

# Chapter 2

## Paradigms and Challenges

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**Abstract** The purpose of this introductory part is to present an overall view of what MCDA is today. In Sect. 2.1, I will attempt to bring answers to questions such as: what is it reasonable to expect from MCDA? Why decision aiding is more often multicriteria than monocriterion? What are the main limitations to objectivity? Sect. 2.2 will be devoted to a presentation of the conceptual architecture that constitutes the main keys for analyzing and structuring problem situations. Decision aiding cannot and must not be envisaged jointly with a hypothesis of perfect knowledge. Different ways for apprehending the various sources of imperfect knowledge will be introduced in Sect. 2.3. A robustness analysis is necessary in most cases. The crucial question of how can we take into account all criteria comprehensively in order to compare potential actions between them will be tackled in Sect. 2.4. In this introductory part, I will only present a general framework for positioning the main operational approaches that exist today. In Sect. 2.5, I will discuss some more philosophical aspects of MCDA. For providing some aid in a decision context, we have to choose among different paths which one seems to be the most appropriate, or how to combine some of them: the path of realism which leads to the quest for a discussion for discovering, the axiomatic path which is often associated with the quest of norms for prescribing, or the path of constructivism which goes hand in hand with the quest of working hypothesis for recommending.

**Keywords** Multiple criteria decision aiding • Imperfect knowledge • Aggregation procedures

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## **2.1 What Are the Expectations That Multicriteria Decision Aiding (MCDA) Responds To?**

The purpose of this introductory chapter is to present an overview of what MCDA is today. Since the 60s, this discipline has produced, and it still produces, a great number of theoretical as well as applied papers and books. The major part of them will be presented in the following chapters of this book. It is important at the outset to understand their specific contributions are in terms of enlarging the operations research field and, more generally, to bringing light to decision making contexts. That is why I shall begin this chapter by considering the three following questions: what is reasonable to expect from MCDA? Why is decision aiding is more often multicriteria than monocriterion? What are the main limitations to objectivity which must be taken into account? The next section will be devoted to a brief presentation of three basic concepts which can be viewed as initial and fundamental keys for analyzing and structuring problem situations. In practice, it is very important to draw attention to questions such as: what is the quality of the information which can be obtained? What is the meaning of the data which are available or can be elaborated? In Sect. 2.3, I shall examine how the existing models and procedures take into account various types of answers to such questions which refer to a given problem's real world context.

Another difficulty in an MCDA context comes from the fact that comparisons between potential actions must be made comprehensively, with respect to all criteria. Various aggregation techniques which will be described in detail throughout the successive chapters of this book have been proposed and used in order to overcome this kind of difficulty. In Sect. 2.4, I shall present a general framework for positioning the main operational approaches in which these aggregation techniques come into play. Some more general philosophical considerations will complete this introductory chapter.

### ***2.1.1 What Is Reasonable to Expect from Decision Aiding (DA)?***

Decision aiding can be defined (see [63]) as follows: Decision aiding is the activity of people using models (not necessarily completely formalized ones) to help to obtain elements of responses to the questions asked by a stakeholder in a decision process. These elements work towards clarifying the decision and usually towards recommending, or simply favoring, a behavior that will increase the consistency between the evolution of the process and this stakeholder's objectives and value system. In this definition, the word "recommending" is used to draw attention to the fact that both analyst and decision maker are aware that the decision maker is completely free to behave as he or she sees fit after the recommendation is made.

This term is increasingly used in DA to replace “prescription”. The latter is, in many cases, inappropriate (see [36, 60]) for designating what a team of analysts accompanying a decision making process might achieve.

Thus defined, DA aims at establishing, on recognized scientific bases, with reference to working hypotheses, formulations of propositions (elements of responses to questions, a presentation of satisfying solutions or possible compromises, ...) which are then submitted to the judgment of a decision maker and/or the various actors involved in the decision making process. According to the case, DA can thus reasonably contribute to:

- Analyzing the decision making context by identifying the actors, the various possibilities of action, their consequences, and the stakes;
- Organizing and/or structuring how the decision making process will unfold, to increase consistency between the values underlying the objectives and goals and the final decision;
- Getting the actors to cooperate, by proposing keys to better mutual understanding and a framework conducive to debate;
- Drawing up recommendations based on results from models and computational procedures designed within the framework of a working hypothesis;
- Participating in the process to legitimate the final decision.

