2

I&I Stabilisation

This chapter contains the basic result for I&I stabilisation, namely a set of sufficient conditions for the construction of globally asymptotically stabilising, static, state feedback control laws for general, control affine, nonlinear systems. Note, however, that similar considerations can be given to non-affine systems and to tracking problems, while local versions follow *mutatis mutandis.*

To illustrate this result and some of the extensions cited above, we provide several academic and physically motivated examples. The former include singularly perturbed systems and systems in feedback and feedforward forms, while the latter consist of a mechanical system with flexibility modes, an electromechanical system with parasitic actuator dynamics, and an underactuated mechanical system.

2.1 Main Stabilisation Result

Theorem 2.1. *Consider the system*

$$\dot{x} = f(x) + g(x)u, \tag{2.1}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and an equilibrium point $x^* \in \mathbb{R}^n$ to be stabilised. Assume that there exist smooth mappings $\alpha : \mathbb{R}^p \to \mathbb{R}^p$, $\pi : \mathbb{R}^p \to \mathbb{R}^n$, $\phi : \mathbb{R}^n \to \mathbb{R}^{n-p}$, $c : \mathbb{R}^p \to \mathbb{R}^m$ and $\upsilon : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m$, with $p < n$, such that the following hold.

(A1) The target system

$$\dot{\xi} = \alpha(\xi), \tag{2.2}$$

with $\xi \in \mathbb{R}^p$ has a globally asymptotically stable equilibrium at $\xi^* \in \mathbb{R}^p$ and

$$x^* = \pi(\xi^*).$$

(A2) For all $\xi \in \mathbb{R}^p$,

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi}\alpha(\xi). \tag{2.3}$$

(A3) The set identity

$$\{x \in \mathbb{R}^n \,|\, \phi(x) = 0\} = \{x \in \mathbb{R}^n \,|\, x = \pi(\xi),\, \xi \in \mathbb{R}^p\} \tag{2.4}$$

holds.

(A4) All trajectories of the system

$$\dot{z} = \frac{\partial \phi}{\partial x}\left(f(x) + g(x)v(x,z)\right), \tag{2.5}$$

$$\dot{x} = f(x) + g(x)v(x,z), \tag{2.6}$$

are bounded and (2.5) has a uniformly globally asymptotically stable equilibrium at $z = 0$.

Then x^* *is a globally asymptotically stable equilibrium of the closed-loop system*

$$\dot{x} = f(x) + g(x)v(x, \phi(x)). \tag{2.7}$$

Proof. We establish the claim in two steps. First, it is shown that the equilibrium x^* is globally attractive, then that it is Lyapunov stable. Let $z(0) = \phi(x(0))$ and note that the right-hand side of (2.5) is $\dot{\phi}$, hence by (A4) any trajectory of the closed-loop system (2.7) is bounded and it is such that $\lim_{t\to\infty} z(t) = 0$, *i.e.,* it converges towards the manifold $\phi(x) = 0$, which is well defined by (A3). Moreover, by (A1) and (A2) the manifold is invariant and internally asymptotically stable, hence all trajectories of the closed-loop system converge to the equilibrium x^*.

Note now that, since (2.5) has a stable equilibrium at the origin, we have that, for any $\epsilon_1 > 0$, there exists $\delta_1 > 0$ such that $|\phi(x(0))| < \delta_1$ implies $|\phi(x(t))| < \epsilon_1$, for all $t \geq 0$. By (A1) and (A3) we also have that $\phi(x^*) = 0$. Consider now a projection of $x(t)$ on the manifold $\mathcal{M} = \{x \in \mathbb{R}^n \,|\, \phi(x) = 0\}$, denoted by $x_P(t)$, such that $|x(t) - x_P(t)| = \gamma(|\phi(x(t))|)$, for some class-$\mathcal{K}$ function $\gamma(\cdot)$, and such that $x_P(t) = \pi(\xi(t))$. By (A1) we have that, for any $\epsilon_2 > 0$, there exists $\delta_2 > 0$ such that $|x_P(0) - x^*| < \delta_2$ implies $|x_P(t) - x^*| < \epsilon_2$, for all $t \geq 0$. Selecting $\epsilon_1 = \epsilon_2 = \frac{1}{2}\epsilon$, and using the triangle inequality

$$|x(t) - x^*| \leq \gamma(|\phi(x(t))|) + |x_P(t) - x^*|,$$

it follows that, for any $\epsilon > 0$, there exists $\delta > 0$ (dependent on δ_1 and δ_2) such that $|x(0) - x^*| < \delta$ implies $|x(t) - x^*| < \epsilon$, for all $t \geq 0$, which proves the claim. □

The result summarised in Theorem 2.1 lends itself to the following interpretation. Given the system (2.1) and the target dynamical system (2.2), the goal is to find a manifold $\mathcal{M}$, described implicitly by $\{x \in \mathbb{R}^n \mid \phi(x) = 0\}$, and in parameterised form by $\{x \in \mathbb{R}^n \mid x = \pi(\xi), \xi \in \mathbb{R}^p\}$, which can be rendered invariant and asymptotically stable, and such that the (well-defined) restriction of the closed-loop system to $\mathcal{M}$ is described by $\dot{\xi} = \alpha(\xi)$.

Notice that the control u that renders the manifold invariant is not unique, since it is uniquely defined only on $\mathcal{M}$, *i.e.,* $\upsilon(\pi(\xi), 0) = c(\pi(\xi))$. From all possible controls we select one that drives to zero the *off-the-manifold* coordinates z and keeps the system trajectories bounded, *i.e.,* such that (A4) holds.

The following observations concerning the assumptions (A1)–(A4) of Theorem 2.1 are in order.

