2

Graph Theory and Matrix Approach as a Decision-making Method

2.1 Introduction

A graph G = (V, E) consists of a set of objects V = {v_1, v_2, ...} called vertices or nodes, and another set E = {e_1, e_2, ...}, of which the elements are called edges, such that each edge e_k is identified with a pair of vertices. The vertices v_i and v_j associated with edge e_k are called the end vertices of e_k. The most common representation of a graph is by means of a diagram, in which the vertices are represented by small points or circles, and each edge as a line segment joining its end vertices.

The application of graph theory was known centuries ago, when the long-standing problem of the Konigsberg bridge was solved by Leonhard Euler in 1736 by means of a graph. Since then, graph theory has proved its mettle in various fields of science and technology such as physics, chemistry, mathematics, communication science, computer technology, electrical engineering, sociology, economics, operations research, linguistics, internet, *etc.* Graph theory has served an important purpose in the modeling of systems, network analysis, functional representation, conceptual modeling, diagnosis, *etc.* Graph theory is not only effective in dealing with the structure (physical or abstract) of the system, explicitly or implicitly, but also useful in handling problems of structural relationship. The theory is intimately related to many branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology, and combinatorics. The advanced theory of graphs and their applications are well documented (Harary, 1985; Wilson and Watkins, 1990; Chen, 1997; Deo, 2000; Jense and Gutin, 2000; Liu and Lai, 2001; Tutte, 2001; Pemmaraju and Skiena, 2003; Gross and Yellen, 2005; Biswal, 2005).

This chapter presents the details of graph theory and the matrix approach as a decision-making method in the manufacturing environment. To demonstrate the approach, an example of machinability evaluation of work materials for a given machining operation is considered. Machinability is a measure of ease with which a work material can satisfactorily be machined. The machinability aspect is of considerable importance for the manufacturing engineer to know in advance, so that the processing can be planned in an efficient manner. The study can also be a basis for cutting tool and cutting fluid performance evaluation, and machining parameter optimization. In the process of product design, material selection is important for realizing the design objective, and for reducing the production cost. The machinability of engineering materials, owing to the marked influence on the production cost, needs to be taken into account in the product design, although it will not always be a criterion considered top priority in the process of material selection. If there is a finite number of work materials from among which the best material is to be chosen, and if each work material satisfies the required design and functionality of the product, then the main criterion to choose the work material is its operational performance during machining, *i.e.*, machinability.

Machinability evaluation is based on the evaluation of certain economic and technical objectives (*e.g.*, higher production rate, low operational cost, good product quality, *etc.*), which are the consequences of the machining operation on a given work material. Machining process output variables (*e.g.*, cutting tool life, cutting tool wear, cutting forces, power consumption, processed surface finish, processed dimensional accuracy, *etc.*) are nothing but the behavioral properties of the work materials during machining operations in terms of economic and technical consequences and are directly related to machining operations, and hence to machinability. Thus, the machining process output variables are the pertinent and most commonly accepted measures of machinability, and are also called pertinent machinability attributes.

2.2 Machinability Attributes Digraph

A directed graph (or a digraph) is nothing but a graph with directed edges. A machinability attributes digraph models the machinability attributes and their interrelationship for a given machining operation. This digraph consists of nodes and edges. A node $\{V_i\}$ represents presence or measure of an i-th machinability attribute. The number of nodes considered is equal to the number of machinability attributes considered for a given machining operation. The directed edge represents the relative importance among the attributes. If node ‘i’ has a relative importance over another anode ‘j’ in the machinability evaluation of work materials for the given machining operation, then a directed edge or arrow is drawn from node i to node j (*i.e.*, e_{ij}). If j has relative importance over i, then the directed edge or arrow is drawn from node j to node i (*i.e.*, e_{ji}).

To demonstrate a machinability attributes digraph, an example of machinability evaluation of work materials in cylindrical grinding operation is considered. Grinding is a machining process of material removal in the form of small chips by the mechanical action of abrasive particles bonded together in a grinding wheel. In this operation, wheel wear is most important, so as to reduce the cost of production. The wheel wear is measured in terms of a ratio known as ‘grinding ratio’, which is defined as the ratio of amount of work material removed to the amount of wheel wear. Higher values of grinding ratio are desired for economic reasons. Two components of the cutting force, namely, normal force and tangential force, significantly affect the grinding process. Higher values of normal force increase the roughness of the processed surfaces, and the geometric and dimensional inaccuracy of the processed parts. Tangential force affects the rating of the motors driving the wheel and the work piece, and higher values of tangential force mean increased power consumption. The grinding process imparts high-grade surface finish and good dimensional accuracy to the job. However, the temperature encountered in the grinding process is very high, and adversely affects the process. So, every care is to be taken to reduce the grinding temperature. All these variables described are the machining process output variables and are the pertinent machinability attributes and these attributes refer to the performance of work material during machining operations in terms of technical and economic consequences, and can be used for objective comparison. A work material is said to possess good machinability in cylindrical grinding operation if it offers higher grinding ratio, and lower values of normal force, tangential force, surface roughness, dimensional inaccuracy, and grinding temperature.