For a deeper understanding of the bases reviewed above, the reader can refer to [13, 14, 20, 21, 42, 50, 61, 72, 74].

### ***2.1.2 Why Is DA More Often Multicriteria than Monocriterion?***

Even when DA is provided for a single decision maker, it is rare for her or him to have in mind a single clear criterion. Thus, when DA takes place in a multi-actor decision making process, it is even rarer for there to be a priori a single, well-defined criterion deemed acceptable by all actors to guide the process. This process is often not very rational. Each actor plays a more or less well defined role which gives priority to her or his own objectives and value system.

In both cases, it is necessary to take into consideration various points of view dealing with, for example, finance, human resources, environmental aspects, delays, security, quality, ethics, ... By considering each pertinent point of view separately, independently from the others, it is generally possible to arrive at a clear and common elicitation of preferences regarding the single point of view considered. This naturally leads to associating a specific criterion to each pertinent point of view. Each of these criteria is used to evaluate any potential action on an appropriate qualitative or quantitative scale. In most cases, there is no obvious and acceptable arithmetic rule which can keep account of these heterogeneous scales by substituting a single scale based on a common unit for each of them (see Sect. 2.4 below).

Such a scale, bringing a common unit into play, must be introduced a priori when we want to avoid a multicriteria approach, i.e., when we prefer to choose what is called a *monocriterion* approach. This choice, in many decision making contexts, might:

- lead to wrongly neglecting certain aspects of realism;
- facilitate the setting up of equivalencies, the fictitious nature of which remains invisible;
- tend to present features of one particular value system as objective.

On the contrary, a multicriteria approach contributes to avoiding such dangers by:

- delimiting a broad spectrum of points of view liable to structure the decision process with regard to the actors involved;
- constructing a family of criteria which preserves, for each of them, without any fictitious conversion, the original concrete meaning of the corresponding evaluations;
- facilitating debate on the respective role (weight, veto, aspiration level, rejection level, . . .) that each criterion might be called upon to play during the decision aiding process.

Additional considerations about relative advantages on monocriterion and multicriteria approaches can be found in [12, 15, 22, 58, 63].

### 2.1.3 *Can MCDA Be Always Totally Objective?*

In many cases, those who claim to shed light objectively on a decision in fact take a stand—consciously or unconsciously—for an a priori position or for a prevailing hypothesis which they then seek to justify. Arguments for making a decision are thus put forward more in the spirit of advocacy than in that of an objective search (see [4, 34]).

In what follows, we will only consider situations in which DA is motivated by a strong desire for objectivity. Even in such situations, it is important to be sensitive to the existence of some fundamental limitations on objectivity. Their origins lie in the following facts:

- (a) The borderline between what is and what is not feasible is often fuzzy in real decision making contexts. Moreover, this borderline is frequently modified in the light of what is found through the study itself.
- (b) Even in cases for which DA is provided for a well-defined decision maker, his or her preferences very seldom seem well-shaped. In and among areas of firm convictions lie hazy zones of uncertainty, half held belief, or indeed conflicts and contradictions. Such sources of ambiguity or arbitrariness concerning preferences which are to be elicited and modeled are even more present when

the decision maker (entity for whom or in the name of whom decision aiding is provided for) is a mythical person, or when decision aiding is provided in a multicriteria context. We have to admit, therefore, that the study itself contributes to eliminating questioning, solving conflicts, transforming contradictions and destabilizing certain convictions. Any interaction and questioning between the analyst and the decision maker, or any actors involved into the decision making process, may have some an unpredictable or imperceptible effect.

- (c) Much of the data is imprecise, uncertain or ill defined. Making data say more than it actually means is a risk. Moreover, some data only reflects the value system of a given individual.
- (d) It is impossible to say that a decision is good or bad only by referring to a mathematical model. The organizational, pedagogical and cultural aspects of the entire decision-making process leading to a particular decision also contribute to its quality and success.

Rather than dismissing or canceling the subjectivity which results from the limitations of objectivity described above, decision aiding must make an objective place for it. (For a pedagogical overview of MCDA approaches, see [60, 61, 66, 70, 72].)

