Preface

This book grew out of lectures given by the first author at Queen's University during 2006 and lectures by the second author at the Chennai Mathematical Institute during 2008. These constitute the first 18 chapters of the book intended to be an introductory course aimed at senior undergraduates and first-year graduate students. The primary goal of these chapters is to give a quick introduction to some of the beautiful theorems about transcendental numbers. We begin with some earliest transcendence theorems and thereafter move to the Schneider–Lang theorem. This requires some rudimentary background knowledge in complex analysis, more precisely the connection between the growth of an analytic function and the distribution of its zeros. Since this constitutes an essential ingredient of many of the transcendence results, we discuss the relevant features in Chap. 5. We also require some familiarity with elementary algebraic number theory. But we have tried our best to recall the required notions as and when we require them. Having proved the Schneider–Lang theorem, we introduce some of the accessible and essential features of the theory of elliptic curves and elliptic functions so that the reader can appreciate the beauty of the primary applications. Thus Chaps. 1–18 essentially comprise the material for an introductory course.

The second part of the book, namely Chaps. 19–28, are additional topics requiring more maturity. They grew out of seminar lectures given by both authors at Queen's University and the Institute of Mathematical Sciences in Chennai, India. A major part of these chapters treats the theorem of Baker on linear independence of linear forms in logarithms of algebraic numbers. We present a proof of Baker's theorem following the works of Bertrand and Masser. Thereafter, we briefly describe some of the applications of Baker's theorem, for instance to the Gauss class number problem for imaginary quadratic fields. In Chap. 21, we discuss Schanuel's conjecture which is one of the central conjectures in this subject. We devote this chapter to derive a number of consequences of this conjecture.

From Chaps. 22 to 26, we concentrate on some recent applications of Baker's theorem to the transcendence of special values of L-functions. These L-functions arise from various arithmetic and analytic contexts. To begin, we give a detailed treatment of the result of Baker, Birch and Wirsing. This is perhaps the first instance when transcendental techniques are employed to address the delicate issue of non-vanishing of a Dirichlet series at special points. In Chap. 25, we specialise to questions of linear independence of special values of Dirichlet L-functions. In Chap. 26, we consider analogous questions for class group L-functions.

In Chap. 27, we focus on applications of Schneider's theorem and Nesterenko's theorem to special values of modular forms. These modular forms are a rich source of transcendental functions and hence potential candidates to generate transcendental numbers (hopefully "new"). Of course, one can ask about the possibility of applying transcendence tools not just to modular forms but also to their L-functions. But this will force us to embark upon a different journey which we do not undertake here.

Finally, the last chapter is intended to give the reader an introduction to the emerging theory of periods and multiple zeta values. This is not meant to be an exhaustive account, but rather an invitation to the reader to take up further study of these elusive objects. This chapter is essentially self-contained and can be read independent of the other chapters.

To summarise, we hope that the first part of this book would be suitable for undergraduates and graduate students as well as non-experts to gain entry into the arcane topic of transcendental numbers. The last ten chapters would be of interest to the researchers keen in pursuing the interrelation between special values of L-functions and transcendence.

To facilitate practical mastery, we have included in each chapter basic exercises that would be helpful to the beginning student.

http://www.springer.com/978-1-4939-0831-8

Transcendental Numbers
Murty, M.R.; Rath, P.
2014, XIV, 217 p., Softcover
ISBN: 978-1-4939-0831-8