Preface

The volume "Analytic Number Theory, Approximation Theory, and Special Functions" consists of 35 articles written by eminent scientists from the international mathematical community, who present both research and survey works. The volume is dedicated to professor Hari M. Srivastava in honor of his outstanding work in mathematics.

Professor Hari Mohan Srivastava was born on July 5, 1940, at Karon in District Ballia of the province of Uttar Pradesh in India. He studied at the University of Allahabad, India, where he obtained his B.Sc. in 1957 and M.Sc. in 1959. He received his Ph.D. from the Jodhpur University (now Jai Narain Vyas University), India, in 1965. He begun his university-level teaching career at the age of 19.

H. M. Srivastava joined the Faculty of Mathematics and Statistics of the University of Victoria in Canada as associate professor in 1969 and then as a full professor in 1974. He is an Emeritus Professor at the University of Victoria since 2006.

Professor Srivastava has held several visiting positions in universities of the USA, Canada, the UK, and many other countries.

He has published 21 books and monographs and edited volumes by well-known international publishers, as well as over 1,000 scientific research journal articles in pure and applied mathematical analysis He has served or currently is an active member of the editorial board of several international journals in mathematics. It is worth mentioning that he has published jointly with more than 385 scientists, including mathematicians, statisticians, physicists, and astrophysicists, from several parts of the world. Professor Srivastava has supervised several graduate students in numerous universities towards their master's and Ph.D. degrees.

Professor Srivastava has been bestowed several awards including most recently the NSERC 25-Year Award by the University of Victoria, Canada (2004), the Nishiwaki Prize, Japan (2004), Doctor Honoris Causa from the Chung Yuan Christian University, Chung-Li, Taiwan, Republic of China (2006), and Doctor Honoris Causa from the University of Alba Iulia, Romania (2007).

The name of professor Srivastava has been associated with several mathematical terms, including Carlitz–Srivastava polynomials, Srivastava–Panda multivariable

H-function, Srivastava–Agarwal basic (or q-) generating function, Srivastava–Buschman polynomials, Chan–Chyan–Srivastava polynomials, Srivastava–Wright operators, Choi–Srivastava methods in Analytic Number Theory, and Wu–Srivastava inequality for higher transcendental functions.

In this volume dedicated to professor Srivastava an attempt has been made to discuss essential developments in mathematical research in a variety of problems, most of which have occupied the interest of researchers for long stretches of time. Some of the characteristic features of this volume can be summarized as follows:

– Presents mathematical results and open problems in a simple and self-contained manner.
– Contains new results in rapidly progressing areas of research.
– Provides an overview of old and new results, methods, and theories towards the solution of long-standing problems in a wide scientific field.

The book consists of the following five parts:

1. Analytic Number Theory, Combinatorics, and Special Sequences of Numbers and Polynomials
2. Analytic Inequalities and Applications
3. Approximation of Functions and Quadratures
4. Orthogonality, Transformations, and Applications
5. Special and Complex Functions and Applications

Part I consists of nine contributions. A. Ivić presents a survey with a detailed discussion of power moments of the Riemann zeta function $\zeta(s)$, when s lies on the "critical line" $\Re s = 1/2$. It includes early results, the mean square and mean fourth power, higher moments, conditional results, and some open problems. M. Hassani gives some explicit upper and lower bounds for γ_n, where $0 < \gamma_1 < \gamma_2 < \gamma_3 < \cdots$ are consecutive ordinates of nontrivial zeros $\rho = \beta + i\gamma$ of the Riemann zeta function, including the asymptotic relation $\gamma_n \log^2 n - 2\pi n \log n \sim 2\pi n \log\log n$ as $n \to \infty$. Y. Ihara and K. Matsumoto prove an unconditional basic result related to the value-distributions of $\{(L'/L)(s, \chi)\}_\chi$ and of $\{(\zeta'/\zeta)(s + i\tau)\}_\tau$, where χ runs over Dirichlet characters with prime conductors and τ runs over $\mathbb{R}$. J. Choi presents a survey on recent developments and applications of the simple and multiple gamma functions Γ_n, including results on multiple Hurwitz zeta functions and generalized Goldbach–Euler series. A. A. Bytsenko and E. Elizalde consider a partition function of hyperbolic three-geometry and associated Hilbert schemes. In particular, the role of (Selberg-type) Ruelle spectral functions of hyperbolic geometry for the calculation of partition functions and associated q-series is discussed. Y. Simsek considers families of twisted Bernoulli numbers and polynomials and their applications. Several relationships between Bernoulli functions, Euler functions, some arithmetic sums, Dedekind sums, Hardy Berndt sums, DC-sums, trigonometric sums, and Hurwitz zeta function are given. A. K. Agarwal and M. Rana discuss a combinatorial interpretation of a generalized basic series. Namely, using a bijection between the Bender–Knuth matrices and the n-color partitions established by Agarwal (ARS Combinatoria **61**, 97–117, 2001), they

extend a recent result to a 3-way infinite family of combinatorial identities. A. Sofo proves some identities for reciprocal binomial coefficients, and M. Merca shows that the q-Stirling numbers can be expressed in terms of the q-binomial coefficients and vice versa.

