Chapter 2
Assessment of Transportation Performance:
A Network Structure

Ming-Miin Yu and Li-Hsueh Chen

Abstract Performance measurement is a popular activity of organizations in the transportation sector. Various studies on the performance of transportation organizations with the utilization of data envelopment analysis models have been common. However, based on the unstorable characteristics of transportation services, conventional data envelopment analysis models are not suitable, and then network data envelopment analysis models are proposed. This chapter is dedicated to describe the network operational structure of transportation organizations and the relative network data envelopment analysis model. In order to be closer to real operational situations, four operational characteristics, which are route-based performance evaluation, environmental factors, undesirable outputs, multi-activity framework, are discussed and incorporated into the network data envelopment analysis model, respectively.

Keywords Transportation • Network DEA • Route-based performance evaluation • Environmental factors • Undesirable outputs • Multi-activity framework
2.1 Introduction

The performance measures of the delivery of the primary services of transportation organizations have been the traditional subject of whatever performance studies were made in the past. There are many ways to scrutinize performance in the transportation sector. In early periods, the usually used measures of performance are ratio indicators, such as vehicle hours per employee, vehicle kilometers per active vehicle, passengers per revenue vehicle hour, and revenue vehicle hours per dollar operating cost (Mackie and Nash 1982; Lee 1989; Fielding 1992). Ratio analysis typically involves the use of a number of performance indicators which consider only a subset of inputs used by a decision-making unit (DMU) and sometimes only a subset of outputs. In single-input single-output contexts, a partial measure of performance is a meaningful, easy to use measure of performance.

However, this is not the case where multiple inputs and/or outputs are involved (Hensher 1992). To the extent that a DMU may increase performance with respect to one input at the expense of reducing the performance of other inputs, the difficulty stems from the fact that each partial measure of performance reflects only one input and one output level, and it is also difficult to portray the overall gains/losses in performance (Thanassoulis et al. 1996). Furthermore, it could provide a misleading indication of overall performance when considered in isolation. In recent years, various studies on the theoretical and empirical measurement of performance in the transportation sector with the utilization of the data envelopment analysis (DEA) model have been generated by researchers. There is a large stream of literature on a single-stage DEA. In a regularly studied situation within this context, it is assumed that a transportation organization’s inputs are transformed from a single operation process into their final outputs. Some of those studies focus on production efficiency (e.g., Tulkens 1993; Oben 1994; Kerstens 1996; Nolan et al. 2001; Cowie 2002; Karlaftis 2003; Graham 2008), while some are interested in the measurement of operational efficiency (e.g., Tofallis 1997; Cowie and Asenova 1999; Adler and Golany 2001; Boame 2004; Yu 2007), and others invested both in a single model (e.g., Viton 1998; McMullen and Noh 2007).

While evaluating the performance in the transportation sector, it is worth noting that, unlike the production and consumption processes of the manufacturing sector, a transportation service cannot be stored, and therefore the output consumed (the final output), such as passenger-km, may vary considerably from the output produced (the intermediate output), such as vehicle-km, in a transportation system. Specifically, the consumed services occur concurrently with the produced services. If the produced output is not consumed, it is lost (Tomazinis 1975) (e.g., if a bus runs during the period at half capacity, the bus system cannot store the other half of its inventory (Karlaftis 2004)). This perishability of the produced services and the fact that only a proportion of the produced services are actually consumed is often neglected in performance measures of transportation organizations (Borger et al. 2002). If these unique unstorable characteristics of transportation services
are justified, then it is vitally important to obtain valid estimates of performance of transportation organizations that include them. Hence, an adequate performance measurement for a transportation organization should consider the network structure that services are produced and consumed concurrently, and interactions in this structure.

In addition, other operational issues, such as route-based performance evaluation, environmental factors, undesirable outputs, multi-activity framework, etc., will also impact the assessment of performance in the transportation sector. In order to construct a more reasonable performance measurement for transportation organizations, these four issues mentioned above will also be explored and incorporated into the network structure.

The remainder of this chapter is organized as follows. In the second section, we describe the transportation performance; the specifications of the network DEA model in transportation appear in the third section; in the fourth section, we explore other issues for transportation applications; the fifth section provides three examples; and concluding remarks are given in the final section.