## 2.2 Three Basic Concepts

From the beginning to the end of work in MCDA, three concepts usually play a fundamental role for analyzing and structuring the decision aiding process in close connection with the decision process itself. The presentation of these concepts in the three following sub-sections is obviously succinct. It nevertheless aims to draw attention to some important features.

### 2.2.1 *Alternative, and More Generally, Potential Action*

By *potential action*, we usually designate that which constitutes the object of the decision, or that which decision aiding is directed towards. The concept of *action* does not a priori incorporate any notion of feasibility, or possible implementation. An action is qualified as potential when it is deemed possible to implement it, or simply when it deserves some interest within the decision aiding process.

The concept of *alternative* corresponds to the particular case in which modeling is such that two distinct potential actions can in no way be conjointly put into operation. This mutual exclusion comes from a way of modeling which in a comprehensive way tackles that which is the object of the decision, or that towards which DA is directed. Many authors implicitly suppose that potential actions are, by definition, mutually exclusive. Nevertheless, such an hypothesis is in no way compulsory. In many real world decision aiding contexts, it can be more appropriate to adopt another way of modeling such that several potential actions can be implemented conjointly (see examples in [57, 63]).

In all cases,  $A$  will denote the set of potential actions considered at a given stage of the DA process. This set is not necessarily stable, i.e., it can evolve throughout the decision aiding process. Such an evolution may come from the study's environment, but also from the study itself. The study may shed light on some aspects of the problem, which could lead to revising some of the data and then, possibly, to modifying the borderline between what is and what is not feasible.

By  $a$ , we will denote any potential action or alternative. When the number of actions is finite ( $|A| = m$ ) we shall let:

$$A = \{a_1, a_2, \dots, a_m\}.$$

When modeling of actions can be done by referring to some variables  $x_1, x_2, \dots$  it is possible to write:

$$a = (x_1, x_2, \dots).$$

In such cases,  $A$  is generally defined by a set of analytic constraints which characterize the borderline between what is feasible and is not feasible. Multiobjective mathematical programming constitutes an important particular case of this type of modeling (see [26] and Part VI).

In another type of modeling, the value of each variable  $x_i$  ( $i = 1, 2, \dots, n$ ) designates a possible score on an appropriate scale  $X_i$  built for evaluating actions according to a specified point of view or criterion. In such cases,  $A$  can be viewed as a subset of the Cartesian product  $X = \prod_{i=1}^n X_i$ . This type of modeling is commonly used in multiattribute utility theory (MAUT) (see Part IV). Let us observe that this type of modeling necessitates some precautions: since each potential action is identified with the  $n$  components of its evaluation, it loses all concrete identity; in particular, two actions having the same evaluations  $x_1, \dots, x_n$  are no longer distinguishable.

More details and illustrations of the concepts and ways of modeling presented above could be found in [63, Chap. 5], [90, Chap. 1], and [38].

### 2.2.2 *Criterion and Family of Criteria*

The reader, who is not yet familiar with some of the terms used in this subsection, will find more precise definitions in [66, Chap. 1, Appendix 1, Glossary].

Let us remember that a criterion  $g$  is a tool constructed for evaluating and comparing potential actions according to a point of view which must be (as far as it is possible) well-defined. This evaluation must take into account, for each action  $a$ , all the pertinent effects or attributes linked to the point of view considered. It is denoted by  $g(a)$  and called the *performance* of  $a$  according to this criterion.

Frequently,  $g(a)$  is a real number, but in all cases, it is necessary to define explicitly the set  $X_g$  of all the possible evaluations to which this criterion can lead.

For allowing comparisons, it should be possible to define a complete order  $<_g$  on  $X_g$ : ( $<_g, X_g$ ) is called the *scale* of criterion  $g$ . To be accepted by all stakeholders, a criterion should not bring into play, in a way which might be determinant, any aspects reflecting a value system that some of these stakeholders would find necessary to reject. This implies in particular that the direction to which the preferences increase along the scale (and more generally the complete order  $<_g$ ) is not open to contest.

