Preface

Linear canonical transforms (LCTs) are a three-parameter family of linear integral transformations, which have a quadratic-phase kernel. For this reason, they have also been called quadratic-phase transforms or quadratic-phase systems (as well as other names). They are unitary transforms that correspond to linear, area-preserving distortions in phase space, a fact which underlies certain invariance properties. Combinations of LCTs are again LCTs. The family includes important operations or transforms such as chirp multiplication, chirp convolution (Fresnel transforms), fractional Fourier transforms, and of course the ordinary Fourier transform, as special cases. Arbitrary LCTs can be written as combinations of these simpler transforms. This leads to fast algorithms for approximately calculating LCTs, much as the ordinary Fourier transform can be calculated with fast algorithms.

LCTs have been rediscovered many times in different contexts, a fact we consider evidence of their ubiquity. Their significance in optics was recognized at least as early as the 1970s. Later, interest in the fractional Fourier transform during the 1990s led to renewed interest in LCTs from new perspectives.

This book deals with LCTs primarily from the perspective of signal and image processing, and optical information processing. Part I presents the mathematical theory of LCTs in the style of signal theory and analysis, as well as the foundations of how LCTs are related to optical systems. Part II deals with issues of degrees of freedom, sampling, numerical implementation, and fast algorithms. Part III is a survey of various applications. No attempt is made here to discuss canonical transformations as they appear in classical Hamiltonian mechanics and symplectomorphisms. These are well-established subjects in physics. However, we note that it is quite possible that a crossover of concepts and techniques between the different approaches to these transforms may be quite fruitful, and we hope this book may contribute to that end, in addition to being useful for its primary audience in the areas of signal processing and optics.

Overview

The opening chapters cover a range of fundamental topics. We start with a discussion of the twin discovery of LCTs in two different areas: paraxial optics and nuclear physics. This provides a fascinating window into more than 40 years of parallel scientific progress. This chapter also contrasts two parallel efforts to define a discrete counterpart to the LCTs—one based on group theory, the other on sampling theory. Chapter 2 provides a self-contained introduction to LCTs and their properties, so the reader who just wishes to dip into the subject may be advised to start here. Chapter 3 discusses the eigenfunctions of the LCTs. These functions are important for analyzing the characteristics of the transforms. Since the LCT can be used to describe wave propagation, they also play important roles in the analysis of self-imaging and resonance phenomena. Chapter 4 continues the theme of key properties of the transform with a discussion of the uncertainty principle. Heisenberg's principle provides a lower bound on the spread of signal energy in the time and frequency domains, and there has been a good deal of work on extending this work to LCTs. The first part of the book is rounded out by Chaps. 5 and 6 that discuss the relationship of LCTs to optics. These chapters deal with both how LCTs can be used to model and analyze optical systems and how LCTs can be optically implemented.

The modern age is digital, whether we are working with spatial light modulators and digital cameras or processing the resulting signals with a computer. In the second part of the book, we have a number of chapters on topics relevant to discrete signals and their processing. Chapter 7 discusses a modern interpretation of the relationship between sampling and information content of signals. Chapter 8 discusses sampling theory and builds up to a discrete transform. Periodic gratings have long been known to produce discrete signals at certain distances, and in Chap. 9 this Talbot effect and hence the relationship between discrete and periodic signals are examined. Just as the fast Fourier transform is key to the utility of conventional spectral analysis, corresponding fast algorithms are critical to our ability to use LCTs in a range of applications. Chapter 10 examines how to calculate the LCT numerically in a fast and accurate fashion.

In the final part of the book, we turn to a series of chapters in which linear canonical transforms are used in a variety of optical applications. One of the fundamental problems in optics is that our detectors are insensitive to phase. Chapter 11 discusses phase retrieval from the field intensity captured in planes separated by systems that can be described using LCTs, focusing particularly on non-iterative techniques. Another way to find the full wave field (amplitude and phase) is to record a hologram, a topic which experienced a revival in the past 20 years due to the rapid improvement in digital cameras. Digital holography is the focus of Chap. 12. Chapter 13 examines optical encryption by means of random phase encoding in multiple planes separated by systems that may be described using LCTs. Coherent light reflected from a rough surface develops laser speckle, a characteristic of the wave field, which may be beneficial in metrology or a nuisance in display technologies. Chapter 14 examines complex-parametered LCTs as a means of modelling speckle fields propagating through apertured optical systems. With Chap. 15, the book is rounded off with a discussion of the use of LCTs in quantum optics.

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