
Contents

Part I Modern Tools of Computational Finance

1	Basics of the Finite Difference Method	3
1.1	Finite Difference Approximation of Derivatives	3
1.1.1	Construction of Finite Differences	4
1.1.2	Higher-Order Approximations	5
1.1.3	Higher-Order Derivatives	7
1.1.4	Mixed Derivatives	8
1.2	Finite Difference Method for Solving PDEs	10
1.3	Stability Analysis	14
	References	19
2	Modern Finite Difference Approach	21
2.1	Introduction	21
2.2	Discretization of $e^{A\tau}\mathcal{L}$ on a Temporal Grid	24
2.2.1	Examples	24
2.3	Discretization of the Operator \mathcal{L} on a Spatial Grid	26
2.3.1	Uniform Grid	26
2.3.2	Nonuniform Grid	27
2.4	Requirements of Modern FD Schemes	34
2.4.1	Order of Approximation	34
2.4.2	Stability	36
2.4.3	Nonnegativity of the Solution	38
2.4.4	Complexity	40
2.5	Operator Splitting Technique	41
2.5.1	General Approach	41
2.5.2	Splitting for a Convection–Diffusion PDE	46
2.A	Appendix: Examples of Some HOC Schemes for Pricing	
	American Options	48
2.A.1	Finite Difference Scheme	50
2.A.2	Higher-Order FD Schemes in Time	52
		xv

2.A.3	L-Stable Scheme of Fifth Order in Time	54
2.A.4	Boundary Conditions for a High-Order Uniform FD Scheme...	55
	References.....	56
3	An M-Matrix Theory and FD	59
3.1	M-Matrices and Metzler Matrices	60
3.2	The Operator \mathcal{L} as a Generator	63
3.3	EM-Matrices.....	65
3.3.1	Some Useful Theorems.....	66
3.4	Mixed Derivatives and Positivity	68
3.4.1	Rate of Convergence of Picard Iterations.....	75
3.4.2	Second Order of Approximation in Space.....	76
	References.....	81
 Part II Pricing Derivatives Using Lévy Processes		
4	A Brief Introduction to Lévy Processes	85
4.1	Preliminaries.....	85
4.2	Main Definitions	86
4.3	Lévy–Khinchin Formula	89
4.4	Lévy Measure, Path, and Moments.....	92
4.5	Semimartingales and Itô’s Lemma	95
4.6	PIDE for Pricing European Options	98
	References.....	100
5	Pseudoparabolic and Fractional Equations of Option Pricing	101
5.1	Introduction.....	101
5.2	Lévy Models and Backward PIDE	105
5.3	From PIDE to PDE: A Basic Example	106
5.4	A More Sophisticated Example: The GTSP Model	109
5.4.1	Transforming a PIDE to a Pseudoparabolic Equation	112
5.5	Solution of the Pseudoparabolic Equation	115
5.5.1	Numerical Method When $\alpha \in \mathbb{I}, \alpha < 0$	116
5.5.2	$\alpha \in \mathbb{R}$: Interpolation	119
5.5.3	Numerical Examples	120
5.5.4	The Case $\alpha_R = 0$ or $\alpha_L = 0$	126
5.6	Jump Integral as a Pseudodifferential Operator.....	130
	References.....	132
6	Pseudoparabolic Equations for Various Lévy Models.....	135
6.1	Introduction.....	135
6.2	Solution of a Pure Jump Equation	138

6.3	Merton Model	141
6.4	Exponential Jumps	143
6.5	Kou Model	145
6.5.1	Numerical Experiments	146
6.6	CGMY Model	148
6.6.1	The Case $\alpha_R < 0$	151
6.6.2	The Case $0 < \alpha_R < 1$	153
6.6.3	The Case $\alpha_R = 1$	154
6.6.4	The Case $1 < \alpha_R < 2$	155
6.6.5	Approximations of \mathcal{L}_L	158
6.7	Other Numerical Experiments	159
6.8	Pure Jump Models	161
6.8.1	Normal Inverse Gaussian Model (NIG)	163
6.8.2	Generalized Hyperbolic Models	167
6.8.3	Meixner Model	175
	References	179
7	High-Order Splitting Methods for Forward PDEs and PIDEs	183
7.1	Introduction	183
7.2	LSV Model with Jumps	185
7.3	Backward and Forward FD Scheme for the Diffusion Part	187
7.3.1	Backward Scheme	187
7.3.2	Forward Scheme	188
7.4	Forward Scheme for Jumps	191
7.4.1	Details of Numerical Implementation	192
7.4.2	Parameters of the Finite Difference Scheme	196
7.5	Construction of the Backward and Forward Evolution Operators	199
	References	201

Part III 2D and 3D Cases and Correlated Jumps

8	Multidimensional Structural Default Models and Correlated Jumps	205
8.1	Introduction	205
8.2	Interbank Mutual Obligations in a Structural Default Model	208
8.3	Correlated Jumps and Structured Default Models	213
8.4	Pseudodifferential Equations and Jump Integrals	215
8.5	Construction of an FD scheme	217
8.6	Benchmark: 1D Structural Default Model with Exponential Jumps	222
8.6.1	A Generalized Fourier–Laplace Transform Approach	223
8.6.2	Inversion of the Laplace Transform: No Jumps	225

8.7	Numerical Experiments	228
8.7.1	The One-Dimensional Problem	228
8.7.2	The Two-Dimensional Problem	230
8.8	The Three-Dimensional Case	235
8.8.1	Numerical Experiments	240
	References	243
9	LSV Models with Stochastic Interest Rates and Correlated Jumps	247
9.1	Introduction	247
9.2	Model	249
9.3	Solution of the PIDE	252
9.3.1	Idiosyncratic Jumps	252
9.3.2	Common Jumps	253
9.4	Numerical Experiments	255
	References	263
10	Stochastic Skew Model	265
10.1	Introduction	265
10.2	Pricing Barrier Options under SSM	267
10.3	A Sufficient Condition for the Matrix of Second Derivatives to Be Positive Semidefinite	270
10.4	Splitting Method	272
10.4.1	Structure of the Numerical Algorithm	276
10.5	Numerical Experiments	279
10.5.1	Test 1	281
10.5.2	Test 2	283
10.5.3	Test 3	284
	References	295
	Glossary	297
	References	302
	Index	305



<http://www.springer.com/978-1-4939-6790-2>

Pricing Derivatives Under Lévy Models
Modern Finite-Difference and Pseudo-Differential
Operators Approach

Itkin, A.

2017, XX, 308 p. 64 illus., 62 illus. in color., Softcover

ISBN: 978-1-4939-6790-2

A product of Birkhäuser Basel