In this chapter, we present the basic concepts of J. We introduce some of J’s built-in functions and show how they can be applied to data objects. The principles presented in this book are supported by examples. The reader therefore is strongly advised to download and install a copy of the J interpreter from www.jsoftware.com.

2.1 Data Objects

Here, we give a brief introduction to data objects in J. Data objects come in a number of forms: scalars, vectors, matrices and higher-dimensional arrays. Scalars are single numbers (or characters); the examples below show the values assigned to variables names:

```
i := 3  NB. integer
j := _1  NB. negative integer
x := 1.5983  NB. real
y := 22r7  NB. rational
z := 2j3  NB. complex number
m := _  NB. infinity
n := __  NB. negative infinity
v := 5x2  NB. exponential notation 5 * exp(2)
```

Note that negative numbers are denoted by an underscore (_) preceding the value rather than by a hyphen (-). An underscore on its own denotes infinity ∞, and two underscores denotes negative infinity −∞. Variables are not limited to numerical values:

```
c := ’hello world’
```

Variables can be evaluated by entering the variable name at the command prompt:
Scalars are called *atoms* in J or *0-cells*, and vectors are called lists or *1-cells*. The sequence of numbers below is an example of a list and is assigned to a variable name:

```j
   dvc =: 1 1 2 3 5 8 13 21 34 55
```

The `i.` verb generates ascending integers from zero \( n - 1 \), where \( n \) is the value of the argument. For example:

```j
   z1 =: i.10 NB. generate list 0 to 9
```

The associated verb `i:` generates integers from \( -n \) to \( n \), thus:

```j
   z2 =: i:5 NB. generate list -5 to 5
```

Matrices (tables in J) are 2-cell objects. Here is a table of complex numbers:

```j
   j =: (i.6) j./ (i.6)
```

Matrices (of ascending integers) can be generated with the `i.` verb. The example below shows a \( 3 \times 2 \) matrix:

```j
   i. 2 3 NB. generate a matrix
```

Higher-dimensional arrays are also possible. The expression below generates an array of reciprocals with ascending denominators:
This 3-cell data object is a three-dimensional array with three columns, four rows and two planes, where the planes are delimited by a blank line.

Scalars/atoms, vector/lists and matrices/tables are just special instances of arrays with respective ranks zero, one and two. Table 2.1 shows the J terms for data objects and their mathematical equivalents. Throughout this book, we will use the J and mathematical terms interchangeably.

<table>
<thead>
<tr>
<th>J term</th>
<th>Mathematical term</th>
<th>Dimension</th>
<th>Object type</th>
</tr>
</thead>
<tbody>
<tr>
<td>atom</td>
<td>scalar</td>
<td>0</td>
<td>0-cell</td>
</tr>
<tr>
<td>list</td>
<td>vector</td>
<td>1</td>
<td>1-cell</td>
</tr>
<tr>
<td>table</td>
<td>matrix</td>
<td>2</td>
<td>2-cell</td>
</tr>
<tr>
<td>array</td>
<td>array</td>
<td>n</td>
<td>n-cell</td>
</tr>
</tbody>
</table>

Table 2.1. J versus mathematical terms

2.2 J Verbs

In J, functions are called verbs. J has a number of built-in verbs/functions; for example, the basic arithmetic operators are: +, −, *, % and ^, which are addition, subtraction, multiplication, division and power functions, respectively. Here are some (trivial) examples:

```
2 + 3   NB. addition
5
2 − 3   NB. subtraction
−1
7 * 3   NB. multiplication
21
2 % 7   NB. division: "%" is used instead of "/"
0.285714
```
In J, there is a subtle (but important) difference between _1 and -1. The term _1 is the value minus one, whereas -1 is the negation function applied to (positive) one.