1. In most applications of Theorem 2.1 described in this chapter, the target system is *a priori* defined, hence condition (A1) is automatically satisfied.
2. If the control objective is to track a given trajectory, then Theorem 2.1 has to be rephrased in terms of tracking errors and the target system should *generate* the reference trajectory.
3. Given the target system (2.2), equation (2.3) of condition (A2) defines a partial differential equation (PDE) in the unknown $\pi(\cdot)$, where $c(\cdot)$ is a free parameter[1]. Finding the solution of this equation is (in general) a difficult task. Despite this fact, as shown by numerous examples in the book, a suitable selection of the target dynamics, *i.e.,* following physical and system theoretic considerations, allows to simplify this task. In some cases (see Example 2.6) it is possible to interlace the steps of definition of the target dynamics (2.2) and generation of the manifold (2.4) by viewing the PDE (2.3) as an algebraic equation relating $\alpha(\cdot)$ with $\pi(\cdot)$ (and its partial derivatives) and then selecting suitable expressions for $\pi(\cdot)$ that ensure the desired stability properties of the target dynamics.
4. Assumption (A3) states that the image of the mapping $\pi(\cdot)$ can be expressed as the zero level set of a (smooth) function $\phi(\cdot)$. Roughly speaking, this is a condition on the invertibility of the mapping that translates into a rank restriction on $\frac{\partial \pi}{\partial \xi}$. In the linear case, where $\pi(\xi) = T\xi$ with T some constant $(n \times p)$ matrix, we have $\phi(x) = T^{\perp}x$, where $T^{\perp}T = 0$, and (A3) holds if and only if T is full rank. In general, if $\pi : \mathbb{R}^p \to \mathbb{R}^n$ is an injective and proper[2] immersion then the image of $\pi(\cdot)$ is a submanifold of $\mathbb{R}^n$. (A3) thus requires that such a submanifold can be described (globally) as the zero level set of the function $\phi(\cdot)$. Note, finally, that if there exists a

[1]Note that, if the linearisation of (2.1) at $x = x^*$ is controllable and all functions are locally analytic, it has been shown in [109], using Lyapunov's auxiliary theorem and under some *non-resonance* conditions, that there always exists $c(\cdot)$ such that the solution exists locally.

[2]An immersion is a mapping $\pi(\cdot) : \mathbb{R}^p \to \mathbb{R}^n$, with $p < n$. It is injective if $\mathrm{rank}(\pi) = p$, and it is proper if the inverse image of any compact set is also compact [1, Chapter 3].

partition of $x = \mathrm{col}(x_1, x_2)$, with $x_1 \in \mathbb{R}^p$ and $x_2 \in \mathbb{R}^{n-p}$, and a corresponding partition of $\pi(\xi) = \mathrm{col}(\pi_1(\xi), \pi_2(\xi))$ such that $\pi_1(\xi)$ is a global diffeomorphism, then the function $\phi(x) = x_2 - \pi_2(\pi_1^{-1}(x_1))$ is such that (A3) holds.

5. In many cases of practical interest, to have asymptotic convergence of $x(t)$ to x^* it is sufficient to require that the system (2.5) has a uniformly globally stable equilibrium at $z = 0$ and

$$\lim_{t\to\infty} g(x(t))\,(v(x(t), z(t)) - v(x(t), 0)) = 0, \tag{2.8}$$

   i.e., it is not necessarily required that the manifold is reached. This fact, which distinguishes the present approach from others, such as sliding mode, is instrumental to the development of the adaptive and output feedback control theory in Chapters 3, 4 and 6.
6. In Theorem 2.1 a stabilising control law is derived starting from the selection of a target dynamical system. A different perspective can be taken: given the mapping $x = \pi(\xi)$, hence the mapping $z = \phi(x)$, find (if possible) a control law which renders the manifold $z = 0$ invariant and asymptotically stable, and a vector field $\dot{\xi} = \alpha(\xi)$, with a globally asymptotically stable equilibrium ξ^*, such that equation (2.3) holds. If this goal is achieved then the system (2.1) with output $z = \phi(x)$ is (globally) minimum-phase and its zero dynamics—*i.e.,* the dynamics on the output zeroing manifold $\mathcal{M}$—are given by (2.2). In this respect, the result in Theorem 2.1 can be regarded as a *dual* of the classical stabilisation methods based on the construction of passive or minimum-phase outputs[3].

Note 2.1. The partial differential equation (2.3) also arises in nonlinear regulator theory[4]. However, equation (2.3) and its solution are used in the present context in a new form. First of all, in classical regulator theory, the system $\dot{\xi} = \alpha(\xi)$ is assumed Poisson stable [34], whereas in the I&I framework it is required to have an asymptotically stable equilibrium. Second, while in regulator theory the mapping $\pi(\xi)$ is needed to define a controlled invariant manifold for the system composed of the plant and the exosystem, in the present approach the mapping $\pi(\xi)$ is used to define a parameterised controlled invariant manifold which is a submanifold of the state-space of the system to be stabilised. Finally, in regulator theory the exosystem $\dot{\xi} = \alpha(\xi)$ is driving the plant to be controlled, whereas in the I&I approach the closed-loop system contains a *copy* of the dynamics $\dot{\xi} = \alpha(\xi)$. Finally, equation (2.3) arises also in the feedback linearisation problem studied in [109]. Therein, the goal is to obtain a closed-loop system which is locally equivalent to a target linear system. Unlike the present context, the target system in [109] has the same dimension as the system to be controlled, *i.e.,* the mapping $\pi(\cdot)$ is a (local) diffeomorphism rather than an immersion. $\triangleleft$

[3]See [36, 163] and the survey paper [22].

[4]To be precise, two equations arise in nonlinear regulator theory. The first one is equation (2.3), the second is an equation expressing the fact that the tracking error is zero on the invariant manifold defined via the solution of equation (2.3), see [34].

The following definition is used in the rest of the book to provide concise statements.

Definition 2.1. *A system described by equations of the form (2.1) with target dynamics (2.2) is said to be (locally) I&I stabilisable if the assumptions (A1)–(A4) of Theorem 2.1 hold (locally).*

2.2 Systems with Special Structures

In this section we show that some of the systems with special structures considered in the literature are I&I stabilisable. In particular we consider singularly perturbed systems and systems in feedback and feedforward forms.

Example 2.1 (Singularly perturbed systems). We present an academic example, with the twofold objective of putting in perspective the I&I formulation and giving the flavour of the required computations. Consider the two-dimensional system[5]

$$\begin{aligned} \dot{x}_1 &= x_1 x_2^3, \\ \epsilon \dot{x}_2 &= x_2 + u, \end{aligned} \tag{2.9}$$

with $x_1(0) \geq 0$[6], and the target dynamics

$$\dot{\xi} = -\xi^5, \tag{2.10}$$

where $\xi \in \mathbb{R}$. Fixing $\pi_1(\xi) = \xi$, equations (2.3) become

$$\begin{aligned} \xi \pi_2(\xi)^3 &= -\xi^5, \\ \pi_2(\xi) + c(\xi) &= -\epsilon \frac{\partial \pi_2}{\partial \xi} \xi^5. \end{aligned} \tag{2.11}$$

From the first equation we obtain $\pi_2(\xi) = -\xi^{4/3}$, which is defined for $\xi \geq 0$, while the mapping $c(\cdot)$ is defined by the second equation. The manifold $x = \pi(\xi)$ can be implicitly described by $\phi(x) = x_2 + x_1^{4/3} = 0$ and the off-the-manifold dynamics (2.5) are given by

$$\epsilon \dot{z} = v(x, z) + x_2 + \frac{4}{3} \epsilon x_1^{4/3} x_2^3.$$

The I&I design is completed by choosing $v(x, z) = -x_2 - \frac{4}{3} \epsilon x_1^{4/3} x_2^3 - z$, which yields the closed-loop dynamics

$$\begin{aligned} \epsilon \dot{z} &= -z, \\ \dot{x}_1 &= x_1 x_2^3, \\ \epsilon \dot{x}_2 &= -\tfrac{4}{3} \epsilon x_1^{4/3} x_2^3 - z. \end{aligned} \tag{2.12}$$

[5]This example has been adopted from [114], where the composite control approach has been used, see Note 2.2.

