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Inside Interesting Integrals

A Collection of Sneaky Tricks, Sly Substitutions, and Numerous Other Stupendously Clever, Awesomely Wicked, and Devilishly Seductive Maneuvers for Computing Nearly 200 Perplexing Definite Integrals From Physics, Engineering, and Mathematics (Plus 60 Challenge Problems with Complete, Detailed Solutions)
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