Arithmetic can also be performed on lists of numbers:

```
2 % 0 1 2 3 4 NB. divide scalar by a vector
_2 1 0.666667 0.5
3 2 1 0 - 0 1 2 3 NB. pointwise vector subtraction
3 1 _1 _3
```

The example above demonstrates how J deals with divide by zero. Any division by zero returns a _ (∞). In addition to the basic arithmetic operators, J has many more primitives, for example:

```
+: 1 2 3 4 NB. double
2 4 6 8
-: 1 2 3 4 NB. halve
0.5 1 1.5 2
%: 1 4 9 16 NB. square root
1 2 3 4
*: 1 2 3 4 NB. squared
1 4 9 16
>: _1 0 1 2 3 NB. increment
0 1 2 3 4
<: _1 0 1 2 3 NB. decrement
_2 _1 0 1 2
```

Verbs are denoted by either a single character (such as addition +) or a pair of characters (such as double +:). Some primitives do use alphabetic characters, for example, the integers verb i. and complex verb j., which were introduced above.

### 2.3 Monadic and Dyadic functions

Each verb in J possesses the property of *valance*, which relates to how many arguments a verb takes. *Monadic* verbs take one argument (and are therefore of valance one), whereas *dyadic* verbs take two arguments (valance two).

Monadic verbs are expressed in the form: \( f \ x \). The (single) argument \( x \) is passed to the right of the function \( f \). In functional notation, this is equivalent to \( f(x) \). In its dyadic form, \( f \) takes two arguments and are passed to the function on either side: \( y \ f \ x \), equivalent to \( f(y, x) \) in functional notation.
J’s verb symbols are overloaded; that is, they implement two separate (often related, but sometimes inverse) functions depending upon the valance. We use the % primitive to demonstrate. We have already seen it used in its dyadic form as a division operator. However, in its monadic form, % performs a reciprocal operation:

\[
\begin{align*}
% 2 & \quad \text{NB. used monadically is reciprocal} \\
& \quad 0.5 \\
3 \% 2 & \quad \text{NB. used dyadically is division} \\
& \quad 1.5
\end{align*}
\]

Let us look at a few more examples. The monadic expression \( \sim x \) is the exponential function of \( x: e^x \). The dyad \( y \sim x \), however, performs \( y \) to the power \( x \), that is: \( y^x \). To illustrate:

\[
\begin{align*}
^e 0 & \quad 1 \quad 2 \quad 3 \quad \text{NB. used monadically is } \exp(x) \\
1 & \quad 2.71828 \quad 7.38906 \quad 20.0855 \\
2 \quad ^e 0 & \quad 1 \quad 2 \quad 3 \quad \text{NB. used dyadically is } y^x \\
1 & \quad 2 \quad 4 \quad 8
\end{align*}
\]

Used monadically <: performs a decrement function:

\[
\begin{align*}
\textcolor{blue}{\text{::< 1 2 3 4 5 6}} \\
& \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\end{align*}
\]

However as a dyad it performs less-than-or-equal-to:

\[
\begin{align*}
4 \textcolor{blue}{\text{::< 1 2 3 4 5 6}} \\
& \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1
\end{align*}
\]

2.4 Positional Parameters

The meaning of a positional parameter is given by virtue of its relative position in a sequence of parameters. J does not really have a concept of positional parameters; however, we can pass positional parameters to functions as an ordered list of arguments. In this section, we introduce verbs for argument processing: left [ and right ]. These verbs return the left and right arguments, respectively:

\[
\begin{align*}
2 \left[ 3 \\
2 \\
2 \right] 3 \\
3
\end{align*}
\]

Similarly >: performs increment and greater-than-or-equal-to.
The *right* provides a convenient means of displaying the result of an assignment:

```
x =: i.10
0 1 2 3 4 5 6 7 8 9
```

The *left* verb can be used to execute two expressions on one line:

```
x =: i.10 { n =: 2 NB. assign x and n
x ^ n
0 1 4 9 16 25 36 49 64 81
```

The verbs *head* {. and *tail* {: return the first element and the last element of a list:

```
{. x
0
{: x
10
```