Based on the above discussion, the machinability attributes considered for the cylindrical grinding operation are: grinding ratio (GR), normal force (NF), tangential force (TF), surface finish (SF), dimensional accuracy of the produced job (DA), and grinding temperature (GT). A machinability attributes digraph for the cylindrical grinding operation is shown in Figure 2.1. As six machinability attributes are considered here, there are six nodes in the machinability attributes digraph with nodes 1, 2, 3, 4, 5, and 6 representing the machinability attributes GR, NF, TF, SF, DA, and GT, respectively. The attribute GR is more important than the other machinability attributes in cylindrical grinding. Every effort should be made to increase the grinding ratio, as it greatly affects the cost of production. So, directed edges are drawn for the attribute GR (*i.e.*, node 1) to the other attributes (*i.e.*, nodes 2, 3, 4, 5, and 6). NF is more important than the attributes TF, SF, DA, and GT in cylindrical grinding operation, as it affects the surface roughness, and the geometric and dimensional accuracy of the processed parts. So, directed edges are drawn from node 2, representing NF, to the nodes 3, 4, 5, and 6. SF is more important than TF, so a directed edge is drawn from node 4 to node 3. DA is more important than TF, so a directed edge is drawn from node 5 to node 3. GT is more important than TF, SF, and DA in cylindrical grinding operation, so directed edges are drawn from node 6 to the nodes 3, 4, and 5 representing TF, SF, and DA, respectively.

A machinability attributes digraph gives a graphical representation of the attributes and their relative importance for quick visual appraisal. As the number of nodes and their interrelations increase, the digraph becomes more complex. In such a case, the visual analysis of the digraph is expected to be difficult and complex. To overcome this constraint, the digraph is represented in a matrix form.

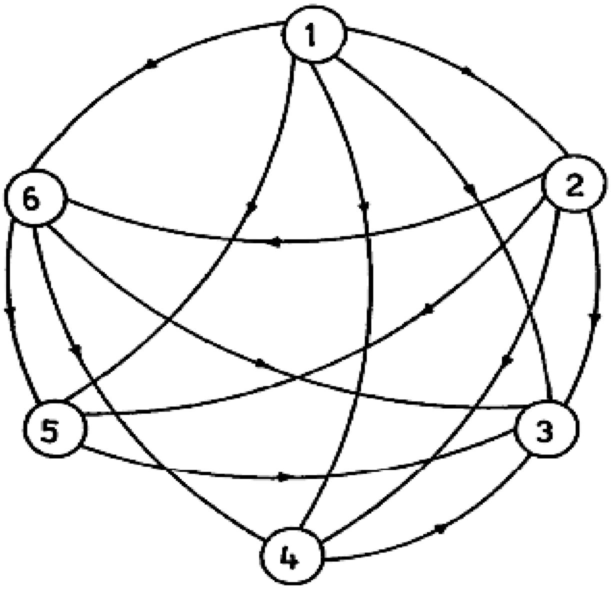

Figure 2.1. Machinability attributes digraph for the cylindrical grinding operation (attributes: 1. grinding ratio, 2. normal force, 3. tangential force, 4. surface finish, 5. dimensional accuracy, and 6. grinding temperature)

2.3 Matrix Representation of the Digraph

Matrix representation of the machinability attributes digraph gives one-to-one representation. A matrix called the machinability attributes relative importance matrix is defined. This is represented by a binary matrix (a_{ij}), where a_{ij} represents the relative importance between attributes i and j such that,

a_{ij} = 1, if the i-th machinability attribute is more important than the j-th machinability attribute for a given machining operation

= 0, otherwise.

It is noted that a_{ii} = 0 for all i, as an attribute can not have relative importance over itself. The machinability attributes relative importance matrix (RIM) for the machinability attributes digraph shown in Figure 2.1 is written as:

$$B = \begin{array}{c|cccccc} \text{Attributes} & GR & NF & TF & SF & DA & GT \\ \hline GR & 0 & 1 & 1 & 1 & 1 & 1 \\ NF & 0 & 0 & 1 & 1 & 1 & 1 \\ TF & 0 & 0 & 0 & 0 & 0 & 0 \\ SF & 0 & 0 & 1 & 0 & 0 & 0 \\ DA & 0 & 0 & 1 & 0 & 0 & 0 \\ GT & 0 & 0 & 1 & 1 & 1 & 0 \end{array} \tag{2.1}$$

The machinability attributes relative importance matrix (RIM) is analogous to the adjacency matrix in graph theory. It is noted from the RIM that all diagonal elements have value 0 and all off-diagonal elements have value either 0 or 1. This means that in this matrix only relative importance among the machinability attributes is considered, and the measures of the machinability attributes is not considered. To incorporate this, another matrix, called 'characteristic machinability attributes presence and relative importance matrix (CPRIM)', is defined and this, for the machinability attributes digraph of Figure 2.1, is written as C given by:

$$C = [AI\text{-}B] = \begin{array}{c} \text{Attributes} \\ \text{GR} \\ \text{NF} \\ \text{TF} \\ \text{SF} \\ \text{DA} \\ \text{GT} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{GR} & \text{NF} & \text{TF} & \text{SF} & \text{DA} & \text{GT} \end{array} \\ \begin{bmatrix} A & -1 & -1 & -1 & -1 & -1 \\ 0 & A & -1 & -1 & -1 & -1 \\ 0 & 0 & A & 0 & 0 & 0 \\ 0 & 0 & -1 & A & 0 & 0 \\ 0 & 0 & -1 & 0 & A & 0 \\ 0 & 0 & -1 & -1 & -1 & A \end{bmatrix} \end{array} \quad (2.2)$$

where I is an identity matrix, and A is a variable representing the measure of the machinability attribute. Matrix C is analogous to the characteristic matrix in graph theory. Referring to the matrix in Equation 2.2, it is noted that the value of all diagonal elements is identical, *i.e.*, the presence or measure of each machinability attribute is taken to be the same. In practice, this is not true. Also, the relative importance of one machinability attribute over the other machinability attribute, *i.e.*, a_{ij}, may take any value other than the extreme value 0 or 1. Thus, there is a need for considering a general attribute value representing attribute presence or measure as well as relative importance value to develop a matrix equation leading to a broad-based machinability evaluation. To consider these aspects, another matrix, D, called 'variable characteristic machinability attributes presence and relative importance matrix (VCPRIM)', is developed.