2.2 Transportation Performance

Since transportation services cannot be stored, the output consumption may be substantially different from the output production. For instance, an airline uses aircraft, employees, and fuel to provide service products, flights, and seat-miles, which are produced and sold to passengers concurrently. Once the service products are not consumed (that is, seats are not sold), they are wasted. So service products function as intermediate inputs (the intermediate outputs in the production process) and used internally in consumption process. To accommodate unstoreable characteristics, Fielding et al. (1985) introduced three performance indicators for a transit system: cost efficiency, service effectiveness, and cost effectiveness. They defined cost efficiency as the ratio of outputs to inputs, service effectiveness as the ratio of consumption to outputs, and cost effectiveness as the ratio of consumption to inputs. Hence, cost effectiveness is the integration of cost efficiency and service effectiveness measures. This transit performance concept is portrayed in Fig. 2.1.

However, the definition of “cost efficiency” used by Fielding et al. (1985) could cause some confusion, because, in the economic theory and DEA context, cost efficiency is defined by the product of technical efficiency and allocative efficiency. If the input factor prices are not available, it would be more appropriate to use the terms of production efficiency, service effectiveness and operational effectiveness instead of cost efficiency, service effectiveness and cost effectiveness, respectively.

Most studies about performance measurement used separate models to measure the interrelated processes, and evaluate sub-process efficiency independently (Chu et al. 1992; Viton 1998; Nolan et al. 2002; Lan and Lin 2003, 2005; Karlaftis 2004; Chiou and Chen 2006). They distinguished the production process from the consumption process, from which one can gain more insight into the firms’ operations.
However, since outputs are consumed concurrently with their production, measuring the performance of the transportation organizations using two models is likely to be unreasonable. In addition, these models mentioned above assume different production technologies without interacting each other and cannot deal formally with intermediate products. It ignores effects of the inter-relationship between sub-processes and then yields an incomplete version of operational performance measurement (Sheth et al. 2007). In any realistic situation, the transportation sector has a feature of unstorable series, which means that intermediate products are presented both in production and consumption processes. Usually, the feature within a transportation organization’s operation should take into account all the complex and interrelated flows between these two processes. Assuming that transportation frequencies are given by a particular schedule for serving their passengers, inefficiency occurs when the actual level of input consumption, for a given level of provided capacity (e.g., frequencies and/or seat-miles), exceeds the optimal level of input requirement as specified by the production function. This observed production inefficiency, however, does not mean service ineffectiveness, since a transportation organization could search for better ways to maximize its ridership to raise its service effectiveness. In other words, service effectiveness may be seen as how a transportation organization efficiently transforms capacity provided to ridership in the consumption process. In making performance comparisons, they must take into account the multistage representation of the technology, otherwise the performance measures would reflect not merely differences in efficiency but also the relative efficiency by which individual processes and the whole operation system are operating. In addition, for a transportation organization which is obliged to provide a stable timetable in a given time period, it implies that if the predetermined timetable is violated, then the violation may result in the waste/decrease of input costs and the loss/gain of consumed outputs with respect to some referenced efficient transportation organizations since the changes in the timetable may increase or reduce cost and/or passengers may feel comfortable/uncomfortable.
using it. Hence, a transportation organization possesses a network structure including a set of interdependent technologies in the whole operational process. By separating the effects of the complex and interrelated technologies, we can explore if the source of observed performance differs. Identification of such sources is essential to the implementation of operational policies and management strategies designed to improve performance. Therefore, it seems more realistic and reasonable to use a unified network model to estimate the performance of transportation organizations. This performance evaluation in network structure is shown in Fig. 2.2.

2.3 Network Data Envelopment Analysis in Transportation

Traditionally, DEA has treated each DMU as a “black box” by considering only the inputs consumed and final outputs produced by this “black box” (Färe and Grosskopf 2000). However, in most real situations, the DMUs may perform several different functions and can also be separated into different components in series. In such situations, some components play important roles in producing outputs through the use of intermediate outputs obtained from their previous components. In this case, the conventional DEA model cannot impose restrictions on the inter-relationships among intermediate products when measuring the DMU’s overall performance together with that of its components. If this “black box” consists of a set of sub-units which are connected serially, then such an approach provides no insights regarding the inter-relationships among the components’ inefficiencies and cannot provide specific process guidance to DMU managers to help them improve the DMU’s efficiency.
In transportation organizations, the operational process in a DMU usually contains two processes in which some outputs produced in former process are used as inputs in a latter process. Färe and Grosskopf (1996, 2000) proposed a network DEA model for measuring performance of those DMUs with multiple processes. The object of this proposed method was to provide a solution to deal with a weakness, which treats the operational process as a “black box”, in the conventional DEA model. In order to represent production and consumption processes in a transportation organization’s operating technology, a network DEA model based on the directional distance function proposed by Luenberger (1992) is constructed as below.