Conversely, *drop* {. and *curtail* }: remove the head and tail of a list and return the remaining elements:

```
}. x
2 4 6 8 10
): x
0 2 4 6 8
```

The *from* verb {. is used to extract a particular element (or elements) within a list, by passing the index of the required element in the list as a left argument:

```
0 { x
0
2 { x
4
3 1 5 { x
6 2 10
0 0 0 1 2 2 2 3 3 4 5 5 5 5 { x
0 0 0 2 4 4 4 6 8 10 10 10 10
```

Lists can be combined with *raze* ,. and *laminate* ,: in columns or rows, respectively. The J expressions below yield two matrices $m_1$ and $m_2$:

```
]m1 =: 1 2 3 4 ,. 5 6 7 8
1 5
2 6
3 7
4 8
```
We cover higher-dimensional objects in more detail in Section 2.6. Here, we look briefly at applying the \textit{from} verb to matrices:

\begin{verbatim}
 ] m2 =: 1 2 3 4 , : 5 6 7 8
1 2 3 4
5 6 7 8
\end{verbatim}

Here, \textit{from} returns the third row of \textit{m}1 in the first example and the second row of \textit{m}2 in the second example. If we wish to reference an individual scalar element, then we need to use \textit{from} twice:

\begin{verbatim}
0 { 1 { m2
5
\end{verbatim}

In order to reference a column, we need to be able to change the \textit{rank} of the verbs. The concept of rank will be covered in the next section. Two data objects can be concatenated with \textit{ravel} (,,), for example:

\begin{verbatim}
 v1 =: 1 2 3 4
 v2 =: 5 6
 ] v3 =: v1 , v2
1 2 3 4 5 6
0 3 4 5 { v3
1 4 5 6
\end{verbatim}

This creates a single list of eight elements (\textit{v}3). There is no separation between the two original lists \textit{v}1 and \textit{v}2. If we wished to retain the separation of the two initial lists, then we combine them with the \textit{link} verb ;, for example:

\begin{verbatim}
 ] v4 =: v1 ; v2
-------+-+
|1 2 3 4|5 6|
-------+-+
\end{verbatim}

The lists are “boxed” and therefore exist as separate data objects. We can reference the two lists in the usual way:

\begin{verbatim}
0 { v4
-------+
|1 2 3 4|
-------+
\end{verbatim}
The data object returned is a single element; we cannot get at any of the individual scalar elements in the box:

```
1 { 0 { v4
|index error
| 1 {0{v4
```

Use `open >` to unbox the object:

```
> 0 { v4 NB. unbox v1
1 2 3 4
1 { > 0 { v4
2
```

There is a corresponding inverse function, namely the monadic verb `box <`, which “groups” elements:

```
<1 2 3 4
+-------+
| 1234 |
+-------+
```

Using the primitives described above, we define a number of functions for referencing positional parameters. These functions will be used a great deal in developing functions later in this book. Note that a couple of conjunctions are used here (`&` and `@`) will be covered later, in Section 3.1.4

```
lhs0 =: [\ NB. all left arguments
lhs1 =: 0&{@lhs0 NB. 1st left argument
lhs2 =: 1&{@lhs0 NB. 2nd left argument
lhs3 =: 2&{@lhs0 NB. 3rd left argument

rhs0 =: ]\ NB. all right arguments
rhs1 =: 0&.@rhs0 NB. 1st right argument
rhs2 =: 1&.@rhs0 NB. 2nd right argument
rhs3 =: 2&.@rhs0 NB. 3rd right argument
```