[6]Similarly to [114], we consider the system on the invariant set $x_1 \geq 0$.

From the first equation in (2.12) it is clear that the z-subsystem has a globally asymptotically stable equilibrium at zero, hence, to complete the proof, it only remains to show that all trajectories of the system (2.12) are bounded. To this end, consider the (partial) change of co-ordinates $\eta = x_2 + x_1^{4/3}$ yielding

$$\begin{aligned} \epsilon\dot{z} &= -z, \\ \dot{x}_1 &= x_1 \left(\eta - x_1^{4/3}\right)^3, \\ \epsilon\dot{\eta} &= -z, \end{aligned}$$

and note that $z(t)$ and $\eta(t)$ are bounded for all t. Finally, boundedness of $x_1(t)$ can be proved observing that the dynamics of x_1 can be expressed in the form

$$\dot{x}_1 = -x_1^5 + \rho(x_1, \eta),$$

for some function $\rho(\cdot)$ satisfying $|\rho(x_1, \eta)| \leq k(\eta)|x_1|^4$, for some $k(\eta) > 0$ and for $|x_1| > 1$. The control law is obtained as[7]

$$u = v(x, \phi(x)) = -2x_2 - x_1^{4/3}\left(1 + \frac{4}{3}\epsilon x_2^3\right).$$

As a result, the system (2.9) is I&I stabilisable with target dynamics (2.10) in its domain of definition.

A *global* stabilisation result can be obtained if instead of (2.10) we choose the target dynamics $\dot{\xi} = -|\xi|^3\xi$. Then the solution of (2.11) is $\pi_1(\xi) = \xi$ and $\pi_2(\xi) = -|\xi|$. As a result, the manifold takes the form $\phi(x) = x_2 + |x_1| = 0$ and it can be rendered globally attractive, while keeping all trajectories bounded, by the control law $u = -2x_2 - \epsilon|x_1|x_2^3 - |x_1|$. ■

Note 2.2. The composite control approach of [114] is applicable for singularly perturbed systems of the form

$$\begin{aligned} \dot{x}_1 &= f(x_1, x_2, u), \\ \epsilon\dot{x}_2 &= g(x_1, x_2, u), \end{aligned}$$

for which a slow manifold, defined by the function $x_2 = h(x_1, u, \epsilon)$, exists and results from the solution of the PDE

$$\epsilon\left(\frac{\partial h}{\partial x_1} + \frac{\partial h}{\partial u}\frac{\partial u}{\partial x_1}\right) f(x_1, h(x_1, u, \epsilon), u) = g(x_1, h(x_1, u, \epsilon), u).$$

In [114] it is proposed to expand $h(x_1, u, \epsilon)$ and u in a power series of ϵ, namely $h(\cdot) = \bar{h} + \mathcal{O}(\epsilon^k)$, where $\bar{h} = h_0 + \epsilon h_1 + \cdots + \epsilon^k h_k$, and $u = \bar{u} + \mathcal{O}(\epsilon^k)$, where $\bar{u} = u_0 + \epsilon u_1 + \cdots + \epsilon^k u_k$, taking as u_0 the control law that stabilises some equilibrium of the slow subsystem. Collecting the terms with the same powers of ϵ yields equations relating the terms h_i and u_i, which can be iteratively solved to approximate (up to any order $\mathcal{O}(\epsilon^k)$) the solution of the PDE. The control law is then constructed as $u = \bar{u} - K\left(x_2 - \bar{h}\right)$, where the last term, with $K > 0$, is a fast control that *steers* the trajectories onto the slow manifold. ◁

[7] For comparison we note that the composite controller obtained in [114] is $u = -2x_2 - x_1^{4/3}(1 + \frac{4}{3}\epsilon x_1^4)$.

We conclude this section by showing that for the systems in feedback and feedforward forms presented in Examples 1.4 and 1.5, respectively, the I&I method can be used to construct (globally) stabilising control laws that are different from those obtained by the standard (backstepping and forwarding) approaches.

Example 2.2 (Systems in feedback form). Consider the class of systems in feedback form introduced in Example 1.4. The following statement summarises the application of the I&I approach to this special case.

Proposition 2.1. *Consider a system described by equations of the form (1.14) with* $f(0,0) = 0$ *and suppose that the system* $\dot{x}_1 = f(x_1, 0)$ *has a globally asymptotically stable equilibrium at zero. Then the system (1.14) is (globally) I&I stabilisable with target dynamics (1.15).*

Proof. To establish the claim we need to prove that conditions (A1)–(A4) of Theorem 2.1 hold. To begin with, note that (A1) is trivially satisfied, whereas the mappings

$$\begin{bmatrix} \pi_1(\xi) \\ \pi_2(\xi) \end{bmatrix} = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \qquad c(\pi) = 0, \qquad \phi(x_1, x_2) = x_2$$

are such that conditions (A2) and (A3) hold.

Note now that the off-the-manifold variable $z = x_2$ can be used as a partial co-ordinate, hence instead of verifying (A4) we simply need to show that it is possible to select u such that the trajectories of the closed-loop system are bounded and $z = x_2$ converges to zero.