We denote inputs for the production process by $x^P \in R_+^A$. Here inputs $x$ are employed in the production process ($P$) to produce intermediate outputs, $m^{(P, C)} \in R_+^B$, where $(P, C)$ represents the intermediate output of $P$ flowing into the consumption process ($C$). Intermediate outputs from the production process act as intermediate inputs to the consumption process. The intermediate products are produced in production and consumed in consumption processes concurrently, resulting in final outputs $y^C \in R_+^D$. To formulate a network DEA model, we need to introduce intensity variables $z^P_j$ and $z^C_j$, $j = 1, \ldots, J$, for production and consumption processes of each DMU $j$, respectively. Hence, the network DEA model has a production possibility set and a consumption possibility set, $A^P$ and $A^C$, which can be defined as follows:

$$A^P = \left\{ \left( x^P, m^{(P, C)} \right) : m^{(P, C)} \text{ can be produced from } x^P \right\}.$$  

(2.1)

$$A^C = \left\{ \left( m^{(P, C)}, y^C \right) : y^C \text{ can be produced from } m^{(P, C)} \right\}.$$  

(2.2)

If $A^P$ is the smallest set which satisfies the convexity, the constant returns to scale, free disposability, and minimum extrapolation postulates (Tsai and Mar Molinero 2002), subject to the condition that each input–output observations $\left( x^P, m^{(P, C)} \right) \in A^P$, then the input set in the production process, $P^P(m^{(P, C)})$, for each $m^{(P, C)}$ can be defined as $P^P(m^{(P, C)}) = \left\{ x^P : (x^P, m^{(P, C)}) \in A^P \right\}$. Similarly, the output set in the consumption process, $P^C(m^{(P, C)})$, for each $m^{(P, C)}$ can be defined as $P^C(m^{(P, C)}) = \left\{ y^C : (m^{(P, C)}, y^C) \in A^C \right\}$.

An overall network operational possibility set in terms of the input and output set is defined as follows:
\[ T^N \left\{ (x^P, m^{(P, C)}, y^C) : \sum_{j=1}^J z_j^P x_{aj}^P \leq x_a^P, \ a = 1, \ldots, A, \right. \]
\[ \sum_{j=1}^J z_j^P m_{bj}^{(P, C)} \geq m_b^{(P, C)}, \ b = 1, \ldots, B, \]
\[ \sum_{j=1}^J z_j^C y_{dj}^C \geq y_d^C, \ d = 1, \ldots, D, \]  
\[ \sum_{j=1}^J z_j^C m_{bj}^{(P, C)} \leq m_b^{(P, C)}, \ b = 1, \ldots, B, \]
\[ z_j^P \geq 0, \ z_j^C \geq 0, \ j = 1, \ldots, J \}

We introduce two functions: \( \beta_k^P(x^P, m^{(P, C)}) \) and \( \beta_k^C(m^{(P, C)}, y^C) \), which provide measures of how efficient a firm \( k \) is in production process and consumption process, respectively. The efficiency score of each part could be calculated as follows:

\[ D(x^P, m^{(P, C)}) = \beta_k^P(x^P, m^{(P, C)}) = \max \{ \beta_k^P : (1 - \beta_k^P)x^P \in P^P(m^{(P, C)}), \ \beta_k^P \geq 0 \}, \]  
\[ D(m^{(P, C)}, y^C) = \beta_k^C(m^{(P, C)}, y^C) = \max \{ \beta_k^C : (1 + \beta_k^C)y^C \in P^C(m^{(P, C)}), \ \beta_k^C \geq 0 \}. \]

For an illustration of the network performance measurement, we choose to evaluate firm \( k \) relative to the network technology (2.3) by means of a directional distance function. The objective function of the network model is taken as the form:

\[ \text{Max} \ \beta^k = w_k^P \beta_k^P + w_k^C \beta_k^C, \]  

where \( \beta_k^P \) and \( \beta_k^C \) are the performance scores of production and consumption processes, respectively; \( w_k^P \) and \( w_k^C \) are positive numbers which represent the relative importance of these processes respectively, and \( w_k^P + w_k^C = 1 \).