Part II is dedicated to analytic inequalities and several applications. Ibrahim and Dragomir give a survey of some recent results for the celebrated Cauchy–Bunyakovsky–Schwarz inequality for functions defined by power series with nonnegative coefficients. G. D. Anderson, M. Vuorinen, and X. Zhang provide a survey of recent results in special functions of classical analysis and geometric function theory, in particular the circular and hyperbolic functions, the gamma function, the elliptic integrals, the Gaussian hypergeometric function, power series, and mean values. M. Merkle gives a collection of some selected facts about the completely monotone (*CM*) functions that can be found in books and papers devoted to different areas of mathematics. In particular, he emphasizes the role of representation of a *CM* function as the Laplace transform of a measure and also presents and discusses a little known connection with log-convexity. S. Abramovich considers results on superquadracity, especially those related to Jensen, Jensen–Steffensen, and Hardy's inequalities. S. Ding and Y. Xing present an up-to-date account of the advances made in the study of L^p theory of Green's operator applied to differential forms, including L^p-estimates, Lipschitz and *BMO* norm inequalities, as well as inequalities with $L^p (\log L)^\alpha$ norms. B. Yang uses methods of weight coefficients and techniques of real analysis to derive a multidimensional discrete Hilbert-type inequality with a best possible constant factor. F. Qi, Q.-M. Luo, and B.-N. Guo establish sufficient and necessary conditions such that the function $(e^{\alpha t} - e^{\beta t})/(e^{\lambda t} - e^{\mu t})$ is monotonic, logarithmic convex, logarithmic concave, 3-log-convex, and 3-log-concave on $\mathbb{R}$. P. Cerone obtains approximation and bounds of the Gini mean difference and provides a review of recent developments in the area. Finally, M. A. Noor considers the parametric nonconvex variational inequalities and parametric nonconvex Wiener–Hopf equations. Using the projection technique, he establishes the equivalence between them.

Approximation of functions and quadratures is treated in Part III. N. K. Govil and V. Gupta discuss Stancu-type generalization of operators introduced by Srivastava and Gupta (Math. Comput. Modelling **37**, 1307–1315, 2003). M. Mursaleen and S. A. Mohiuddine prove the Korovkin-type approximation theorem for functions of two variables, using the notion of statistical summability $(C, 1, 1)$, recently introduced by Moricz (J. Math. Anal. Appl. **286**, 340–350, 2003). In 1961, Baker, Gammel, and Wills formulated their famous conjecture that if a function f is meromorphic in the unit ball and analytic at 0, then a subsequence of its diagonal Padé approximants converges uniformly in compact subsets to f. This conjecture was disproved in 2001, but it generated a number of related unresolved conjectures. D. S. Lubinsky reviews their status. A. R. Hayotov, G. V. Milovanović, and K. M. Shadimetov construct the optimal quadrature formulas in the sense of Sard, as well as interpolation splines minimizing the semi-norm in the space $K_2(P_2)$, where $K_2(P_2)$ is a space of functions φ which φ' is absolutely continuous and φ'' belongs to $L_2(0, 1)$ and $\int_0^1 (\varphi''(x) + \omega^2 \varphi(x))^2 dx < \infty$. Finally, a survey on some specific

nonstandard methods for numerical integration of highly oscillating functions, mainly based on some contour integration methods and applications of some kinds of Gaussian quadratures, including complex oscillatory weights, is presented by G. V. Milovanović and M. P. Stanić.

In Part IV, C. Ferreira, J. L. López, and E. Pérez Sinusía study asymptotic reductions between the Wilson polynomials and the lower-level polynomials of the Askey scheme, and K. Castillo, L. Garza, and F. Marcellán analyze a perturbation of a nontrivial probability measure $d\mu$ supported on an infinite subset on the real line, which consists of the addition of a time-dependent mass point. A. F. Loureiro and S. Yakubovich consider special cases of Boas–Buck-type polynomial sequences and analyze some examples of generalized hypergeometric-type polynomials. P. W. Karlsson gives an analysis of Goursat's hypergeometric transformations, and A. Kılıçman considers partial differential equations (PDE) with convolution term and proposes a new method for solving PDE. Finally, using the fixed point method, C. Park proves the Hyers–Ulam stability of the orthogonally additive–additive functional equation.

Special functions and complex functions with several applications are presented in Part V. Á. Baricz, P. L. Butzer, and T. K. Pogány have provided a generalization of the complete Butzer–Flocke–Hauss (BFH) Ω-function in a natural way by using two approaches and obtain several interesting properties. Á. Baricz and T. K. Pogány consider properties of the product of modified Bessel functions and establish discrete Chebyshev-type inequalities for sequences of modified Bessel functions of the first and second kind. S. Porwal and D. Breaz investigate the mapping properties of an integral operator involving Bessel functions of the first kind on a subclass of analytic univalent functions. V. V. Mityushev introduces and uses the Poincaré α-series ($\alpha \in \mathbb{R}^n$) for classical Schottky groups in order to solve Riemann–Hilbert problems for n-connected circular domains. He also gives a fast algorithm for the computation of Poincaré series for disks that are close to each other. N. E. Cho considers inclusion properties for certain classes of meromorphic multivalent functions. He introduces several new subclasses of meromorphic multivalent functions and investigates various inclusion properties of these subclasses. Finally, I. Lahiri and A. Banerjee discuss the influence of Gross-problem on the set sharing of entire and meromorphic functions. It is hope that the book will be particularly useful to researchers and graduate students in mathematics, physics, and other computational and applied sciences.

Finally, we wish to express our deepest appreciation to all the mathematicians from the international mathematical community, who contributed their papers for publication in this volume dedicated to Hari M. Srivastava, as well as to the referees for their careful reading of the manuscripts. A thank also goes to professor Marija Stanić (University of Kragujevac, Serbia), for her help during the technical preparation of the manuscript. Last but not least, we are very thankful to Springer for its generous support for the publication of this volume.

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