizing the Mind As with the Difference Engine, an important aspect of the Analytical Engine was its demonstration of how an apparently mental process, that of evaluating algebraic formulae, was in fact mechanizable. Menabrea had argued that mathematics could be divided into a mechanical part, "subjected to precise and invariable laws, that are capable of being expressed by the operations of matter",

[71] Lovelace (1843), p. 719; the sequence $(1, 2, \ldots, m)$ here denotes a group of unspecified operations.

[72] Lovelace (1843), p. 720.

and a part "demanding the intervention of reason" which "belongs more specially to the domain of the understanding". Because it implemented not simply the four basic arithmetical operations, but also the operations that were involved in applying the sequence of operations required to evaluate a formula, the Analytical Engine was of a much wider scope than its predecessor. As we have seen, Menabrea considered it to be a "system of mechanism whose operations should themselves possess all the generality of algebraical notation".[73]

The greater capabilities of the Analytical Engine brought with them a temptation to use increasingly anthropomorphic language in talking about it, as Babbage noted apologetically in 1837:

> In substituting mechanism for the performance of operations hitherto executed by intellectual labour it is continually necessary to speak of contrivances by which certain alterations in parts of the machine enable it to execute or refrain from executing particular functions. The analogy between these acts and the operations of mind almost forced upon me the figurative employment of the same terms. They were found at once convenient and expressive and I prefer continuing their use rather than substituting lengthened circumlocutions.
> For example, the expression 'the engine *knows*, etc.' means that one out of many possible results of its calculations has happened and that a certain change in its arrangement has taken place by which it is compelled to carry on the next computation in a certain appointed way.[74]

In particular, it seems to have been the apparent ability of the Engine to choose between alternative courses of action which raised questions about what the most appropriate way to describe it was. Menabrea tended to emphasize its mechanical nature, and to stress that it could only proceed in a determinate manner. He pointed out that Babbage had had to devise a method for carrying out division which did not employ the usual "method of guessing indicated by the usual rules of arithmetic", and wrote that the machine "must exclude all methods of trial and guesswork, and can only admit the direct processes of calculation. It is necessarily thus; for the machine is not a thinking being, but simply an automaton which acts according to the laws imposed upon it".[75]

At times, Lovelace expressed herself with a greater freedom, however, and her comment on Babbage's comments above was that "[t]his must not be understood in too unqualified a manner. The engine is capable, under certain circumstances, of feeling about to discover which of two or more possible contingencies has occurred, and of then shaping its future course accordingly".[76] This is reminiscent of the passage to which the above quote from Babbage is a footnote, in which he describes his so-called "anticipatory carriage" mechanism by saying that he has designed the machine to "foresee" the effect of a carry operation in addition.

At other times, however, Lovelace too emphasized the machinic nature of the Engine, as in the following remark, later to be discussed in detail by Turing:

[73] Menabrea (1842), pp. 669, 674.

[74] Babbage (1837b), p. 31, footnote.

[75] Menabrea (1842), p. 675.

[76] Lovelace (1843), p. 675, footnote.

> The Analytical Engine has no pretensions whatever to *originate* anything. It can do whatever we *know how to order it* to perform.[77]

However, this did not mean that every step and result had to be known in advance: it is "by no means necessary that a formula proposed for solution should ever have been actually worked out, as a condition for enabling the engine to solve it. Provided we know the *series of operations* to be gone through, that is sufficient".[78]

Time and Economy Babbage paid great attention in the design of the Analytical Engine to making its operations as fast as possible, going so far as to say that "the whole history of the invention has been a struggle against time"; he gave a detailed account of the way in which the timing of the machine had been estimated and optimized.[79] This applied at the most detailed levels of the mechanism, as well as in more general concerns, such as the choice of the algorithm to be implemented for the basic operations of multiplication and division.

Menabrea referred to the "economy of time" that the machine enabled, quoting Babbage's estimate that the multiplication of two 20-digit numbers could be completed in three minutes. He further identified "economy of intelligence" as one of the machine's advantages, writing that "the engine may be considered as a real manufactory of figures".[80]

Lovelace reinforced this perception of a connection between the machine and contemporary views about political economy when describing its productive nature:

> In the case of the Analytical Engine we have undoubtedly to lay out a certain capital of analytical labour in one particular line; but this is in order that the engine may bring us in a much larger return in another line.[81]

The means by which this profit was to be realized was made clear by drawing an analogy between the internal structure of the engine and current practices of labour organization:

> The columns which receive [intermediate and temporary combinations of the primitive data] are rightly named *working Variables*, for their office is in its nature purely *subsidiary* to other purposes.[82]

In both its use and its internal structure, then, the Analytical Engine was viewed as having a close relationship with the capitalist economy of the factory system, and embodying its labour relations, just as its predecessor had been.

Avoidance of Error A significant advantage of the engine was what Menabrea called its "rigid accuracy", a property which stemmed not only from the supposed

[77] Lovelace (1843), p. 722.

[78] Lovelace (1843), p. 721.