In the network DEA model, we can identify these two sub-technologies. Hence, (2.6) is subject to these following constraints:

The production process consists of

\[ \sum_{j=1}^J z_j^P x_{aj}^P \leq (1 - \beta_k^P)x_{ak}^P, \ a = 1, \ldots, A, \]  

(2.6.1)
\[
\sum_{j=1}^{J} z_j^p m_{bj}^{(p,c)} \geq m_{bk}^{(p,c)}, \quad b = 1, \ldots, B,
\]

(2.6.2)

\[
\beta_k^p \geq 0, \quad z_j^p \geq 0, \quad j = 1, \ldots, J.
\]

(2.6.3)

The consumption process is given by,

\[
\sum_{j=1}^{J} z_j^c m_{bj}^{(p,c)} \leq m_{bk}^{(p,c)}, \quad b = 1, \ldots, B,
\]

(2.6.4)

\[
\sum_{j=1}^{J} z_j^c y_{dj}^c \geq (1 + \beta_k^c) y_{dk}^c, \quad d = 1, \ldots, D,
\]

(2.6.5)

\[
\beta_k^c \geq 0, \quad z_j^c \geq 0, \quad j = 1, \ldots, J,
\]

(2.6.6)

The network directional distance function in (2.6) is zero if and only if the transportation organization’s production process is technically efficient, and its consumption process is simultaneously serviced effectively. However, its value is greater than zero if and only if the transportation organization is technically inefficient in at least one of the two processes. The network DEA model has several attractive features compared to the conventional one. In particular, it provides individual managers with specific information regarding the sources of inefficiency within their DMUs.

### 2.4 Other Issues for Transportation Applications

In order to resemble the real operational characteristics of transportation organizations, besides the network structure of transportation services, other operational issues must be considered. In this section, we mention four issues that transportation organizations often confront, but not all are included. These four issues are:

- Route-based performance evaluation
- Environmental factors
- Undesirable outputs
- Multi-activity framework
2.4.1 Route-Based Performance Evaluation

Most studies measure the performance of transportation organizations from a whole-company perspective. They treat individual firms as individual DMUs. However, different transportation organizations may operate different routes, such as operational routes vary among different shipping companies or airlines, even in the same country. A whole-company perspective may lead to a different operational benchmark. In order to avoid heterogeneity, some studies have used the route-based performance evaluation to substitute for the company-based performance evaluation (Chiou and Chen 2006; Lin et al. 2010; Yu and Chen 2011; Chiou et al. 2012).

2.4.2 Environmental Factors

Since firms run in different environments, their operation outcome will be affected by the environmental factors that they face. If environmental factors are ignored, performance measures would be seriously biased against firms that generate a misleading performance evaluation profile. For example, the population at the airport would affect its outputs. Higher utilization of an airport does not guarantee more efficient management, since some of the effects may be caused by higher population around the airport. It is appropriate to adjust for environmental conditions before credible results could be presented. Although, environmental factors usually cannot be controlled by the administrator, they may influence how we measure efficiency in the use of capacity. Standard DEA assumes that the assessed units are operated in similar operational environments (Golany and Roll 1989). Often the assumption of homogeneous environments is violated. Hence, it is essential that, if the model is to be used in this manner, factors which establish the operational environments need to be incorporated into the model. A number of different approaches have been developed to overcome this weakness (Syrrjanen 2004). In this section, the approach introduced by Banker and Morey (1986) is described.

According to Banker and Morey (1986), a DMU should be compared with its peers under a similar operational environment. In order to capture the effects of environmental factors on the production and consumption process, we include the environmental variables as non-discretionary inputs by adding the following constraints into the network DEA model illustrated in Sect. 2.3:

\[
\sum_{j=1}^{J} z_{j}^{P} e_{fj}^{P} \leq e_{fj}^{P}, \quad f = 1, \ldots, F, \quad (2.6.7)
\]

\[
\sum_{j=1}^{J} z_{j}^{C} e_{gj}^{C} \leq e_{gj}^{C}, \quad g = 1, \ldots, G, \quad (2.6.8)
\]

where \( e_{f}^{P} \in R_{+}^{F} \) and \( e_{g}^{C} \in R_{+}^{G} \) represent environmental factors \( f \) and \( g \) associated only with the production and consumption processes of firm \( j \), respectively.
2.4.3 Undesirable Outputs

Since undesirable outputs are often produced together with desirable outputs, the more complete performance evaluation of a transportation organization should consider the trade-off between the utilization of desirable output and the control of undesirable output. For example, aircraft noise has the greatest influence on the community surrounding the airport (Morrell and Lu 2000). If the effect of aircraft noise is ignored, the rank of airport performance in capacity utilization may be severely distorted. Thus, when the efficiency of airports is evaluated, the provision of desirable outputs like the number of passengers should be credited, but the provision of undesirable outputs like noise pollution should be penalized.