Elements  $x \in X_g$  are called *degrees* or *scores* of the scale. Each degree can be characterized by a number, a verbal statement or a pictogram. When in order to compare two actions according to criterion  $g$  we compare the two degrees used for evaluating their respective performances, it is important to analyze the concrete meaning in terms of preferences covered by such degrees. This leads to distinguishing various types of scales:

- (a) *Purely ordinal scale*: Scale such that the gap between two degrees does not have a clear meaning in terms of difference preferences; this is the case with:
- a *verbal scale* when nothing allows us to state that the pairs of consecutive degrees reflect equal preference differences all along the scale;
  - a *numerical scale* when nothing allows us to state that a given difference  $y$  between two degrees reflects an invariant preference difference when we move the pair of degrees considered along the scale.

This type of scale is often called a qualitative scale.

- (b) *Quantitative scale*: Numerical scale whose degrees are defined by referring to a clear, concrete defined quantity in a way that it gives meaning, on the one hand, to the absence of quantity (degree 0), and on the other hand, to the existence of a unit allowing us to interpret each degree as the addition of a given number (integer or fractional) of such units. In such conditions, the ratio between two degrees can receive a meaning which does not depend on the two particular degrees considered; this is another way of defining quantitative scales which are also called *cardinal* or *ratio scales*.
- (c) *Other types*: In MCDA, we do not always work with scales belonging to one of the above two extreme types (especially interval scales). The most interesting intermediate types are presented in [43, Sect. 2] and [66].

In MCDA, it is essential to know which type of scale we are working with to be sure of using its degrees in a meaningful way. According to the type of scales considered, certain kinds of reasoning and arithmetical operations are significant in terms of preference (see Chap. 3).

Moreover, the use of the degrees in a significant way must take the following fact into account: the difference between two degrees that are sufficiently close together may be non significant for justifying an indisputable preference in favor of one of the two actions. This stems from the procedure used to position the two actions on the scale considered. This procedure can appear insufficiently precise (with regard to the complexity of the reality in question), or insufficiently reliable (with regard to uncertainty concerning the future) for founding such an indisputable preference on such a small difference. I will come back to this subject in the next section.

In most cases, the first step of DA consists of building  $n$  criteria with  $n > 1$  (see Sect. 2.1.2 above). They constitute what we call the *family  $F$  of criteria*. In order to be sure that  $F$  is able to play its role in the DA process correctly, i.e., in laying the foundations for convictions, communicating concerning the latter, debating and orienting the process towards the decision, and in contributing in some cases to legitimating this decision, it is necessary to verify that:

- what is apprehended by each criterion is sufficiently intelligible for each stakeholder;
- each criterion is perceived as a relevant instrument for comparing potential actions on the scale associated with this criterion, without prejudging its relative importance (importance that may vary considerably from one stakeholder to the next);
- the  $n$  criteria considered all together satisfy some logical requirements (exhaustiveness, cohesiveness, and non redundancy) which insure coherence of the family (for more details, see [66, Chap. 1, Appendix 1, Glossary], [73]).

It is important to observe that none of the above requirements implies that the  $n$  criteria of  $F$  must be independent. The concept of independence is very complex, and if dependence is desirable, it is necessary to specify what type of independence is needed. Multicriteria analysis has led to important distinctions between structural independence, preferential independence, and utility independence (see [39, 56], [63, Chap. 10], and [73, Chap. 2]).

Additional developments concerning this basic concept of criterion can be found in [2, 3, 5, 6, 8, 69].

### 2.2.3 *Problematic as a Way in Which DA May Be Envisaged*

The word “problematic” is used here in the sense indicated by the heading. Other expressions such as “statement”, “problem formulation” or “problem type” have been used as substitutes, but in my view, they are inappropriate and may lead to misunderstanding.

Let us underline first that DA must not be envisaged solely in the perspective of solving a problem of choice. In some cases, DA consists only of elaborating an appropriate set  $A$  of potential actions, building a suitable family  $F$  of criteria, and determining, for all or some  $a \in A$ , their performances sometimes completed by additional information (possible values for discriminating thresholds, aspiration and/or rejection levels, weights, . . .). For designating this manner of conceiving of DA’s aim without seeking to elaborate any prescription, or recommendation, we use the term *description problematic* often coded  $P.\delta$ .