$$D = [E\text{-}F] = \begin{array}{c} \text{Attributes} \\ \text{GR} \\ \text{NF} \\ \text{TF} \\ \text{SF} \\ \text{DA} \\ \text{GT} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{GR} & \text{NF} & \text{TF} & \text{SF} & \text{DA} & \text{GT} \end{array} \\ \begin{bmatrix} A_1 & -a_{12} & -a_{13} & -a_{14} & -a_{15} & -a_{16} \\ 0 & A_2 & -a_{23} & -a_{24} & -a_{25} & -a_{26} \\ 0 & 0 & A_3 & 0 & 0 & 0 \\ 0 & 0 & -a_{43} & A_4 & 0 & 0 \\ 0 & 0 & -a_{53} & 0 & A_5 & 0 \\ 0 & 0 & -a_{63} & -a_{64} & -a_{65} & A_6 \end{bmatrix} \end{array} \quad (2.3)$$

where E is a diagonal matrix with diagonal element A_i representing a variable of presence or measure of the i-th machinablity attribute. If a machinability attribute is excellent, then it is assigned a maximum value. If a machinability attribute is not very significant, then it is assigned a minimum value. In general, most of the machinability attributes are assigned intermediate values of the interval scale, as attributes may be moderately present. These judgments are to be made based on an appropriate test of the machinability attribute. In the absence of this test, a subjective value based on experience is assigned. F is a matrix of which the off-diagonal elements are represented as a_{ij}, instead of 1, wherever the i-th machinability attribute has more relative importance than the j-th machinability attribute.

It may be noted that the matrix VCPRIM considers the presence or measures of the machinability attributes, and their relative importance for the given machining operation. The characteristic multinomial of the matrix VCPRIM is nothing but the determinant of the matrix VCPRIM, and may be written as:

$$\det (D) = A_1\, A_2\, A_3\, A_4\, A_5\, A_6 \qquad (2.4)$$

Equation 2.4 contains only one term, *i.e.*, $A_1\, A_2\, A_3\, A_4\, A_5\, A_6$, which is a set of six machinability attributes measures. It is evident that the relative importance among the machinability attributes is not represented by this characteristic multinomial. It is therefore necessary to look into the aspect of relative importance representation in the machinability attributes digraph and its matrix to identify the reasons. If the i-th machinablity attribute is more important than the j-th machinability attribute, then a directed edge is drawn from i to j to represent this relative importance. Similarly, if the j-th machinability attribute is more important than the i-th machinability attribute, then a directed edge is drawn from j to i to represent their relative importance. But if the i-th machinability attribute is less important than the j-th machinability attribute, then no directed edge is drawn from i to j, and *vice versa*. In that case, a_{ij} (or a_{ji}) becomes 0 in the matrix representation of the digraph. This 0 causes many terms of the characteristic multinomial to become 0 (as there are no relative importance loops in the corresponding machinability attributes digraph), thus leading to the loss of a fair amount of information useful during the machinability evaluation. Hence, the relative importance between i, j and j, i is distributed on a scale 0 to L and is defined as:

$$a_{ji} = L - a_{ij} \qquad (2.5)$$

It means that a scale is adapted from 0 to L on which the relative importance values are compared. If a_{ij} represents the relative importance of the i-th machinability attribute over the j-th machinability attribute, then the relative importance of the j-th machinability attribute over the i-th machinability attribute is evaluated using Equation 2.5. The modified machinability attributes digraph showing the presence or measures of the machinabilty attributes, and all the possible relative importance among these is shown in Figure 2.2.

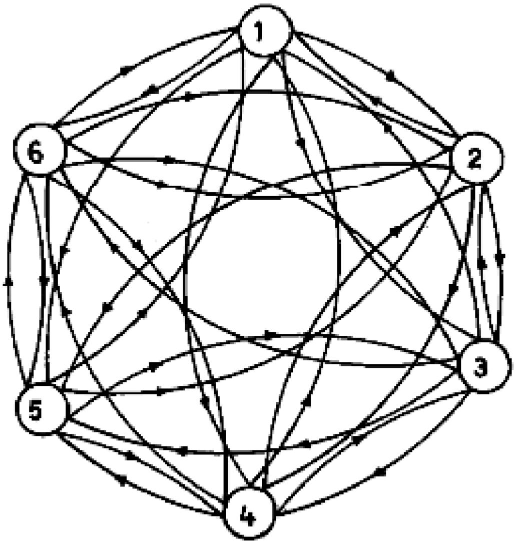

Figure 2.2. Modified machinability attributes digraph for the cylindrical grinding operation (attributes: 1. grinding ratio, 2. normal force, 3. tangential force, 4. surface finish, 5. dimensional accuracy, and 6. grinding temperature)

The modified VCPRIM for this digraph for the cylindrical grinding operation is represented as:

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \text{GR} & \text{NF} & \text{TF} & \text{SF} & \text{DA} & \text{GT} \end{array} \\
G = \begin{array}{c} \text{GR} \\ \text{NF} \\ \text{TF} \\ \text{SF} \\ \text{DA} \\ \text{GT} \end{array} &
\begin{bmatrix}
A_1 & -a_{12} & -a_{13} & -a_{14} & -a_{15} & -a_{16} \\
-a_{21} & A_2 & -a_{23} & -a_{24} & -a_{25} & -a_{26} \\
-a_{31} & -a_{32} & A_3 & -a_{34} & -a_{35} & -a_{36} \\
-a_{41} & -a_{42} & -a_{43} & A_4 & -a_{45} & -a_{46} \\
-a_{51} & -a_{52} & -a_{53} & -a_{54} & A_5 & -a_{56} \\
-a_{61} & -a_{62} & -a_{63} & -a_{64} & -a_{65} & A_6
\end{bmatrix}
\end{array}
\quad \text{Attributes}
\tag{2.6}
$$

where A_i is the measure of the i-th machinability attribute represented by node v_i, and a_{ij} the relative importance of the i-th machinability attribute over the j-th, represented by the edge e_{ij}. The characteristic multinomial of this matrix G is defined as 'variable characteristic machinability function (VCF)', and is written as Equation 2.6.

