Preface

The development of mathematics has not followed a smooth or continuous curve, although in hindsight we may think so. As the mathematician and historian of mathematics Eric Temple Bell (1883–1960) said: "Nothing is easier … than to fit a deceptively smooth curve to the discontinuities of mathematical invention" [1, p. viii]. In fact, there have been dramatic insights and breakthroughs in mathematics throughout its history, as well as what seemed for a time to be insurmountable stumbling blocks—both leading to major shifts in the subject. And then—for the most part—there have been relatively "routine" developments, from whose importance we do not wish to detract.

Here are two "nonroutine" examples:

a. The invention (discovery?) of noneuclidean geometry—a breakthrough which was about two millennia in the making (ca 300 BC—ca 1830), and which culminated in the resolution of "the problem of the fifth postulate." This brought about a reevaluation of the nature of geometry and its relationship to the physical world and to philosophy, as well as a reconsideration of the nature of axiomatic systems. See ▶ Chapter 7.
b. The introduction, around the mid-eighteenth century, of "foreign objects", such as irrational and complex numbers, into number theory, to be followed in the late nineteenth century by the founding of a new subject—algebraic number theory. These developments paved the way for splendid achievements of modern mathematics, including, to take a familiar example, the resolution of the problem, stated in the 1630s, concerning the unsolvability in integers of Fermat's equation $x^n + y^n = z^n$, $n > 2$. The proof of unsolvability, given by Andrew Wiles in 1994, required most of the grand ideas which number theory had evolved during the twentieth century. See ▶ Chapter 6 and [4].

We aim in this book to discuss some of these major turning points—transitions, shifts, breakthroughs, discontinuities, revolutions (if you will)—in the history of mathematics, ranging from ancient Greece to the present [2, 3]. Among those which we consider are the rise of the axiomatic method (▶ Chapter 1), the wedding of algebra and geometry (▶ Chapter 4), the taming of the infinitely small and the infinitely large (▶ Chapter 5), the passage from algebra to algebras (▶ Chapter 8), and the revolutions resulting in the late nineteenth and early twentieth centuries from Cantor's creation of transfinite set theory (▶ Chapters 9 and 10). The historical origin of each turning point is discussed, as well as some of the resulting mathematics.

The above examples, and others discussed in this book, highlight the great drama inherent in the evolution of mathematics. Teachers of this grand subject will benefit from reflecting on this important aspect of it, focusing on the big ideas in its development—though not, of course, to the neglect of "routine" mathematics. They should pass on to students—at some point in their studies—at least the spirit, if not always the content, of these ideas. In particular, students should be made aware that not every fact, technique, idea, or theory is as important, and should receive as much emphasis, as every other. If this thought is not conveyed to them, our teaching will do justice neither to the students nor to the subject.

The book contains ten chapters, more or less of equal length, though not of equal difficulty. They describe only a small number of "turning points" in the history of mathematics, and we have appended an 11th chapter which suggests "Some Further Turning Points" to pursue. Each chapter contains about ten "problems and projects", most of which are intended to deepen or extend the material in the text. At the end of each chapter, there is a substantial list of references, whose aim is to elaborate, enhance, and exemplify the material in the text proper. Finally, the book has a comprehensive index.

This book can be read by a person with some mathematical background who is interested in getting a nontraditional look at aspects of the history of mathematics. It can also be used in history-of-mathematics courses, especially those centered around the important idea of "turning points." Moreover, since appreciation of the historical development of the central ideas of mathematics enhances, we strongly believe, one's understanding and appreciation of the subject, this book can serve as a text in a capstone course for mathematics majors, a course that will integrate and "humanize" at least some of their knowledge of mathematics by placing it in historical perspective. In any such course our book will probably need to be supplemented by additional technical material; a teacher will know best when and how to use this "extra" material in his or her particular classroom setting. Teachers are resourceful and will likely use the book in ways we have not anticipated.

One of the reviewers of our book said the following: "I see the value of the manuscript in its role as a 'starter' to ignite love for the history of maths and to give a first overview. It is a good 'teaser.'" We hope that readers' experiences will justify this assessment.

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