In MCDA, the word *problematic* refers to the way in which DA is envisaged. This means that the problematic deals with answers to questions such as the following: in what terms should we pose the problem?, what type of results should we try to obtain?, how does the analyst see himself fitting into the decision process to aid in



arriving at these results?, what kind of procedure seems the most appropriate for guiding his investigation? In addition to  $P.\delta$ , three other reference problematics are currently used in practice. They can be briefly described as follows (for more details, see [54], [63, Chap. 6]):

- The *choice problematic* ( $P.\alpha$ ): The aid is oriented towards and relies on a *selection* of a small number (as small as possible) of “good” actions in such a way that a single alternative may finally be chosen; this does not mean that the selection is necessarily oriented towards the determination of one or all the actions of  $A$  which can be regarded as optimum; the selection procedure can also, more modestly, be based on comparisons between actions so as to justify the elimination of the greatest number of them, the subset  $N$  of those actions which are selected (which can be viewed as a first choice) containing all the most satisfying actions, which remain non-comparable between one another.
- The *sorting problematic* ( $P.\beta$ ): The aid is oriented towards, and relies on an *assignment* of each action to the one category deemed the most appropriate for receiving it among a family of predefined categories. Categories are designed a priori to receive actions which will be or might be processed in the same way during the following step. For instance, a family of four categories can be based on a comprehensive appreciation leading to distinguishing between: actions for which implementation (1) is fully justified, (2) could be advised after only minor modifications, (3) can only be advised after major modifications, (4) is unadvisable. Let us observe that categories are not necessarily ordered as it is the case in the above example.
- The *ranking problematic* ( $P.\gamma$ ): The aid is oriented towards and relies on a *partial* or *complete* ordering (pre-order) on  $A$  that may be considered as an appropriate instrument for comparing actions pairwise. This pre-order is the result of a *classifying* procedure allowing us to put together in classes actions which can be judged as indifferent, and to rank these classes (some of them may remain non-comparable).

The four problematics described above are not the only possible ones (see [10, 12]). Whatever the problematic adopted, the result arrived at by exploiting a given set of data through a single procedure is (except under unusual conditions) not sufficient for founding a prescription or a recommendation (see Sect. 2.4 below).

### 2.3 How to Take into Account Imperfect Knowledge and Ill-Determination?

DA cannot be correctly provided without trying to analyze and to take into account reasons and factors which can be responsible for contingency, arbitrariness, and ignorance in the way the problem is addressed and procedures implemented. In addition to their subjective characteristics, these reasons and factors may take on

various forms whose presence and/or importance greatly depends on the decision making context considered. Their presence comes essentially from three sources (for more details, see [17, 59, 70, 71]):

- *Source  $\alpha$  ( $S.\alpha$ )*: The imprecise, uncertain and, more generally, poorly understood or ill-defined nature of certain specific features or factual quantities or qualities present in the problem.
- *Source  $\beta$  ( $S.\beta$ )*: the conditions for implementing the decision taken; these will be influenced by:
  - The state of the context at the time the decision is implemented if it is a once-and-for-all decision;
  - The successive states of the context if the decision is sequential.
- *Source  $\gamma$  ( $S.\gamma$ )*: the fuzzy or incomplete, sometimes unstable and easily influenced character of the system or systems of values to be taken into account; these values involve, in particular, the system and most often the systems of preferences which should prevail in order to evaluate the feasibility and relative interest of diverse potential actions, by considering the conditions envisaged for implementing these actions.

Once the problem is formulated, and during the entire decision aiding process, special attention should be paid to the three sources. Their examination must highlight what is imprecise, uncertain, unstable or ill determined. This can lead, for instance:

- starting from  $S.\alpha$ , to delimiting a domain of reasonable instantiation values for various data and parameters;
- from  $S.\beta$ , to building a set of scenarios describing different possible future contexts;
- from  $S.\gamma$ , to eliciting a set of weight vectors; for this purpose, it is important to remember that it makes no sense and is theoretically incorrect to specify measures of relative importance for the criteria without considering the nature of the overall evaluation model which will be used, i.e., without having defined the type of mathematical aggregation rules (see next section) which allow us to derive comprehensive preferences.