$$
\det(G) = \prod_{i=1}^{6} A_i - \sum_{i=1}^{5} \sum_{j=i+1}^{6} \sum_{k=1}^{3} \sum_{l=k+1}^{4} \sum_{m=l+1}^{5} \sum_{n=m+1}^{6} (a_{ij}a_{ji})A_kA_lA_mA_n
$$

$$
k,l,m,n \neq \text{pus}
$$

$$
- \sum_{i=1}^{4} \sum_{j=i+1}^{5} \sum_{k=j+1}^{6} \sum_{l=1}^{4} \sum_{m=l+1}^{5} \sum_{n=m+1}^{6} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji})A_lA_mA_n
$$

$$
k,l,m,n \neq \text{pus}
$$

$$+\Big[\sum_{i=1}^{3}\sum_{j=i+1}^{6}\sum_{k=i+1}^{5}\sum_{l=i+2}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{ji})(a_{kl}a_{lk})A_mA_n \quad k,l,m,n \neq pus$$

$$-\sum_{i=1}^{3}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=j+1}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{kl}a_{li}+a_{il}a_{lk}a_{kj}a_{ji})A_mA_n\Big] \quad k,l,m,n \neq pus$$

$$+\Big[\sum_{i=1}^{4}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=1}^{5}\sum_{m=l+1}^{6}\sum_{n=1}^{6}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})(a_{lm}a_{ml})A_n \quad k,l,m,n \neq pus$$

$$-\sum_{i=1}^{2}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=i+1}^{6}\sum_{m=j+1}^{6}\sum_{n=1}^{6}(a_{ij}a_{jk}a_{kl}a_{lm}a_{mi}+a_{im}a_{ml}a_{lk}a_{kj}a_{ji})A_n\Big] \quad k,l,m,n \neq pus$$

$$+\Big[\sum_{i=1}^{3}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=j+1}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{kl}a_{li}+a_{il}a_{lk}a_{kj}a_{ji})(a_{mn}a_{nm}) \quad k,l,m,n \neq pus$$

$$+\sum_{i=1}^{1}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=1}^{4}\sum_{m=l+1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})(a_{lm}a_{mn}a_{nl}+a_{ln}a_{nm}a_{ml}) \quad k,l,m,n \neq pus$$

$$-\sum_{i=1}^{1}\sum_{j=i+1}^{6}\sum_{k=i+1}^{3}\sum_{l=i+2}^{6}\sum_{m=k+1}^{5}\sum_{n=k+2}^{6}(a_{ij}a_{ji})(a_{kl}a_{lk})(a_{mn}a_{nm}) \quad k,l,m,n \neq pus$$

$$-\sum_{i=1}^{1}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=i+1}^{6}\sum_{m=i+1}^{6}\sum_{n=j+1}^{6}(a_{ij}a_{jk}a_{kl}a_{lm}a_{mn}a_{ni}+a_{in}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji})\Big] \quad k,l,m,n \neq pus \qquad (2.7)$$

'pus' stands for 'previously used subscripts', *i.e.*, in Equation 2.7, k, l, m and n take those subscripts that are other than previously used subscripts. The multinomial Equation 2.7 in symbolic form is a complete expression for the considered cylindrical grinding operation, as it considers measures of the attributes and all possible relative importance among the attributes. Mathematically, each term is a product of six different matrix elements. If this function is interpreted from a combinatorial point of view, it is found that different terms are the sets of distinct diagonal elements (A_i) and loops of off-diagonal elements of different sizes (*i.e.,* $a_{ij}a_{ji}$, $a_{ij}a_{jk}a_{ki}$, *etc.*).

The variable characteristic machinability function (VCF) contains terms arranged in (6 + 1) groupings and these groupings represent the measures of attributes and the relative importance loops. The first grouping represents the measures of the machinability attributes. The second grouping is absent, as there is no self-loop in the digraph. The third grouping contains 2-attribute relative importance loops and measures of four attributes. Each term of the fourth grouping represents a set of a 3-attribute relative importance loop, or its pair, and measures of three attributes. The fifth grouping contains two sub-groupings. Each term of the first sub-grouping is a set of two 2-attribute relative importance loops and the measures of two attributes. Each term of the second sub-grouping is a set of a 4-attribute relative importance loop, or its pair, and the measures of two attributes. The sixth grouping contains two sub-groupings. Each term of the first sub-grouping is a set of a 3-attribute relative importance loop, or its pair, and a 2-attribute relative importance loop and the measure of one attribute. Each term of the second sub-grouping is a set of 5-attribute relative importance loop, or its pair, and the measure of one attribute. The seventh grouping contains four sub-groupings. Each term of the first sub-grouping is a set of a 4-attribute relative importance loop, or its pair, and a 2-attribute relative importance loop. Each term of the second sub-grouping is a set of a 3-attribute relative importance loop, or its pair, and another 3-attribute relative importance loop, or its pair. Each term of the third sub-grouping is a set of three 2-attribute relative importance loops. Each term of the fourth sub-grouping is a set of a 6-attribute relative importance loop, or its pair. After identifying these combinatorial terms, and by associating a proper physical meaning with these, a new mathematical meaning of the multinomial is obtained.