The functions `lhs0` and `rhs0` evaluate the left and right arguments, respectively. The other functions are programmed to return positional parameters, thus `lhs1` (respectively `rhs1`) returns the first positional parameter on the left-hand side (respectively right-hand side). We illustrate the use of positional parameters with the following example. Consider the classical M/M/1 queuing model given in Equation (1.6). We can write a function that takes the parameters $\mu$ and $\rho$ as left-hand arguments and right-hand arguments, respectively:

```
mm1 =: %@lhs1 % 1:-rhs0
3 mm1 0.5 0.6 0.7 0.8 0.9
0.666667 0.833333 1.11111 1.66667 3.33333
```
2.5 Adverbs

The (default) behaviour of verbs can be altered by combining them with *adverbs*. We have already encountered an adverb with the summation function `/`. The application of `/` causes + to be inserted between the elements of the argument, in this case, the individual (scalar) numbers in the list.

```
+/ i.6
```

```
0 + 1 + 2 + 3 + 4 + 5  NB. as we’ve seen before
```

```
+/ i.6
```

```
0 + 1 + 2 + 3 + 4 + 5  NB. and is equivalent to this
```

The dyadic case results in a matrix of the sum of the elements of the left argument to each element of the right argument.

```
(i.6) +/ (i.6)
```

```
0 1 2 3 4 5
1 2 3 4 5 6
2 3 4 5 6 7
3 4 5 6 7 8
4 5 6 7 8 9
5 6 7 8 9 10
```

The *prefix* adverb \ causes the data object to be divided into sublists that increase in size from the left; the associated verb is then applied to each sublist in turn. We can see how the sublist is generated using the *box* verb:

```
<\ i.5
```

```
+-----------+---------+-------+-----+---+-+
|0 1 2 3 4 5|1 2 3 4 5|2 3 4 5|3 4 5|4 5|5|
+-----------+---------+-------+-----+---+-+
```

A cumulative summation function can be implemented using the *insert* and *prefix* verbs:

```
+/\ i.6
```

```
0 1 3 6 10 15
```