To this end, let $u = -K(x_1, x_2)x_2$, with $K(x_1, x_2) \geq k > 0$ for any (x_1, x_2) and for some k, and consider the system

$$\begin{aligned} \dot{x}_1 &= f(x_1, x_2), \\ \dot{x}_2 &= -K(x_1, x_2)x_2. \end{aligned}$$

Note that x_2 converges to zero. To prove boundedness of x_1 pick any $M > 0$ and let $V(x_1)$ be a positive-definite and proper function such that

$$\frac{\partial V}{\partial x_1} f(x_1, 0) < 0, \tag{2.13}$$

for all $|x_1| > M$. Note that such a function $V(x_1)$ exists, by global asymptotic stability of the zero equilibrium of the system $\dot{x}_1 = f(x_1, 0)$, but $V(x_1)$ is not necessarily a Lyapunov function for $\dot{x}_1 = f(x_1, 0)$. Consider now the positive-definite and proper function $W(x_1, x_2) = V(x_1) + \frac{1}{2}x_2^2$ and note that, for some function $F(x_1, x_2)$ and for any smooth function $\gamma(x_1) > 0$, one has

$$\begin{aligned} \dot{W} &= \frac{\partial V}{\partial x_1} f(x_1, 0) + \frac{\partial V}{\partial x_1} F(x_1, x_2)x_2 - K(x_1, x_2)x_2^2 \\ &\leq \frac{\partial V}{\partial x_1} f(x_1, 0) + \frac{1}{\gamma(x_1)} \left| \frac{\partial V}{\partial x_1} \right|^2 + \gamma(x_1)|F(x_1, x_2)|^2 x_2^2 - K(x_1, x_2)x_2^2. \end{aligned}$$

As a result, setting $\gamma(x_1)$ such that

$$\frac{\partial V}{\partial x_1} f(x_1, 0) + \frac{1}{\gamma(x_1)} \left| \frac{\partial V}{\partial x_1} \right|^2 < 0,$$

for all $|x_1| > M$, and selecting

$$K(x_1, x_2) > \gamma(x_1) |F(x_1, x_2)|^2$$

yields the claim. □

It is worth noting that although system (1.14) is stabilisable using standard backstepping arguments[8], the control law obtained using backstepping is different from the control law suggested by Proposition 2.1. The former requires the knowledge of a Lyapunov function for the system $\dot{x}_1 = f(x_1, 0)$ and it is such that, in closed loop, the manifold $x_2 = 0$ is not invariant, whereas the latter requires only the knowledge of the function $V(x_1)$ satisfying equation (2.13) for sufficiently large $|x_1|$ and renders the manifold $x_2 = 0$ invariant and globally attractive.

To illustrate the result in Proposition 2.1 and compare it with the control law resulting from standard backstepping, consider the two-dimensional system

$$\begin{aligned} \dot{x}_1 &= -x_1 + \lambda x_1^3 x_2, \\ \dot{x}_2 &= u. \end{aligned}$$

A backstepping-based stabilising control law is

$$u = -\lambda x_1^4 - x_2,$$

whereas a direct application of the procedure described in the proof of Proposition 2.1 shows that the I&I stabilising control law is

$$u = -\left(2 + x_1^8\right) x_2.$$

The latter does not require the knowledge of the parameter λ, however, it is in general more *aggressive* because of the higher power in x_1.

As a second example consider the system

$$\begin{aligned} \dot{x}_1 &= A(\theta) x_1 + F(x_1, x_2) x_2, \\ \dot{x}_2 &= u, \end{aligned}$$

with $x_1 \in \mathbb{R}^n$ and $\theta \in \mathbb{R}^s$ an unknown constant vector. Assume that the matrix $A(\theta)$ is Hurwitz for all θ. Then the system is I&I stabilisable with target dynamics $\dot{\xi} = A(\theta)\xi$. A simple computation shows that an I&I stabilising control law is

$$u = -\left(1 + x_1^2\right) |F(x_1, x_2)|^2 x_2 - x_2,$$

and this does not require the knowledge of a Lyapunov function for the system $\dot{x}_1 = A(\theta) x_1$, nor the knowledge of the parameter θ. ■

[8] See [123].

Example 2.3 (Systems in feedforward form). We now show that, using the approach pursued in this chapter, a new class of control laws can be constructed for the special class of systems in feedforward form given in Example 1.5.

Proposition 2.2. *Consider a system described by equations of the form (1.16) and suppose that the zero equilibrium of the system* $\dot{x}_2 = f(x_2)$ *is globally asymptotically stable. Assume, moreover, that there exists a smooth function* $M(x_2)$ *such that, for all* x_2, $L_f M(x_2) = h(x_2)$, *the set* $\mathcal{S} = \{x_2 \in \mathbb{R}^n \mid L_g M(x_2) = 0\}$ *is composed of isolated points, and* $0 \notin \mathcal{S}$. *Then the system (1.16) is globally I&I stabilisable with target dynamics (1.17).*

Proof. To begin with note that (A1) in Theorem 2.1 is trivially satisfied and that the mappings

$$\begin{bmatrix} \pi_1(\xi) \\ \pi_2(\xi) \end{bmatrix} = \begin{bmatrix} M(\xi) \\ \xi \end{bmatrix}, \qquad c(\xi) = 0$$

are such that condition (A2) holds. The implicit description of the manifold in (A3) is $z = \phi(x) = x_1 - M(x_2)$ and the off-the-manifold dynamics are

$$\dot{z} = -L_g M(x_2) u.$$

To complete the proof it remains to verify condition (A4), or alternatively the weaker condition (2.8). To this end, let

$$u = \epsilon \frac{1}{1 + |g(x_2)|} \frac{L_g M(x_2)}{1 + |L_g M(x_2)|} \frac{z}{1 + |z|},$$

with $\epsilon = \epsilon(x_2) > 0$, and consider the closed-loop system

$$\begin{aligned} \dot{z} &= -\epsilon \frac{1}{1 + |g(x_2)|} \frac{(L_g M(x_2))^2}{1 + |L_g M(x_2)|} \frac{z}{1 + |z|}, \\ \dot{x}_1 &= h(x_2), \\ \dot{x}_2 &= f(x_2) + \epsilon \frac{g(x_2)}{1 + |g(x_2)|} \frac{L_g M(x_2)}{1 + |L_g M(x_2)|} \frac{z}{1 + |z|}. \end{aligned}$$

Note now that, from the first equation, z is bounded, hence x_2 is bounded, provided that ϵ is sufficiently small. It follows that $L_g M(x_2) z$, and hence x_2, converge to zero. Note now that, if ϵ is sufficiently small, z converges exponentially to zero. As a result, $\eta = x_1 - M(x_2)$ is bounded, hence x_1 is also bounded for all t, which proves the claim. □

To illustrate Proposition 2.2 consider the system

$$\begin{aligned} \dot{x}_1 &= x_{21}^3, \\ \dot{x}_{21} &= x_{22}^3, \\ \dot{x}_{22} &= v, \end{aligned} \tag{2.14}$$

and let $v = -x_{21}^3 - x_{22}^3 + u$. The system (2.14) satisfies all the assumptions of Proposition 2.2, with $M(x_2) = M(x_{21}, x_{22}) = -x_{21} - x_{22}$. As a result its zero equilibrium is stabilised by the control law[9] $u = -z$ with $z = x_1 + x_{21} + x_{22}$.