The variable characteristic machinability function is the characteristic of the work material, and a powerful tool for machinability evaluation. However, a close look at the multinomial reveals that its various characteristic coefficients carry both positive and negative signs. The variable characteristic machinability function may not be able to provide the total objective value, when the numerical values for A_i and a_{ij} are substituted in the multinomial, because some of the information is lost by subtraction and addition operations in the determinant function. Considering these factors, the 'variable permanent machinability function (VPF)' is defined. This function is derived from a new matrix called the 'machinability permanent matrix'. The machinability permanent matrix, H, for the machinability attributes digraph (Figure 2.2) is written as Equation 2.8.

$$
H = \begin{array}{c} \text{Attributes} \\ GR \\ NF \\ TF \\ SF \\ DA \\ GT \end{array}
\begin{array}{c} \begin{array}{cccccc} GR & NF & TF & SF & DA & GT \end{array} \\
\begin{bmatrix}
A_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
a_{21} & A_2 & a_{23} & a_{24} & a_{25} & a_{26} \\
a_{31} & a_{32} & A_3 & a_{34} & a_{35} & a_{36} \\
a_{41} & a_{42} & a_{43} & A_4 & a_{45} & a_{46} \\
a_{51} & a_{52} & a_{53} & a_{54} & A_5 & a_{56} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & A_6
\end{bmatrix} \end{array} \quad (2.8)
$$

The permanent of H may be called the 'variable permanent machinability function (VPF)'.

$$
\mathrm{per}\,(H) = \prod_{i=1}^{6} A_i + \sum_{i=1}^{5} \sum_{j=i+1}^{6} \sum_{k=1}^{3} \sum_{l=k+1}^{4} \sum_{m=l+1}^{5} \sum_{\substack{n=m+1 \\ k,l,m,n \neq pus}}^{6} (a_{ij}a_{ji})A_kA_lA_mA_n
$$

$$+\sum_{i=1}^{4}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=1}^{4}\sum_{m=l+1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})A_lA_mA_n$$
$$k,l,m,n \neq \text{pus}$$

$$+[\sum_{i=1}^{3}\sum_{j=i+1}^{6}\sum_{k=i+1}^{5}\sum_{l=i+2}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{ji})\,(a_{kl}a_{lk})A_mA_n$$
$$k,l,m,n \neq \text{pus}$$

$$+\sum_{i=1}^{3}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=j+1}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{kl}a_{li}+a_{il}a_{lk}a_{kj}a_{ji})A_mA_n]$$
$$k,l,m,n \neq \text{pus}$$

$$+[\sum_{i=1}^{4}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=1}^{5}\sum_{m=l+1}^{6}\sum_{n=1}^{6}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})\,(a_{lm}a_{ml})A_n$$
$$k,l,m,n \neq \text{pus}$$

$$+\sum_{i=1}^{2}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=i+1}^{6}\sum_{m=j+1}^{6}\sum_{n=1}^{6}(a_{ij}a_{jk}a_{kl}a_{lm}a_{mi}+a_{im}a_{ml}a_{lk}a_{kj}a_{ji})A_n]$$
$$k,l,m,n \neq \text{pus}$$

$$+[\sum_{i=1}^{3}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=j+1}^{6}\sum_{m=1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{kl}a_{li}+a_{il}a_{lk}a_{kj}a_{ji})\,(a_{mn}a_{nm})$$
$$k,l,m,n \neq \text{pus}$$

$$+\sum_{i=1}^{1}\sum_{j=i+1}^{5}\sum_{k=j+1}^{6}\sum_{l=1}^{4}\sum_{m=l+1}^{5}\sum_{n=m+1}^{6}(a_{ij}a_{jk}a_{ki}+a_{ik}a_{kj}a_{ji})(a_{lm}a_{mn}a_{nl}+a_{ln}a_{nm}a_{ml})$$
$$k,l,m,n \neq \text{pus}$$

$$+\sum_{i=1}^{1}\sum_{j=i+1}^{6}\sum_{k=i+1}^{3}\sum_{l=i+2}^{6}\sum_{m=k+1}^{5}\sum_{n=k+2}^{6}(a_{ij}a_{ji})\,(a_{kl}a_{lk})\,(a_{mn}a_{nm})$$
$$k,l,m,n \neq \text{pus}$$

$$+\sum_{i=1}^{1}\sum_{j=i+1}^{5}\sum_{k=i+1}^{6}\sum_{l=i+1}^{6}\sum_{m=i+1}^{6}\sum_{n=j+1}^{6}(a_{ij}a_{jk}a_{kl}a_{lm}a_{mn}a_{ni}+a_{in}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji})]$$
$$k,l,m,n \neq \text{pus} \tag{2.9}$$

It may be noted that the only difference between the VPF, *i.e.*, per (H), and the determinant polynomial det (G), *i.e.*, VCF, is that the former does not carry negative signs with its terms, while both positive and negative signs appear in the latter. Comparing Equations 2.8 and 2.9, it is noted that each term of the grouping/ sub-grouping is the same in both cases, the only difference being in the signs of the coefficients. Both the functions are basically the same, and have the same physical meaning, except for the difference in signs. It may be mentioned that the permanent is a standard matrix function, and is used in combinatorial mathematics (Marcus and Minc, 1965; Jurkat and Ryser, 1966; Nijenhuis and Wilf, 1975).

Use of the permanent concept in machinability evaluation will help in representing machinability attributes of work materials as obtained from combinatorial consideration. Application of the permanent concept will lead to a better appreciation of machinability attributes of the work materials. Moreover, using this, no negative sign will appear in the equation, and hence no information will be lost.