The DA process must clearly take into account all the results of this study. To do so, many approaches (formalisms, models, methods, ...) have been proposed. A panorama of such approaches can be found in Chap. 12, and in [41, 82]. These approaches rest upon various concepts, tools and theories; the main ones are:

- probability theory mainly used in MAUT (see Chap. 9), but also used in many other approaches, particularly for building criteria when uncertainty can be characterized by a probabilistic distribution;
- possibility theory [23, 24];
- multi-valued logic (see Chap. 4, [86]);

- concept of discriminating thresholds and quasi or pseudo-criterion (see Chap. 3, and for more details, [37, 76–78, 93]) mainly used in outranking methods (see Chap. 5).
- concept of fuzzy binary relations [19, 25, 28, 29, 46, 55];
- rough sets theory (see Chap. 13);
- ordinal regression for MCDA (see [33]).

Regardless of the tools used for taking into account imperfect knowledge and ill determination, the analyst must seek for obtaining robust solutions and/or robust conclusions.

Solutions and conclusions are qualified as robust if their design takes account of the existence of vague approximations and zones of ignorance, thus enabling the DM to forestall any impacts that he or she may deem regrettable, for instance when the goal of at is effectively reached is far short of expectations, or when the properties the DM wanted to maintain are deteriorated.

For decision aiding to be truly useful, it must provide the decision maker with the kinds of answers that will provide enough information enabling the decision maker (according to his/her subjectivity) to reach a decision in a situation where there are two conflicting risks:

- being poorly protected in the event of very poor performance relative to the impacts that he/she deems regrettable;
- being in a position that leads the DM to abandon any hope of good, or even very good, performance.

For more details, regarding tools that the analyst can use for obtaining robust solutions and/or robust conclusions, and regarding also more generally robustness concerns, the reader can see [1, 27, 35, 40, 52, 53, 64, 65, 68–71, 84, 91, 92].

Let us note that the analyst can make use of sensitivity analysis in the framework of robustness concern. In [64, 65], the reader could find some comments on links and differences between robustness analysis and sensitivity analysis (see also [84]).

## 2.4 An Operational Point of View

As soon as more than one criterion comes into play, a crucial question arises: how can we take into account all criteria comprehensively in order to compare potential actions between them? Let us consider two potential actions  $a$  and  $b$  characterized by their respective performances on the  $n$  criteria considered. More often,  $a$  will be better than  $b$  for some of the criteria, and  $b$  better than  $a$  for others. In such cases, in comparing  $a$  and  $b$ , on what basis can we found a *comprehensive judgment*, i.e., taking into account, in a comprehensive way, the  $n$  performances of  $a$  and the  $n$  performances of  $b$ . This problem is usually called the *aggregation problem*. In many of the chapters in this book, the reader will find a wide variety of solutions to this fundamental problem. In the present introductory chapter, I shall present only

a general framework for positioning the main operational approaches provided for DA today (for more details on what the operational approach concept covers, see, [63, Chap. 11] and [72]).

### 2.4.1 About Multicriteria Aggregation Procedures

Let us suppose that two potential actions characterized by their  $n$  performances are shown to a respondent (for instance the DM) to elicit her or his preference, so that he or she can say:

- “I prefer the first to the second or vice versa”;
- “I am indifferent about the two”;
- “I am unable to compare these two actions”.

What we call the preference system for the respondent refers to the results of such comparisons, as they could be stated (for all pairs of actions) expressed in terms of:

*Preference, indifference or incomparability*

In certain cases, there may be information qualifying the intensity of preference.

Overall, assuming there is a rational, rigorous and stable (implicit or explicit) procedure, capable of defining the totality of the respondent’s preference system in the respondent’s mind, even before the outset of the decision-aiding process, is not very realistic.

What is usually called *preference model* is, on the contrary, a well-defined model, which produces results allowing the comparison of any pair of potential actions in crisp or fuzzy terms denoting preference, indifference, or incomparability.

Above all, a preference model in DA must be a tool for delving deeper into, exploring, interpreting, debating, and even arguing the subject.

It is not necessary (I even think it unrealistic) for the analyst to attempt to design the model so that it becomes a representation, albeit simplified, of what is actually happening in the respondent’s mind when he/she compares two actions.

The most frequently used decision aiding operational approaches make use of a preference model that is based on a mathematical procedure called the *multicriteria aggregation procedure* (MCAP). Let:

$$g_1(a), \dots, g_n(a) \quad \text{and} \quad g_1(b), \dots, g_n(b)$$

denote the performances by which two potential actions  $a$  and  $b$  are characterized.