Consider now the system

$$\begin{aligned}\dot{x}_1 &= x_2^3,\\ \dot{x}_2 &= -x_2^3 + (1 - x_2^2)u.\end{aligned}$$

A simple computation shows that $M(x_2) = -x_2$ satisfies the assumptions of Proposition 2.2 and that $\mathcal{S} = \{x_2 = -1,\, x_2 = 1\}$. Hence, a straightforward application of the procedure outlined in the proof of Proposition 2.2 shows that the control law

$$u = -\frac{1 - x_2^2}{1 + x_2^4}(x_1 + x_2),$$

renders the zero equilibrium of the closed-loop system globally asymptotically stable. ■

Note 2.3. The system (2.14) has been studied in several papers. In [35] a globally stabilising control law has been designed through the construction of a minimum-phase relative degree one output map, whereas in [168] a globally stabilising control law has been obtained using a control Lyapunov function. The control law in [168] is $u = -x_{21}^3 - x_{22}^3 - x_1 - x_{22}$ and it is similar to the one proposed above. Note that the former requires the knowledge of a (control) Lyapunov function, whereas the latter does not. Finally, the zero equilibrium of the system (2.14) can be stabilised with the modified version of forwarding proposed in [142], which is again a Lyapunov-based design methodology, and with homogeneous feedback laws. ◁

2.3 Physical Systems

In this section the I&I stabilisation method is applied to the three physical examples introduced in Section 1.3, namely the magnetically levitated ball with actuator dynamics, the robot manipulator with joint flexibilities, and the cart and pendulum system. The first example is presented to highlight the connections between I&I and other existing techniques, in particular we show how the I&I framework allows to recover the composite control and backstepping solutions, while for the robot problem we prove that with I&I we can generate a novel family of global tracking controllers under standard assumptions. In both examples we start from the assumption that we know a stabilising controller for a nominal reduced-order model, and we robustify it with respect to some higher-order dynamics.

Finally, the objective of the cart and pendulum example is to suggest a procedure to overcome the need to solve the PDE of the immersion condition—*i.e.,* the computation of the function $\pi(\cdot)$ that defines the manifold $\mathcal{M}$. This

[9] Due to the special form of the system (2.14), the control law applied here is simpler than the one given in the proof of Proposition 2.2.

computation is unquestionably the main difficulty for the application of the I&I methodology. Towards this end, we propose to transform the PDE into an algebraic equation where the target dynamics are viewed as a function of $\pi(\cdot)$ (and its partial derivatives), and then select $\pi(\cdot)$ to ensure that the target dynamics have the desired stability properties.

Example 2.4 (Magnetic levitation). Consider the magnetic levitation system (1.8) of Example 1.1 and the problem of (locally) stabilising the ball around a given position. A solution to this problem is expressed by the following proposition.

Proposition 2.3. *The full-order model of the levitated ball system (1.8) is I&I stabilisable with target dynamics (1.7), where $w = w(\xi)$ is any stabilising state feedback such that (1.7) in closed loop with $w(\xi(t)) + c(t)$, where $c(t)$ is a bounded signal, has bounded trajectories.*

Proof. As (A1) is automatically satisfied, we only verify the remaining conditions (A2)–(A4) of Theorem 2.1. First, simple calculations show that a solution of equation (2.3) is given by the mapping $\pi(\xi) = \mathrm{col}(\xi_1, \xi_2, \xi_3, w(\xi))$. This solution can be easily obtained by fixing $\pi_1(\xi) = \xi_1$ and $\pi_3(\xi) = \xi_3$, a choice which captures the control objective. The parameterised manifold $x = \pi(\xi)$ can be implicitly defined as $\phi(x) = x_4 - w(x_1, x_2, x_3) = 0$, hence condition (A3) is also satisfied. Finally, we have to choose a function $v(x, z)$ that preserves boundedness of trajectories and asymptotically stabilises the zero equilibrium of the off-the-manifold dynamics (2.5), which in this example are given by

$$\dot{z} = \frac{1}{Ck}\left(1 - x_1\right) x_3 - \frac{1}{R_1 C}\left(x_4 - v(x, z)\right) - \dot{w}, \tag{2.15}$$

where $\dot{w}$ is evaluated on the manifold, hence it is computable from the full-order dynamics (1.8). An obvious simple selection is

$$v(x, z) = x_4 - z + R_1 C \dot{w} + \frac{R_1}{k}\left(1 - x_1\right) x_3,$$

which yields the system

$$\begin{aligned}
\dot{z} &= -\frac{1}{R_1 C} z, \\
\dot{x}_1 &= \frac{1}{m} x_2, \\
\dot{x}_2 &= \frac{1}{2k} x_3^2 - mg, \\
\dot{x}_3 &= -\frac{R_2}{k}(1 - x_1) x_3 + x_4, \\
\dot{x}_4 &= -\frac{1}{R_1 C} z + \dot{w}.
\end{aligned} \tag{2.16}$$

To complete the proof of the proposition it only remains to prove that, for the given $w(\cdot)$, the trajectories of (2.16) are bounded. To this end, note that in the co-ordinates (z, x_1, x_2, x_3, η), with $\eta = x_4 - w(x_1, x_2, x_3)$, the system is described by the equations

$$\begin{aligned}
\dot{z} &= -\frac{1}{R_1 C} z, \\
\dot{x}_1 &= \frac{1}{m} x_2, \\
\dot{x}_2 &= \frac{1}{2k} x_3^2 - mg, \\
\dot{x}_3 &= -\frac{R_2}{k}(1 - x_1)x_3 + w(x_1, x_2, x_3) + \eta, \\
\dot{\eta} &= -\frac{1}{R_1 C} z,
\end{aligned}$$

from which we conclude that z converges to zero exponentially, hence η is bounded. The proof is completed by invoking the assumed robustness property with respect to bounded input disturbances. The control law is finally obtained as

$$u = w(x_1, x_2, x_3) + R_1 C \dot{w} + \frac{R_1}{k}(1 - x_1)x_3.$$

□

We use this example to compare the I&I formulation with the composite control approach. Taking x_4 as the slow variable and $R_1 C$ as the small parameter the problem reduces to finding a function $h(\cdot)$ and a control u such that $x_4 = h(x_1, x_2, x_3, u)$ describes an invariant manifold. This requires the solution of a PDE of the form

$$\begin{aligned}
-\frac{1}{Ck}(1 - x_1)x_3 + \frac{1}{R_1 C}(u - h) = \frac{\partial h}{\partial x_3}\left(-\frac{R_2}{k}(1 - x_1)x_3 + h\right) \\
+ \frac{1}{m}\frac{\partial h}{\partial x_1} x_2 + \frac{\partial h}{\partial x_2}\left(\frac{1}{2k} x_3^2 - mg\right) + \frac{\partial h}{\partial u}\dot{u}.
\end{aligned}$$