The adjacency matrix, incidence matrix, characteristic matrix, *etc.*, could also be used for machinability evaluation, but these matrices have their own drawbacks. The adjacency matrix makes no provision for parallel-directed edges in both directions (*i.e.*, relative importance in both directions), and the elements of the matrix are either 0 or 1. On expanding the adjacency matrix, only some numbers can be obtained that do not reveal much physical information associated with the machinability attributes and their relative importance. The incidence matrix contains the elements either 0 or 1, and it requires more computer storage than needed for an adjacency matrix, as the number of edges is usually greater than the number of nodes. Moreover, as the incidence matrix is a non-square matrix, its further use for machinability evaluation is not possible. The characteristic matrix is not an invariant of the system, as a new matrix can be obtained by changing the labeling, but one matrix can be obtained from the other by proper permutations of rows and columns. The characteristic multinomial or characteristic function, which is nothing but the determinant of the characteristic matrix, contains both positive and negative signs, and may not be able to provide the total objective value when the numerical values for A_i and a_{ij} are substituted in the multinomial, because some of the information is lost by subtraction and addition operations in the determinant function, as explained above. Due to these reasons, researchers have used the permanent function of a matrix, which does not contain any negative terms, and thus provides the complete information without any loss (Gandhi *et al.*, 1991; Gandhi and Agrawal, 1992, 1994; Venkatasamy and Agrawal, 1996, 1997; Rao and Gandhi, 2001, 2002a, 2002b; Rao, 2004, 2006a, 2006b, 2006c, 2006d; Grover *et al.*, 2004; Rao and Padmanabhan, 2006).

In general, if there is M number of machinability attributes, and the relative importance exists among all the machinability attributes, then the machinability attributes matrix, J, for the considered machinability attributes digraph is written as Equation 2.10.

$$J = \begin{array}{c} \text{Attributes} \\ 1 \\ 2 \\ 3 \\ - \\ - \\ M \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & - & - & M \end{array} \\ \begin{bmatrix} A_1 & a_{12} & a_{13} & - & - & a_{1M} \\ a_{21} & A_2 & a_{23} & - & - & a_{2M} \\ a_{31} & a_{32} & A_3 & - & - & a_{3M} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ a_{M1} & a_{M2} & a_{M3} & - & - & A_M \end{bmatrix} \end{array} \quad (2.10)$$

The VPF for this matrix J contains factorial M (M!) number of terms. In sigma form, it is written as Equation 2.11.

$$
\begin{aligned}
\operatorname{per}(J) = {} & \prod_{i=1}^{M} A_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{ji}) A_k A_l A_m A_n A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^{M} \cdots\cdots \sum_{l=1} \sum_{\substack{M=t+1 \\ k,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji}) A_l A_m A_n A_o \ldots A_t A_M \\
& + \Big[\sum_{i=1}^{M-3} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-1} \sum_{l=i+2}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{ji})(a_{kl}a_{lk}) A_m A_n A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-3} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \sum_{l=j+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji}) A_m A_n A_o \ldots A_t A_M\Big] \\
& + \Big[\sum_{i=1}^{M-2} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^{M} \sum_{l=1}^{M-1} \sum_{m=l+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,m,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji})(a_{lm}a_{ml}) A_n A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-4} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \sum_{l=i+1}^{M} \sum_{m=j+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,m,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{ml}a_{lk}a_{kj}a_{ji}) A_n A_o \ldots A_t A_M\Big] \\
& + \Big[\Big(\sum_{i=1}^{M-3} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \sum_{l=j+1}^{M} \sum_{m=1}^{M-1} \sum_{n=m+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,m,n,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji})(a_{mn}a_{nm}) A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-5} \sum_{j=i+1}^{M-1} \sum_{k=j+1}^{M} \sum_{l=1}^{M-2} \sum_{m=l+1}^{M-1} \sum_{n=m+1}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,m,n,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{ki} + a_{ik}a_{kj}a_{ji})(a_{lm}a_{mn}a_{nl} + a_{ln}a_{nm}a_{ml}) A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-5} \sum_{j=i+1}^{M} \sum_{k=i+1}^{M-3} \sum_{l=i+2}^{M} \sum_{m=k+1}^{M-1} \sum_{n=k+2}^{M} \cdots\cdots \sum_{\substack{M=t+1 \\ k,l,m,n,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{ji})(a_{kl}a_{lk})(a_{mn}a_{nm}) A_o \ldots A_t A_M \\
& + \sum_{i=1}^{M-5} \sum_{j=i+1}^{M-1} \sum_{k=i+1}^{M} \sum_{l=i+1}^{M} \sum_{m=i+1}^{M} \sum_{n=j+1}^{M} \cdots \sum_{\substack{M=t+1 \\ k,l,m,n,\, \ldots,\, M \neq \text{pus}}}^{M} (a_{ij}a_{jk}a_{kl}a_{lm}a_{mn}a_{ni} + a_{in}a_{nm}a_{ml}a_{lk}a_{kj}a_{ji}) A_o \ldots A_t A_M\Big)\Big] \\
& + \text{----------}
\end{aligned}
\tag{2.11}
$$

'pus' stands for 'previously used subscripts', *i.e.*, in the Equation 2.11, k, l, m, n, … , M take those subscripts that are other than previously used subscripts. The VPF contains terms arranged in (M + 1) groups, and these groups represent the measures of attributes and the relative importance loops. The first group represents the measures of M attributes. The second group is absent as there is no self-loop in the digraph. The third group contains 2-attribute relative importance loops and measures of (M-2) attributes. Each term of the fourth group represents a set of a 3-attribute relative importance loop, or its pair, and measures of (M-3) attributes. The fifth group contains two sub-groups. The terms of the first sub-group is a set of two 2-attribute relative importance loops and the measures of (M-4) attributes. Each term of second sub-group is a set of a 4-attribute relative importance loop, or its pair, and the measures of (M-4) attributes. The sixth group contains two sub-groups. The terms of the first sub-group is a set of a 3-attribute relative importance loop, or its pair, and 2-attribute relative importance loop and the measures of (M-5) attributes. Each term of the second sub-group is a set of a 5-attribute relative importance loop, or its pair, and the measures of (M-5) attributes. Similarly other terms of the equation are defined. Thus, the VPF fully characterizes the considered machinability evaluation problem, as it contains all possible structural components of the attributes and their relative importance. It may be mentioned that this equation is nothing but the determinant of an M * M matrix but considering all the terms as positive.