A MCAP is a procedure that produces a result allowing for the comparison of both actions in a comprehensive way.

A MCAP brings into play:

- A *logic of aggregation* that takes account of the conditions under which compensation between bad and good performances are accepted or refused.

- *Various inter-criterion and technical parameters* (e.g., weights, scaling constants, vetoes, aspiration levels, rejection levels). The specific role that each criterion can play with respect to the others is defined by the numerical values assigned to these parameters

Outside of the logic on which the MCAP is based, the inter-criterion and technical parameters have no meaning: they have no real existence in the mind of the decision maker. Thus, no true value that has to be approximated as well as possible exists. Rather, the analyst must strive to assign an appropriate value to each parameter so that the resulting preference model can become a useful tool for decision aiding.

For more details on the above considerations, see [16, 44, 62, 73, 75, 85].

Methods which are based on a mathematically explicit MCAP come under one of two types of operational approaches usually designated by the expressions approach based on a *synthesizing criterion* and approach based on a *synthesizing preference relational system*.

### 2.4.2 Approach Based on a Synthesizing Criterion

This approach is the most traditional. It can be characterized as follows: Formal rules taking account of the  $n$  performances  $g_1(a), \dots, g_n(a)$  are defined to assign a well-defined degree (most often a numerical value  $v(a)$ ) to each  $a \in A$  on an appropriate scale. The comparison between two actions is determined by their respective positions on this scale.

The way the aggregation problem is addressed in this approach leads to defining a complete pre-order on the set  $A$ . Most often, the formal rules consist in mathematical formulas that lead to an explicit definition of a unique criterion synthesizing the  $n$  criteria:

$$v(a) = V[g_1(a), \dots, g_n(a)].$$

This is the case with MAVT, MAUT, SMART, TOPSIS, MACBETH, AHP, and so on (see Chaps. 8–10). The complete preorder on  $A$  can also be obtained by the use of a set of formal rules without any mathematically explicit expression of the synthesizing criterion, which remains implicit (see [7, 8]). In any case, this approach does not allow any incomparability.

Building a synthesizing criterion using such a multicriteria approach is not equivalent to a monocriterion approach. The dangers of the monocriterion approach have been presented above (see Sect. 2.1.2). Nevertheless, even if a multicriteria approach based on a synthesizing criterion contributes to reducing these dangers, it forces us to introduce a common scale (monetary scale, utility scale, ...) on which performances of each of the  $n$  criteria have to be evaluated. Moreover, with this approach, imperfect knowledge and ill-determination (*cf.* Sect. 2.3 above) can be

taken into account essentially through probability distribution, fuzzy numbers, or in some cases through rough set theory but never through preference or indifference thresholds.

### 2.4.3 *The Operational Approach Based on a Synthesizing Preference Relational System*

As is the first, this second operational approach is based on a mathematically explicit MCAP. A major difference with the preceding approach comes from the fact that here the MCAP does not work on each potential action  $a$  separately from the others, but it successively compares  $a$  to each of the other  $b \in A$ .

In other words, the aggregation problem is no longer addressed in terms of defining a complete preorder on  $A$ , it is now addressed in terms of pairwise comparisons so as to design a synthesizing preference relational system. Taking into account the  $n$  performances of  $a$  and the  $n$  performances of  $b$ , the role of the MCAP is to give an answer to the question: what is the preference relation which can be validated between  $a$  and  $b$ , and in some cases with what degree of credibility? Mathematical rules, which lead to answering this question, are based on:

- various inter-criteria parameters, as in the first approach; but also, unlike the first approach, on discriminating thresholds (see Sect. 2.3 above) and veto thresholds;
- a logic of aggregation which easily allows us to take into account (and this is much more difficult with the approach based on a synthesizing criterion), on one hand, some limitations to compensation, and on the other, no quantitative performances.

The synthesizing preference relational system can be reduced to a single binary relation, which can be crisp or fuzzy. But it can also bring into play more than one binary relation. In all instances, the advantages of this second type of MCAP relative to the first cause certain difficulties to arise when we consider the operational approach based on such an MCAP. These difficulties stem from the fact that:

- pairwise comparisons can cause some intransitivities to appear;
- incomparability can be the most appropriate conclusion for comparing certain pairs  $(a, b)$ ;
- consequently, a synthesizing relational preference system is not a tool which is immediately usable for elaborating a recommendation.