In this simple case an exact solution is possible setting $h = w$ and

$$u_1 = \dot{w} + \frac{1}{Ck}(1 - x_1)x_3.$$

This choice ensures $h_k = 0$ for all $k \geq 1$. The resulting invariant manifold $(x_4 = w)$ and the controller are the same obtained via I&I stabilisation. However, while in the composite control approach the function u_1 that determines the controller is essentially imposed, in I&I we have some freedom in the choice of the function $v(x, z)$ to stabilise (2.15). Finally, notice that this controller also results from direct application of backstepping to system (1.8). ■

Example 2.5 (Flexible joints robot). Consider the problem of global tracking for the n-DOF flexible joints robot model introduced in Example 1.2. We present a procedure to robustify an arbitrary global state feedback *tracking* controller designed for the rigid robot.

Proposition 2.4. *The flexible joints robot model (1.10) is globally I&I stabilisable with target dynamics (1.11), where $w = w(\xi, t)$ is any time-varying state feedback that ensures that the solutions $\xi_1(t)$ of (1.11) globally track any bounded, four times differentiable trajectory $\xi_1^*(t)$, and with the additional property that in closed loop with $w(\xi, t) + c(t)$, where $c(t)$ is a bounded signal, trajectories remain bounded*[10]*.*

Proof. As discussed in Section 2.1, since the control objective is to track a reference trajectory, (A1) is replaced by a condition on the trajectory of the target system (1.11). Hence, to establish the claim we verify that the conditions (A2)–(A4) of Theorem 2.1 are satisfied. First, it is easy to see that a solution of equations (2.3) is given by the mapping

$$\pi(\xi, t) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_1 + K^{-1} w(\xi, t) \\ \pi_4(\xi, t) \end{bmatrix},$$

where $\pi_4(\xi, t) = \frac{\partial \pi_3}{\partial \xi} \dot{\xi}$, with $\dot{\xi}$ as defined in (1.11). This solution follows immediately considering (1.10) and fixing $\pi_1(\xi) = \xi_1$, as required by the control objective. An implicit definition of the manifold[11] $\phi(x, t) = 0$ is obtained selecting

$$\phi(x, t) = \begin{bmatrix} x_3 - x_1 - K^{-1} w(x, t) \\ x_4 - x_2 - K^{-1} \left(\frac{\partial w}{\partial x_1} x_2 - \frac{\partial w}{\partial x_2} D^{-1}(x_1) \left(C(x_1, x_2) x_2 \right. \right. \\ \left. \left. + g(x_1) + K(x_1 - x_3) \right) - \frac{\partial w}{\partial t} \right) \end{bmatrix}.$$

It is important to underscore that, while $\phi_1(x, t)$ is obtained from the obvious choice

$$\phi_1(x, t) = x_3 - \pi_3(\xi, t)|_{\xi_1 = x_1, \, \xi_2 = x_2},$$

the term $\phi_2(x, t)$ is not defined likewise. However, the set identity (2.4) is satisfied for the definition above as well. To verify this observe that

[10] A possible selection is the Slotine and Li controller [200, 163], which is given by

$$w(\xi, t) = D(\xi_1)(\ddot{\xi}_1^* - \Lambda \dot{\tilde{\xi}}_1) + C(\xi_1, \xi_2)(\dot{\xi}_1^* - \Lambda \tilde{\xi}_1) - K_p(\dot{\tilde{\xi}} + \Lambda \tilde{\xi}_1) + g(\xi_1),$$

where $K_p = K_p^\top > 0$, $\Lambda = \Lambda^\top > 0$ and $\tilde{\xi}_1 = \xi_1 - \xi_1^*$.

[11] Note that in this case the target dynamics and the equations defining the invariant manifold depend explicitly on t.

$$x_4 - \pi_4(\xi,t)|_{\xi_1=x_1,\,\xi_2=x_2} = \phi_2(x,t) \\ +K^{-1}\frac{\partial w}{\partial x_2}D^{-1}(x_1)K\big(x_3 - x_1 - K^{-1}w(x,t)\big),$$

but the last right-hand term in brackets is precisely $\phi_1(x,t)$. The interest in defining $\phi_2(x,t)$ in this way is that $\dot{\phi}_1 = \phi_2$, simplifying the task of stabilising the zero equilibrium of the off-the-manifold dynamics, which are given by

$$\dot{z}_1 = z_2, \\ \dot{z}_2 = m(x,t) + J^{-1}v(x,z,t),$$

where $m(x,t)$ can be computed via differentiation of $\phi_2(x,t)$. It is then a trivial task to select a control law $v(x,z,t)$ that asymptotically stabilises the zero equilibrium of the z system, an obvious simple selection being

$$v(x,z,t) = -J\left(m(x,t) + K_1 z_1 + K_2 z_2\right),$$

with K_1, K_2 arbitrary positive-definite matrices. To complete the proof it is necessary to show that all trajectories of the closed-loop system with state (x, z_1, z_2) are bounded. To this end, it suffices to rewrite the system in the co-ordinates $(x_1, x_2, \phi_1, \phi_2, z_1, z_2)$ and use arguments similar to those in the proof of Proposition 2.3. □

It must be stressed that the target dynamics (1.11) are *not* the dynamics of the rigid model obtained from a singular perturbation reduction of the full model (1.10) with small parameters $1/k_i$. In the latter model the inertia matrix is $D + J$ and not simply D as in the present case[12]. The motivation to choose these target dynamics is clear noting that, if we take the rigid model resulting from a singular perturbation reduction, the solution of equations (2.3) leads to $\pi_3(\xi) = \xi_1 + K^{-1}(w - \dot{\xi}_2)$, complicating the subsequent analysis. ■

Note 2.4. In [207] the composite control approach is used to derive approximate, feedback linearising, asymptotically stabilising controllers for the full-inertia model. In the case of the block-diagonal inertia matrix considered here the slow manifold equations can be solved exactly and the stabilisation is global. ◁

Example 2.6 (Cart and pendulum system). Consider the problem of upward stabilisation of the underactuated cart and pendulum system (1.12) with target dynamics the fully actuated pendulum (1.13). The following statement describes a procedure to generate I&I stabilising feedback laws without the need to solve the PDE (2.3).