The computer program written in C^{++} language to calculate the permanent function of a square matrix of M * M size is given in Appendix A.

2.4 Machinability Index

The machinability index is a measure of the ease with which a work material can satisfactorily be machined in a given machining operation. The machinability function defined above, *i.e.*, Equation 2.11, contains measures of attributes and their relative importance, and is hence appropriate, and can be used for evaluation of the machinability index. As the machinability function contains only positive terms, higher values of A_i and a_{ij} will result in increased value of the machinability index. To calculate this index, the required information is the values of A_i and a_{ij}.

The value of A_i should preferably be obtained from a standard or specific test. If such objective value is not available, then a ranked value judgment on a scale, *e.g.*, 0 to 1, is adapted. Table 2.1 represents the machinability attribute on a subjective scale. It holds for a given machining operation, some of the A_i will be subjective, and the others objective. Moreover, these objective values will have different units. It is therefore desirable to convert, or normalize, the objective values of A_i on the same scale as the subjective values, *i.e.*, 0 to 1. If A_i has range A_{il} and A_{iu}, the value 0 is assigned to the lowest range value A_{il} and 1 is assigned to the highest range value A_{iu}. The other, intermediate value A_{ii} of the machinability attribute is assigned a value in between 0 and 1, as per the following:

$$A_i = (A_{ii} - A_{il}) / (A_{iu} - A_{il}) \tag{2.12}$$

Equation 2.12 is applicable for general beneficial attributes only. A beneficial attribute (*e.g.*, grinding ratio) is one of which higher attribute value is more desirable for the given machining operation. A non-beneficial attribute (*e.g.*, normal force) is one of which the lower attribute value is desirable. Therefore, in the case of non-beneficial machinability attributes, the attribute value 0, on scale 0 to 1, is assigned to the highest range value A_{iu}, and the value 1 is assigned to the lower range value A_{il}. The other intermediate value A_{ii} of the machinability attribute is assigned a value in between 0 and 1, as per the following:

$$A_i = (A_{iu} - A_{ii}) / (A_{iu} - A_{il}) \tag{2.13}$$

Alternatively, the normalized value A_i can be calculated by A_{ii} / A_{iu} in the case of the beneficial attribute, and by A_{il} / A_{ii} in the case of the non-beneficial attribute. This alternative method is better than the method described by Equations 2.12 and 2.13 as it does not contain '0' as the normalized attribute value, and hence no information will be lost subsequently in machinability index calculation.

The relative importance between two attributes (*i.e.*, a_{ij}) for a given machining operation is also assigned value on the scale 0 to 1, and is arranged into six classes. The relative importance implies that an attribute 'i' is compared with another attribute 'j' in terms of relative importance for the given machining operation. The relative importance between i, j and j, i is distributed on the scale 0 to 1, and is defined similarly to Equation 2.5 in which L is taken as 1. If a_{ij} represents the relative importance of the i-th attribute over the j-th attribute, then the relative importance of the j-th attribute over the i-th attribute is evaluated using Equation 2.5. For example, if the i-th attribute is slightly more important than the j-th attribute, then $a_{ij} = 6$ and $a_{ji} = 4$.

Table 2.2 aids in assigning a_{ij} values based on the above. The relative importance is expressed in six classes, which lead to minimization of subjectivity while deciding the relative importance between machinability attributes.

Table 2.1. Value of attribute

Subjective measure of attribute	Assigned value
Exceptionally low	0.0
Extremely low	0.1
Very low	0.2
Low	0.3
Below average	0.4
Average	0.5
Above average	0.6
High	0.7
Very high	0.8
Extremely high	0.9
Exceptionally high	1.0

Table 2.2. Relative importance of attributes

Class description	Relative importance a_{ij}	$a_{ji} = 1 - a_{ij}$
Two attributes are equally important	0.5	0.5
One attribute is slightly more important over the other	0.6	0.4
One attribute is strongly more important over the other	0.7	0.3
One attribute is very strongly important over the other	0.8	0.2
One attribute is extremely important over the other	0.9	0.1
One attribute is exceptionally more important over the other	1.0	0.0

It may be mentioned that one may choose any scale, *e.g.*, 0 to 1, 0 to 5, 1 to 5, 0 to 10, 1 to 10, 1 to 11, 0 to 50, 0 to 100, 1 to 100, 1 to 110, 0 to 1000, 1 to 1000, or any other scale for A_i and a_{ij}. But the final ranking will not change, as these are relative values. It is, however, desirable to choose a lower scale for A_i and a_{ij} to obtain a manageable value of machinability index. It may be further mentioned that the scales adapted for A_i and a_{ij} can be independent of each other. Whenever the machinability index is calculated for a work material, only the diagonal elements will change, *i.e.*, (A_i), and the off-diagonal elements (a_{ij}) remain the same.

The machinability index for each material is evaluated using Equation 2.11, and substituting the value of A_i and a_{ij}. The work materials are arranged in the descending or ascending order of the machinability index to rank these for a given machining operation. These are called the machinability ranking values of the work materials for the given machining operation. The work material, for which the value of machinability index is highest, is the best choice for the machining operation considered. However, the final decision depends on factors such as cost, availability, environmental constraints, economical constraints, political constraints, *etc.* Compromise, however, should be made to select the work material having the highest value of machinability index.

The next section describes the identification and comparison of work materials.