This function will be useful later on when we wish to convert interval traffic arrival processes to cumulative traffic arrival processes. The *suffix* \. operates on decreasing sublists of the argument:

```
<\. i.6
```

```
+-----------+---------+-------+-----+---+-+
|0 1 2 3 4 5|1 2 3 4 5|2 3 4 5|3 4 5|4 5|5|
+-----------+---------+-------+-----+---+-+
```

```
The monadic reflexive adverb \( \sim \) duplicates the right-hand argument as the left-hand argument. So the J expression \( f \sim x \) is equivalent to \( x f x \), for example:

\[
+/ \sim i.6 \quad \text{NB. } (i.6) +/ (i.6)
\]

0 1 2 3 4 5
1 2 3 4 5 6
2 3 4 5 6 7
3 4 5 6 7 8
4 5 6 7 8 9
5 6 7 8 9 10

The \( \sim \) verb also has a dyadic form (passive). This means that the left and right-hand side arguments are swapped; that is, \( y f x \) becomes \( x f y \), as an illustration:

\[
2 \% \sim i.6 \quad \text{NB. equivalent to } (i.6) \% 2
\]

0 0.5 1 1.5 2 2.5

### 2.6 Rank, Shape and Arrays

Arithmetic can be performed between a scalar and a list or between two lists, for example:

\[
2 * 0 1 2 3 4 5
\]

0 2 4 6 8 10

0 1 2 3 4 5 + 1 2 3 4 5 6

1 3 5 7 9 11

Notice that the lists have to be the same length; otherwise the J interpreter throws a "length error":

\[
9 8 - 0 1 2 3 4 5
\]

|length error |
| 9 8 - 0 1 2 3 4 5 |

J can also perform arithmetic on higher-dimensional objects. In this section, we introduce arrays as well as the concepts of rank and shape. Rank is synonymous with dimensionality; thus a two-dimensional array has rank two, a three-dimensional array has rank three. Verbs have rank attributes which are used to determine at what rank level they should operate on data objects. We will explore this later. First, let us consider at how we can define array objects by using the dyadic shape verb \( \$: \)

\[
]x2 =: 2 3 \$: i.6
\]

0 1 2
3 4 5
As we have already seen, we could have defined this particular array, simply by:

\[
\begin{array}{cccc}
i & 2 & 3 \\
0 & 1 & 2 \\
3 & 4 & 5 
\end{array}
\]

This is fine for defining an array with ascending integers (as returned by \( \text{i.} \)), but if we wanted to form an array using some arbitrary list of values, then we need to use \( \$ \). We will continue to use the \( \$ \) method, although we acknowledge that it is not necessary, as the data objects used in these examples are merely ascending integers. The shape is specified by the left arguments of \( \$ \) and can be confirmed using the \( \$ \) in its monadic form:

\[
\$ \; x2 \\
2 \; 3
\]

The data object \( x2 \) is a \((3 \times 2)\) two-dimensional array, or, in J terms, a rank two object (of shape \( 2 \; 3 \)). Arithmetic can be applied in the usual way. This example shows the product of a scalar and an array:

\[
2 \; \ast \; x2 \\
0 \; 2 \; 4 \\
6 \; 8 \; 10
\]

Here we have the addition of two arrays (of the same shape):

\[
x2 \; + \; (2 \; 3 \; \$ \; 1 \; 2 \; 3 \; 4 \; 5 \; 6) \\
1 \; 3 \; 5 \\
7 \; 9 \; 11
\]

J can handle this:

\[
2 \; 3 \; + \; x2 \\
2 \; 3 \; 4 \\
6 \; 7 \; 8
\]

But apparently not this:

\[
1 \; 2 \; 3 \; + \; x2 \\
\mid \text{length error} \\
\mid \; 1 \; 2 \; 3 \; + \; x2
\]

J of course, can handle this, but we need to understand more about the rank control conjunction \( * \) which will be covered in Section 3.1 below. Consider a \( 3 \times 2 \times 2 \) array:
$x_3$ is a three-dimensional array and, therefore, of rank three. J displays this array arranged into two planes of two rows and three columns, where the planes are delimited by the blank line. We can confirm the structure of $x_3$ by using $\$ $ as a monad, where it performs a shape-of function:

$$x_3$$

2 2 3

Now, let us apply summation to $x_3$:

$$+/x_3$$

6 8 10
12 14 16

Here, the individual elements of the two planes have been summed; that is:

\[
\begin{bmatrix}
0 & 1 & 2 \\
3 & 4 & 5
\end{bmatrix}
\begin{bmatrix}
6 & 7 & 8 \\
9 & 10 & 11
\end{bmatrix}
= \begin{bmatrix}
0 + 6 & 1 + 7 & 2 + 8 \\
3 + 9 & 4 + 10 & 5 + 11
\end{bmatrix}
= \begin{bmatrix}
6 & 8 & 10 \\
12 & 14 & 16
\end{bmatrix}
\]

It is important to understand why $+/ \ $ sums across the planes rather down the columns or along the rows. First consider this example:

$$%/x_3$$

_ 1 0.5
0.333333 0.25 0.2
0.166667 0.142857 0.125
0.111111 0.1 0.0909091

Aside from the difference between the arithmetic functions $+/ \ $ and $\% \ $ perform, they also operate on the argument in a different way. Where as $+/ \ $ operated on the two planes, here $\% \ $ is applied to each individual scalar element. The difference in the behaviour of the two verbs $+/ \ $ and $\% \ $ is governed by their respective rank attributes. We can query the rank attribute of verbs with the expressions below:

$$%/b.$$

0 0 0
0 0
0 0
0 0
0 0 0
0 0 0
Three numbers are returned. The first number (reading left to right) is the rank of the monad form of the verb. The second and third numbers are the ranks of the left and right arguments of the dyadic form of the verb. When a verb performs an operation on an object, it determines the rank of the cell elements on which it will operate. It does this by either using the rank (dimension) of the object or the rank attribute of the verbs, whichever is smaller. In the example above, \(x3\) has a rank of three, and the (monadic) rank attribute of \(\%\) is zero. So \(\%\) is applied to \(x3\) at rank zero. Thus it is applied to each 0-cell (scalar) element. However, the rank attribute of \(+/\) is infinite, and, therefore, the rank, at which the summation is performed, is three. Thus, \(+/\) applies to each 3-cell element of \(x3\), resulting in a summation across the planes.

Consider another example. We define the data object \(x0\) as a list of six elements and then apply the fork \((\$, \#)\) which (simultaneously) returns the shape and the number of elements.