Proposition 2.5. *Let* $\pi_3(\cdot) : \mathcal{S}^1 \times \mathbb{R} \to \mathbb{R}$ *be such that* $\frac{\partial \pi_3}{\partial \xi_2}$ *is a function of* ξ_1 *only and let*

$$V'(\xi_1) = -\frac{\sin(\xi_1)}{\Delta(\xi_1)}, \qquad R(\xi_1,\xi_2) = \frac{\cos(\xi_1)}{\Delta(\xi_1)}\frac{\partial \pi_3}{\partial \xi_1}, \tag{2.17}$$

[12] See the discussion in [207].

with

$$\Delta(\xi_1) = 1 + \cos(\xi_1)\frac{\partial \pi_3}{\partial \xi_2}. \tag{2.18}$$

Assume that the functions $\Delta(\xi_1)$ and $R(\xi_1, \xi_2)$ are such that $\Delta(0) < 0$ and $R(0,0) > 0$. Then the cart–pendulum system (1.12) in closed loop with the I&I controller

$$v(x, \phi(x)) = \frac{1}{\Delta(x_1)}\left(-\gamma\phi(x) + \frac{\partial \pi_3}{\partial x_1}x_2 + \frac{\partial \pi_3}{\partial x_2}\sin(x_1)\right), \tag{2.19}$$

with $\gamma > 0$ and $\phi(x) = x_3 - \pi_3(x_1, x_2)$, has a locally asymptotically stable equilibrium at zero.

Proof. We proceed to verify the hypotheses (A1)–(A4) of Theorem 2.1. By assumption the target system (1.13), namely

$$\begin{aligned}\dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2,\end{aligned}$$

has a locally asymptotically stable equilibrium at the origin, therefore (A1) is satisfied. Given the control objectives and our choice of target dynamics, a natural selection of the mapping $\pi(\cdot)$ is

$$\pi(\xi) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi_1, \xi_2) \end{bmatrix}, \tag{2.20}$$

where $\pi_3(\cdot)$ is a function to be defined. With this choice of $\pi(\cdot)$ and of the target dynamics the PDE (2.3) reduces to

$$\sin(\xi_1) - \cos(\xi_1)c(\pi(\xi)) = -V'(\xi_1) - R(\xi_1, \xi_2)\xi_2, \tag{2.21}$$

$$c(\pi(\xi)) = \frac{\partial \pi_3}{\partial \xi_1}\xi_2 - \frac{\partial \pi_3}{\partial \xi_2}\left(V'(\xi_1) + R(\xi_1, \xi_2)\xi_2\right). \tag{2.22}$$

Replacing $c(\cdot)$ from (2.22) in (2.21) yields the PDE

$$\left(\cos(\xi_1)\frac{\partial \pi_3}{\partial \xi_1} - R(\xi_1, \xi_2)\Delta(\xi_1)\right)\xi_2 = \sin(\xi_1) + \Delta(\xi_1)V'(\xi_1), \tag{2.23}$$

where $\Delta(\cdot)$ is defined in (2.18). Clearly, (2.23) holds by (2.17), hence (A2) holds.

The implicit manifold condition (A3) is verified noting that the manifold $\mathcal{M}$ can be implicitly described by $\mathcal{M} = \{x \in \mathbb{R}^3 \mid \phi(x) = 0\}$, with $\phi(x) = x_3 - \pi_3(x_1, x_2)$. Finally, we prove condition (A4). The off-the-manifold dynamics are

$$\dot{z} = -\frac{\partial \pi_3}{\partial x_1}x_2 - \frac{\partial \pi_3}{\partial x_2}\sin(x_1) + \Delta(x_1)v(x, z).$$

Selecting

$$v(x,z) = \frac{1}{\Delta(x_1)}\left(-\gamma z + \frac{\partial \pi_3}{\partial x_1}x_2 + \frac{\partial \pi_3}{\partial x_2}\sin(x_1)\right),$$

which is well-defined locally around zero by assumption, yields $\dot{z} = -\gamma z$. Finally, replacing $z = \phi(x)$ in the foregoing equation yields the controller (2.19). To complete the proof we show that, for a set of initial conditions containing a neighbourhood of the origin, the trajectories of the system (2.5), (2.6), namely

$$\begin{aligned} \dot{z} &= -\gamma z, \\ \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \sin(x_1) - \cos(x_1)v(x,z), \\ \dot{x}_3 &= v(x,z), \end{aligned} \tag{2.24}$$

with $v(\cdot)$ given by (2.19), are bounded. Towards this end, note that from (2.18) and (2.17) there exist $\epsilon_1 > 0$ and $\epsilon_2 > 0$ such that, locally around zero,

$$\Delta(x_1) \le -\epsilon_1, \qquad R(x_1,x_2) \ge \epsilon_2. \tag{2.25}$$

Note now that, using the functions $V'(\cdot)$ and $R(\cdot)$ defined in (2.17), the first three equations of (2.24) can be rewritten in the form

$$\begin{aligned} \dot{z} &= -\gamma z, \\ \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -V'(x_1) - R(x_1,x_2)x_2 + \frac{\gamma\cos(x_1)}{\Delta(x_1)}z. \end{aligned} \tag{2.26}$$

Consider the function $H(x_1,x_2) = \frac{1}{2}x_2^2 + V(x_1) + \frac{\gamma}{2\epsilon_2\epsilon_1^2}z^2$ and note that, from (2.17), $V'(0) = 0$ and $V''(0) = -\frac{1}{\Delta(0)} > 0$, hence the function $H(x_1,x_2)$ is positive-definite with a local minimum at zero. Differentiating along the trajectories of (2.26) yields

$$\begin{aligned} \dot{H} &= -R(x_1,x_2)x_2^2 + \frac{\gamma\cos(x_1)}{\Delta(x_1)}x_2 z - \frac{\gamma^2}{\epsilon_2\epsilon_1^2}z^2 \\ &\le -\epsilon_2 x_2^2 + \frac{\epsilon_2}{2}x_2^2 + \frac{\gamma^2}{2\epsilon_2\epsilon_1^2}z^2 - \frac{\gamma^2}{\epsilon_2\epsilon_1^2}z^2 \\ &\le -\frac{\epsilon_2}{2}x_2^2 - \frac{\gamma^2}{2\epsilon_2\epsilon_1^2}z^2, \end{aligned}$$

where we have used (2.25) and Young's inequality. It follows that there exists a neighbourhood of the origin such that all trajectories $(x_1(t), x_2(t), z(t))$ starting in this set remain bounded and asymptotically converge to $(0,0,0)$. Finally, boundedness of x_3 follows from the fact that $x_3(t) = z(t) + \pi_3(x_1(t), x_2(t)) - z(0) + x_3(0)$ and the right-hand side is bounded. □

To complete the design we propose functions $\pi_3(\cdot)$ that verify the assumptions $\Delta(0) < 0$ and $R(0,0) > 0$ which, in view of (2.17) and (2.18), impose that $\frac{\partial \pi_3}{\partial x_1}(0) < 0$ and $1 + \frac{\partial \pi_3}{\partial x_2}(0) < 0$. Two simple candidates are

$$\pi_3(x_1, x_2) = -k_1 x_1 - k_2 x_2$$

and

$$\pi_3(x_1, x_2) = -k_1 x_1 - \frac{k_2}{\cos(x_1)} x_2,$$

where $k_1 > 0$ and $k_2 > 1$ are tuning gains.