2.5 Identification and Comparison of Work Materials

2.5.1 Identification of Work Materials

The variable permanent machinability function, *i.e.*, Equation 2.11, is useful for the identification and comparison of work materials for a given machining operation. The number of terms in each grouping of the machinability function for all the work materials for a given machining operation will be the same. However, their values will be different. This aspect is used for the purpose. Let T_{ij} represent the total value of terms of the j-th sub-grouping of i-th grouping of the machinability function. In case there is no sub-grouping, then $T_{ij} = T_i$, *i.e.*, total value of terms of the i-th grouping. The identification set for a work material for the given machining operation is:

$$/ T_1 / T_2 / T_3 / T_4 / T_{51} + T_{52} / T_{61} + T_{62} / \ldots\ldots\ldots \quad (2.14)$$

Two work materials can be compared using Equation 2.14.

2.5.2 Comparison of Work Materials

In general, two work materials are never identical from the performance (*i.e.*, machinability) point of view. If two work materials are similar, then they must be similar in performance, and *vice versa*. Comparison of two work materials is also carried out by evaluating the coefficient of similarity/dissimilarity based on the numerical value of the terms of the machinability function in its grouping/sub-grouping. The coefficient of similarity/dissimilarity lies in the range 0 – 1. If two work materials are of similar performance, then the coefficient of similarity is 1 and coefficient of dissimilarity is 0. In the same manner, if two work materials are of dissimilar performance, then the coefficient of dissimilarity is 1 and coefficient of similarity is 0. Based on performance dissimilarity, the coefficient of dissimilarity for two work materials is proposed as Equation 2.15.

$$C_d = (1/Q) \left(\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \psi_{ij} \right) \quad (2.15)$$

where, Q = maximum of $\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} T_{ij}$ and $\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} T'_{ij}$

T_{ij} and T'_{ij} denote the values of the terms for the machinability function of the two work materials under comparison, and $\psi_{ij} = \left| T_{ij} - T'_{ij} \right|$. It may be noted that the absolute difference between the values of the terms for the machinability function of the two work materials is considered for proposing C_d. The coefficient of similarity is proposed as:

$$C_s = 1 - C_d \quad (2.16)$$

Equations 2.15 and 2.16 are useful for comparing two work materials, based upon their performance in a given machining operation. The coefficients of similarity and dissimilarity, and the identification sets are also useful for work materials documentation, and for easy storage and retrieval of the work materials data for various machining operations.

Thus, graph theory and the matrix approach can be used as a decision-making method for choosing an appropriate alternative work material from amongst the given alternatives, based on machinability. The proposed method offers a general procedure that can be used for any type of decision-making problem involving any number of selection attributes and alternatives. The next section describes the general methodology of graph theory and matrix approach as a decision-making method.

2.6 Methodology of GTMA as a Decision-making Method

The main steps are given below:

Step 1: Identify the pertinent attributes and the alternatives involved in the decision-making problem under consideration. Obtain the values of the attributes (A_i) and their relative importance (a_{ij}). An objective or subjective value, or its range, may be assigned to each identified attribute as a limiting value or threshold value for its acceptance for the considered decision-making problem. An alternative with each of its selection attributes, meeting the acceptance value, may be short-listed. After short-listing the alternatives, the main task in choosing the alternative is to see how it serves the considered attributes.

Step 2:

1. Develop the attributes digraph considering the identified pertinent attributes and their relative importance. The number of nodes shall be equal to the number of attributes considered in Step 1 above. The edges and their directions will be decided upon based on the interrelations among the attributes (a_{ij}). Refer to Section 2.2 for details.
2. Develop the attributes matrix for the attributes digraph. This will be the M*M matrix with diagonal elements as A_i and off-diagonal elements as a_{ij}. Refer to Section 2.3 for details.
3. Obtain the permanent function for the attributes matrix, on the lines of Equation 2.11.
4. Substitute the values of A_i and a_{ij}, obtained in step 1, in Equation 2.11 above to evaluate the index for the short-listed alternatives.
5. Arrange the alternatives in the descending order of the index. The alternative having the highest value of index is the best choice for the decision-making problem under consideration.
6. Obtain the identification set for each alternative, using Equation 2.14. Refer to Section 2.5 for details.
7. Evaluate the coefficients of dissimilarity and similarity using Equations 2.15 and 2.16. List also the values of the coefficients for all possible combinations.
8. Document the results for future analysis/reference.

Step 3: Take a final decision, keeping practical considerations in mind. All possible constraints likely to be experienced by the user are looked into during this stage. These include constraints such as: availability or assured supply, management constraints, political constraints, economic constraints, environmental constraints, *etc.* However, compromise may be made in favor of an alternative with a higher index.

From the above, it is clear that the graph theory and matrix approach as a decision-making method is relatively new, and offers a generic, simple, easy, and convenient decision-making method that involves less computation. The method lays emphasis on decision-making methodology, gives much attention to the issues of identifying the attributes, and to associating the alternatives with the attributes, *etc.* The method enables a more critical analysis and any number of objective and subjective attributes can be considered. In the permanent procedure, even a small variation in attributes leads to a significant difference in the selection index, and hence it is easy to rank the alternatives in the descending order, with clear-cut difference in the selection index. Further, the proposed procedure not only provides the analysis of alternatives, but also enables the visualization of various attributes present and their interrelations, using graphical representation. The measures of the attributes and their relative importance are used together to rank the alternatives, and hence provides a better evaluation of the alternatives. The permanent concept fully characterizes the considered selection problem, as it contains all possible structural components of the attributes and their relative importance.

The decision-making capability of graph theory and the matrix approach can be utilized for making decisions in the manufacturing environment, and Chapters 5-30 of this book present those details.

The next chapter gives an introduction to the multiple attribute decision-making methods.

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