[79] Babbage (1837b), pp. 39; 55–61.

[80] Menabrea (1842), p. 690.

[81] Lovelace (1843), p. 698.

[82] Lovelace (1843), p. 707.

infallibility of mechanical processes but also the fact that it required "no human intervention during the course of its operations".[83] However, there were still many points in the process of using the engine for a complete calculation at which the intervention of human operators would have been required. Babbage pointed out that users would have to insert potentially large amounts of numerical material into the machine, and also to compose the sequences of operation and variable cards required to translate a particular formula and make sure that they were correctly presented to the engine. While noting this, Lovelace claimed that there was nevertheless less chance of error in these tasks than in purely numerical work.[84]

Babbage suggested a number of techniques for verifying the correctness of the engine's results. For example, he suggested that all numbers entered into it should be immediately printed out, so that a subsequent check could be made that the correct numbers had been entered. For what he considered to be the more difficult task of checking the correctness of the cards used, a number of approaches to verification were suggested, including the use of coloured cards which would provide a visual check that the structure of the cards matched that of the formula.

He also suggested running test cases to verify a formula, using "such numerical values to the constant quantities as shall render its value easily computed by the pen", arguing rather optimistically that

> If trials of three or four simple cases have been made, and are found to agree with the results given by the engine, it is scarcely possible that there can be any error among the cards.[85]

Further ideas put forward included the suggestion that a computation could be carried out using two different, but mathematically equivalent, formulae and the results compared, and the use of a machine which would translate a set of cards back into the analytical formula represented before printing it out for checking. Finally, Babbage suggested that the use of combinatorial cards to reduce the number of cards required would simplify the task of preparing the cards for a computation, and even speculated that "the formula printing-machine might by some improvements itself ultimately work out many of such algebraic developments".[86]

Natural Theology In the 1830s, the Earl of Bridgewater sponsored the publication of eight treatises *On the Power, Wisdom, and Goodness of God, as manifested in the Creation*. Written by a number of well-known scientists and divines, the treatises were intended to demonstrate how the results and theories of recent natural science were compatible with an over-arching view of God as a designer. Babbage detected a 'prejudice' in the published treatises, particularly one written by William Whewell on physics and astronomy, to the effect "*that the pursuits of science are unfavourable to religion*" Babbage (1837a). He published his own views on natural theology in what he called an unofficial, 'ninth' Bridgewater treatise.

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[83] Menabrea (1842), p. 689.

[84] Babbage (1837b), pp. 51–54, Lovelace (1843), p. 698.

[85] Babbage (1837b), p. 53.

[86] Babbage (1837b), p. 54.

A central part of his argument focused on the question of miracles. This was an important question for natural theology, as the scientific world view, of a clockwork universe of matter which had been set in motion at the creation and was thereafter governed by unchangeable natural laws, seemed to be at odds with the assumed fact of miraculous events as revealed in the Bible.

Babbage advanced a view which purported to explain how miracles could in fact be produced by the operation of purely mechanical laws of nature. This argument was inspired by the properties of the Analytical Engine. Unlike the mechanism of a clock, in which the same processes would take place repeatedly so long as energy was available to drive them, the Analytical Engine could be set up in such as way that after any prespecified length of time its behaviour would change. Furthermore, any number of such changes could be specified. From the human point of view, these changes, coming perhaps after eons of regular behaviour, would appear miraculous, though they were in fact inevitable, being prefigured in the design of the machinery of the universe.

2.8 Conclusions

Babbage was ultimately unsuccessful in his attempt to build automatic computing engines. A small working prototype of the Difference Engine was completed, and portions of the full-sized machine were built, but work on the Analytical Engine did not proceed beyond the production of design drawings.

In the light of the subsequent history of computers, it is interesting to consider the reasons for this failure. Babbage wrote generally of technological innovation that "[i]t is partly due to *the imperfection of the original trials*, and partly to the gradual improvements in the art of making machinery, that many inventions which have been tried, and given up in one state of art, have at another period been eminently successful".[87] Most historians have followed Babbage's suggestion here, and concluded that Babbage's designs made too great a demand on the mechanical and tool-making expertise available at the time. However, as Babbage elsewhere made clear, the early nineteenth century was a time of great technical innovation, and the work carried out on the Difference Engine was itself responsible for significant advances.

An additional factor in Babbage's failure was certainly financial, in that the costs of the development even of the Difference Engine would in no way be met by the savings that its use would lead to. Babbage was aware of this, believing that the costs of the project should be underwritten by the Government rather than private capital, a position he supported by pointing out the national importance of the application of the Engine to fields such as navigation. The Government ultimately withdrew its support, however, being unconvinced that significant savings would follow from the use of the Engine. Some retrospective support for this position can be drawn from

[87] Babbage (1835b), Sect. 324.

the experience of the American Nautical Almanac Office in trying to make use of Scheutz's completed difference engine in 1859.[88]

Babbage as Pioneer Babbage is commonly taken to be a 'pioneer' of the computer, someone whose designs anticipated modern developments. In contrast, this chapter has attempted to describe Babbage's work as it was presented to and understood by his contemporaries.