Alternatively, to enforce a particular behaviour on the target dynamics, we can also proceed dually, that is, fix the desired potential energy $V(\cdot)$ and then work backwards to compute $\pi_3(\cdot)$, $\Delta(\cdot)$ and $R(\cdot)$. A particularly interesting choice is $V(x_1) = \frac{k_1}{2} \tan^2(x_1)$, with $k_1 > 0$, which has a unique minimum at zero and is radially unbounded on the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. Replacing in (2.17) yields $\Delta(x_1) = -\frac{1}{k_1} \cos^3(x_1)$, which satisfies the assumption $\Delta(0) < 0$. From (2.18), and after some simple calculations, we obtain

$$\pi_3(x_1, x_2) = -\left(\frac{1}{\cos(x_1)} + \frac{1}{k_1} \cos^2(x_1) \right) x_2 + \psi(x_1),$$

where $\psi(\cdot)$ is a free function. As it can be easily shown, $R(0, 0) = -k_1 \psi'(0)$, hence $\psi(\cdot)$ must be such that $\psi'(0) < 0$ to ensure the damping is positive, *e.g.*, $\psi(x_1) = -k_2 x_1$, with $k_2 > 0$.

Simulations have been carried out with $\pi_3(x_1, x_2) = -k_1 x_1 - \frac{k_2}{\cos(x_1)} x_2$, yielding[13]

$$u = \frac{1}{k_2 - 1} \left[\gamma \left(x_3 + k_1 x_1 + \frac{k_2}{\cos(x_1)} x_2 \right) + k_1 x_2 + k_2 \tan(x_1) \left(\frac{x_2^2}{\cos(x_1)} + 1 \right) \right].$$

Note, however, that this controller is not globally defined because $\pi_3(\cdot)$ has a singularity at $\pm\frac{\pi}{2}$. Simulation results are shown in Figures 2.1 and 2.2 for the controller gains $k_1 = 3$, $k_2 = 4$ and for two different values of γ. The initial conditions are set to $x(0) = (\frac{\pi}{2} - 0.1, 0, 0)$, which correspond to zero velocities and with the pendulum close to horizontal. The results clearly show the desired closed-loop behaviour: first, convergence towards the manifold $z = 0$ at a speed determined by γ, and then, once close to the manifold, where the cart–pendulum system behaves like a simple pendulum, convergence towards the equilibrium. Note from Figure 2.1 that increasing γ, hence increasing the speed of convergence to the manifold, does not necessarily lead to a faster overall transient response. This is due to the fact that, even though the closed-loop system (2.26) is the cascade connection of an exponentially stable and an asymptotically stable system, the peaking phenomenon[14] appears when we increase the rate of convergence of the former. ■

[13]The simplicity of this control law should be contrasted with other schemes proposed in the literature, *e.g.*, [31, 30, 2].

[14]See [115].

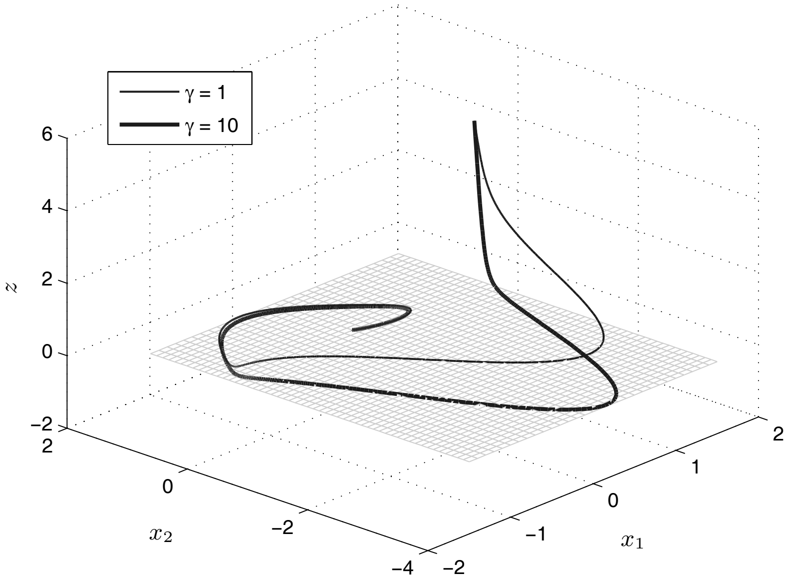

Fig. 2.1. Trajectories of the cart–pendulum system—the shaded area corresponds to the invariant manifold $z = 0$.

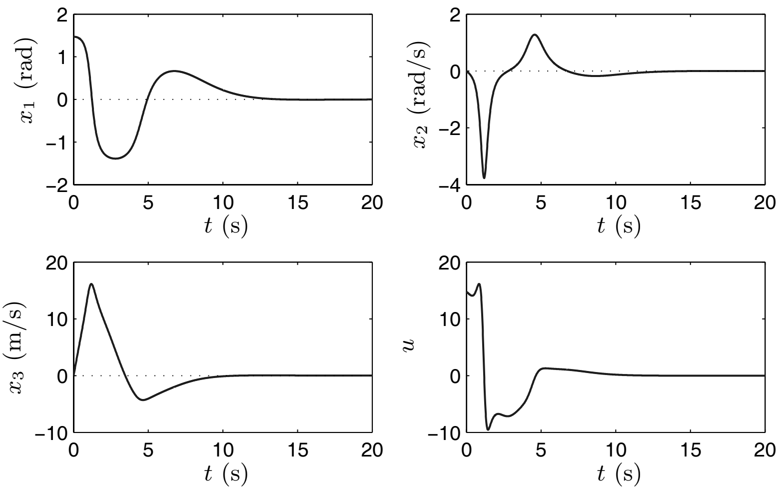

Fig. 2.2. Time histories of the states and the control input of the cart–pendulum system for $\gamma = 1$, $k_1 = 3$ and $k_2 = 4$.

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