```plaintext
]x0 =: i.6 0 1 2 3 4 5
($;#) x0
+-+-+-
|6|6|
+-+-
```

Here both \# and \$ return six as \(x0\) consists of six atoms, or 0-cell elements. Now try this on \(x2\) which was declared earlier:

```plaintext
($;#) x2
+-+-+-+-+-
|2 3|2|
+-+-+-+-
```

The resultant shape is as expected but the number of elements returned by \textit{tally} may not be. To make sense of the result, we need to know what “elements” the \textit{tally} is counting: 0-cell, 1-cell or 2-cell? This depends upon the rank at which \# is operating. The data object \(x2\) is clearly rank two. The command-line below shows us that the (monadic) verb attribute of \# is infinite:

```plaintext
# b.
0 NB. monadic rank attribute is infinite
_ 1 _
```

In this particular case we may ignore the dyadic rank attributes. Tally (\#) is applied to the 2-cell elements which are the rows. Consider this example:

```plaintext
]x1 =: 1 6 $ i.6
0 1 2 3 4 5
($;#) x1
+-+-+-+-+-
```
Data objects $x_0$ and $x_1$ may appear the same but they are actually different by virtue of their shape and rank. $x_1$ is a $(6 \times 1)$ two-dimensional array, and, therefore, of rank two (with shape $1 \times 6$). It also has only one element (one row) because # still operates on the 2-cell elements. In contrast, $x_0$ is a list of six 0-cell elements. In actual fact, $x_0$ is equivalent to $y_0$, defined below:

```j
]y0 =: 6$i.6
0 1 2 3 4 5
```

$y_0$ and $x_0$ are equivalent

```j
x0 = y0 NB. x0 and y0 are equivalent
1 1 1 1 1 1
```

$x_0$ and $x_1$, as we know, are not

```j
|length error
| x0 =x1
```

The difference between $x_0$ and $x_1$ becomes more apparent when we perform some arithmetic operation on them:

```j
x1- x0
```

```j
|length error
| x1 -x0
```

The interpreter is trying to subtract the first element from $x_1$ from the first element of $x_0$, then subtract the second element from $x_1$ from the second element of $x_0$, and so on. However, while $x_0$ has six 0-cell elements, $x_1$ only has one element, which is a 2-cell. However it does not seem unreasonable to want to perform arithmetic operations on $x_0$ and $x_1$, as they both contain six numbers. We can control the rank attribute of a verb, thereby enabling us to perform arithmetic on both $x_0$ and $x_1$. We will return to this example in Chapter 3 when we cover conjunctions.

### 2.7 Summary

In this chapter we have introduced some of the basic concepts of programming in J. J functions are called verbs. The primitive verbs are (mostly) designated by a single punctuation character (+) or a pair of punctuation characters (+:), though a few use alphabetic characters (i.). Verbs can be monadic (one argument) or dyadic (two arguments). Data objects have properties of rank (dimension) and shape. All objects can be thought of as arrays. Atoms (0-cell), lists (1-cell) or tables (2-cell) are merely special instances of arrays ranked (dimensioned) zero, one and two, respectively. Furthermore any $n$ ranked array can be thought of as a list of $n-1$ cell objects. Note that, an atom has an empty shape and is different from a one-item list. Furthermore, a list is different from a $1 \times n$ table. Verbs have rank attributes that determine at what cell level they operate.
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