It is a misleading anachronism to describe the Analytical Engine as a 'computer'. This characterization prevents us from seeing it in its own terms, as Babbage and his collaborators understood it. As argued above, Babbage thought of it primarily as a machine for the numerical evaluation of algebraic formulae, and much of its structure, organization and use can be best understood by thinking of it in these terms. If we think of it as a computer, it is impossible not to notice the ways in which it differs from current designs, and hence to describe it as a limited first attempt, or to wonder at the fact that Babbage did not include certain current design features. This prevents us from understanding it in its own terms, and from forming a just evaluation of Babbage's achievement.

Nevertheless, there are striking similarities between the design of the Analytical Engine and certain features of mid-twentieth century computer architecture. If we conclude, as seems likely, that the later developments were largely independent of the details of Babbage's work, then what is left with is a striking case of design convergence. Bromley points out that because of the inaccessibility of Babbage's design for the Analytical Engine, it is unlikely that direct influence could have been anything other than superficial, and concludes: "I am bothered that the Analytical Engine is *too much* like a modern computer. Do we infer that a computer can be built in fundamentally only one way?".[89]

This seems too strong. It is notable that Babbage and computer developers of the 1930s and 1940s were facing, in many ways, the same material situation, and were trying to develop computers that were essentially calculators. The material practice of computation had been remarkably constant in the intervening years: de Prony would have recognized and understood the industrial practices in a typical computing laboratory of the 1930s. The similarities between Babbage's and the later work can perhaps be understood as similar responses to the desire to automate the same material practice.

Babbage and Programming The Difference Engine was a physical embodiment of a single algorithmic process, and therefore did not need to be programmed. As suggested in Chap. 1, it can been seen to be more closely analogous to a single program than to a computer. Setting the machine up for a calculation would have been a matter of providing certain initial data, by setting the appropriate number wheels with the required differences.

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88 Grier (2005), p. 69.

89 Bromley (1982), p. 217.

The Analytical Engine, on the other hand, was to be capable of evaluating any analytical formula, and so some method was needed to control the operations that the machine would carry out on a particular occasion of use. Its design was based around the idea that the sequence of operations required for a computation was specified explicitly by a deck of operation cards. However, Babbage was also aware that patterns could frequently be found in the sequence of operations to be carried out that meant, for example, that the number of operation cards could be reduced. An example of such a capability was the repeating apparatus, which in conjunction with the combinatorial and index cards would allow the engine to return to an earlier point in the sequence and repeat certain operations.

Babbage occasionally gave descriptions of what he saw as the significant patterns of computation. For example, in the *Ninth Bridgewater Treatise*, he gave a general description of the capabilities of the machine, stating that

> it will calculate the numerical value of any algebraical function—that, at any period previously fixed upon, or contingent or certain events, it will cease to tabulate that algebraic function, and commence the calculation of a different one, and that these changes may be repeated to any extent.[90]

However, he does not appear to have considered in much detail the ways in which decks of cards could conveniently be prepared by the user to specify such structured computations. In part, this was no doubt due to the fact that he put much less effort into the preparation of computational plans that to the mechanical design of the engine. According to Allen Bromley, "[a]side from the Bernoulli numbers program prepared for Ada Lovelace's notes, there is no evidence that Babbage prepared any user programs for the Analytical Engine after his 1840 trip to Turin".[91]

One later development is relevant to this issue. Throughout the 1840s, Babbage continued work on the design of the engine, and at various times produced lists of the basic operations that it would provide. Originally, these had been limited to the basic arithmetical operations, but by 1844 they had been supplemented with operations for checking whether the number at a given location in the store was equal to zero.[92] The proposed implementation of this operation involved the use of index numbers on the operation and variable cards. If the specified number was found to be equal to zero, the decks of cards would be turned back by the number of cards specified by the indexes. A similar operation was specified for detecting whether a number was greater or less than zero. There appear to be no examples of the use of these operations in detailed computation plans, however.

This proposal bears an interesting relationship to the idea that the engine was translating the language of analysis, a key part of which was the correspondence between the operations of analysis and the operations carried out by the mill. The operation of ascertaining if a variable is equal to zero does not act on numbers; rather, its purpose is to act on the sequence of cards being presented to the machine.

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[90] Babbage (1837a), p. 187.

[91] Bromley (2000), p. 11.

[92] Bromley (2000), pp. 8–9.

It therefore represents a new type of operation, and its inclusion means that it is no longer possible to read the sequence of operation cards as a simple translation of an analytical formula.

There is an echo here of Lovelace's suggestions that the Σ in her notation of operation sequences represented a particular non-arithmetical operation of the mill, namely turning back the string of operation cards by a specified amount, and her view that the science of operations was a general theory which could be applied to many different areas. Neither Babbage nor Lovelace was able, however, to cleanly separate the operations required by this new science from the familiar operations of analysis.

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