For these reasons, this second operational approach necessitates completing the MCAP by a second procedure called *exploitation procedure*. This procedure is conditioned by the problematic considered (see above Sect. 2.3).

This second operational approach has led to various methods, most of which are covered by the label of *outranking methods* (mainly PROMETHEE and ELECTRE methods). The second part of this book is devoted to them. Other works related to this approach are presented in Part IV.

#### ***2.4.4 About Other Operational Approaches***

All the operational approaches which are based on a mathematically explicit MCAP are not exactly in accordance with one of the two preceding approaches. Regarding this subject, the reader can refer to [9, 18, 32, 45, 81].

Finally, let us mention the existence of operational approaches which are not based on a mathematically explicit MCAP. In such approaches the analyst takes account of the DM's preference system by interacting with him or her.

This type of operational approaches (called interactive) is based on a formal procedure organizing a dynamic sequence of questions that the DM (or his/her representative) must answer.

The respondent is asked questions involving actions characterized by their performance in the criteria space. Each question only refers to a small area in the criteria space. The answers are given in terms of preference, indifference or incomparability.

The procedure is designed so that, through trial-and-error, the respondent progresses to the point of reaching one or several satisfying actions.

For more details on this kind of approach, see Chap. 22, [30, 47, 49], [73, Chap. 7], [83, 87–90, 94, 96].

In any case, whatever the operational approach considered, there is a possible confusion which should be avoided. Except under very unusual conditions, the results arrived at by treating a set of data through any appropriate procedure should not be confused with a well founded scientific recommendation. Repeated calculations using different but equally realistic versions of the DA problem (sets of data, scenarios, . . .) are generally necessary to elaborate a recommendation on the basis of robust conclusions stemming from the multiple results thus obtained. The statement of the proposals which make up the recommendation should be submitted to the assessment and discernment of the decision maker and/or the actors involved in the DA process (see [36, 48, 65–68, 84]).

## **2.5 Conclusion**

The final objective of MCDA is, of course, to help managers to make “better” decisions. But what is the meaning of better? This meaning depends, in part, on the process by which the decision is made and implemented. This, combined with limitations on objectivity described above (see Sect. 2.1.3), shows that we cannot hope to prove scientifically, in a decision making context, that a given decision is the best one. So, one cannot consider that, in every situation, the right selection, the right assignment, the right ranking—all of which decision aiding strives to discover, or to approach as closely as possible—exists somewhere. This implies that the concepts, models and procedures presented in this book must not be viewed as being conceived from the perspective of discovering, with a better or a worst

good approximation, a pre-existing truth which could be universally accepted. They have to be seen as keys capable of opening doors giving access to answers and/or expectations as described in Sect. 2.1.1.

Designed and implemented in this manner, decision aiding based on appropriate concepts, models and procedures can play a significant and beneficial role in guiding a decision making process.

Above all, the purpose of MCDA is to enable us to enhance the degree of conformity and consistency between the evolving decision-making process and the value system and the objectives of the people involved in the process. For this purpose, concepts, tools and procedures must be designed to help us make our way on a road fraught with ambiguity, uncertainty and countless crossroads.

To achieve this goal, three non exclusive paths can be envisaged:

- the path of realism which leads to the quest for a description for discovering;
- the axiomatic path which is often associated with the quest for norms for prescribing;
- the path of constructivism which goes hand in hand with the quest for a working hypothesis for recommending.

(for more details on each of these paths, see [60]). In a DA process, it is important, when following one or a combination of such paths, to shed light on:

- those aspects of reality which give meaning, value and order to facts;
- the influence exerted upon this reality by observing it, organizing it, provoking within it certain forms of debate, or even having certain tools placed there.

Personally, I consider that the path of realism can only play a role in producing certain descriptions of physical, institutional, socio-economic, financial or psychological systems which form the decision making context. Insofar as such descriptions are produced by other disciplines than DA strictly speaking, the contribution of DA comes essentially, in my opinion, from the constructivism path taken in conjunction with (observing certain precautions) the axiomatic path. Interesting developments and other points of view can be found in [11, 13, 31, 33, 38, 42, 51, 79, 80, 95, 97–99].

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