Following Färe et al. (1989) and Chung et al. (1997), we use a directional distance function to construct the efficiency measurement model that simultaneously credits a decrease in undesirable outputs and an increase in desirable outputs. Let $u^C \in R^H_+$ denote an undesirable output vector in the consumption process. Since, in the consumption process, DMUs seek to increase the desirable outputs and decrease the undesirable outputs simultaneously, the objective function of the network model still is (2.6). However, in the consumption process, an additional constraint must be added to present the deflation of undesirable outputs. This constraint is written as the form:

$$\sum_{j=1}^{J} z_j^C u^C_{hj} = (1 - \beta_k^C) u^C_{hk}, \quad h = 1, \ldots, H,$$

(2.6.9)

By applying the objective function identified in (2.6) and the constraints identified in Equations (2.6.1)–(2.6.9), we could compute the efficiency of transportation organizations based on the network structure with these undesirable outputs.

2.4.4 Multi-activity Framework

In many instances, organizations of any complexity typically consist of a number of individually identifiable units (Beasley 2003). For example, within a bus transit firm/railway company these units may correspond to various transportation services. Bus transit firms/railway companies may operate both highway and urban bus services/passenger and freight transportation services, what is efficient in a highway bus service/passenger transportation service may not be efficient in an urban bus service/freight transportation service, and thus different efficiency ratings for various activities should be distinguished. Units are linked by allocating resources, such as management labor and mechanics, to individual activities. The total amount of resources that the firm can allocate will be limited and unseparated. To allocate those unseparated shared resources is plainly important in a number of
firms. However, the conventional DEA model evaluates the efficiency that a DMU transforms inputs into outputs. It assumes that a DMU is equally efficient in all its activities. Hence, the problem of a firm’s efficiency which faces different production functions using shared inputs needs to be solved.

Many studies have been engaged to deal with this shared input problem in a practical organizational standpoint and a cost perspective (Golany et al. 1993; Golany and Tamir 1995; Beasley 1995, 2003; Mar Molinero 1996; Thanassoulis 1996; Färe et al. 1997, 2002; Mar Molinero and Tsai 1997; Tsai and Mar Molinero 1998, 2002; Cook and Kress 1999; Cook et al. 2000). The multi-activity DEA model, a novel refinement of the conventional DEA approaches, for the joint determination of efficiencies in the DEA context, was proposed by Beasley (1995) and subsequently revised by Mar Molinero (1996) and Tsai and Mar Molinero (1998, 2002). Specifically, the multi-activity model is used to evaluate efficiencies of organizations that engage in several activities simultaneously and some inputs and outputs are utilized and produced among all the activities.

In order to capture characteristics of the multi-activity model based on the network structure, we construct a multi-activity network DEA model by taking the railway companies, which generally provide passenger and freight transportation services in the production process, as example. A schematic of the performance evaluation in multi-activity network structure for a particular railway company is depicted in Fig. 2.3. In Fig. 2.3, the production process is divided into two sub-processes by passenger and freight transportation activities and those shared inputs are allocated to these two sub-processes.

Similarly, suppose there are \( J \) railway companies to be evaluated. We denote that \( x^{PP} \in R^I \) and \( m^{PP, C} \in R^N \) are (dedicated) inputs and intermediate outputs

![Fig. 2.3 Performance evaluation in the multi-activity network structure](image)
associated solely with the passenger production process (PP), $x^{\text{PP}} \in \mathbb{R}^+_+$ and $m^{(\text{PP}, \text{C})} \in \mathbb{R}^N_+$ are (dedicated) inputs and intermediate outputs associated solely with the freight production process (FP), but $x^{\text{PP}} \in \mathbb{R}^N_+$ are shared inputs associated in part with PP and in part with FP. Railway companies use (dedicated and shared) inputs to produce intermediate outputs in the production process. The intermediate products are consumed in consumption processes to produce final outputs, $y^{\text{C}} \in \mathbb{R}^D_+$. In the situation where there are inputs associated with both activities, we assume that these shared inputs can be apportioned between PP and FP. In this way, each joint input contributes to the determination of the passenger efficiency and the freight efficiency in the production process. Assuming that the proportions of the shared inputs assigned to each one of the said activities are $\alpha^{\text{PP}}$ and $1 - \alpha^{\text{PP}}$. Thus the objective function of the multi-activity network DEA model is revised as follows:

$$\text{Max } \beta_k^* = w^{\text{PP}}_k \beta^{\text{PP}}_k + w^{\text{FP}}_k \beta^{\text{FP}}_k + w^{\text{C}}_k \beta^{\text{C}}_k,$$

(2.7)

where $\beta^{\text{PP}}_k$ and $\beta^{\text{FP}}_k$ measure the maximum deflation of inputs in the passenger and freight production processes, respectively; $\beta^{\text{C}}_k$ measure the maximum inflation of outputs in the consumption processes; $w^{\text{PP}}_k$, $w^{\text{FP}}_k$ and $w^{\text{C}}_k$ are positive numbers which represent the relative importance of these activities/processes respectively, and $w^{\text{PP}}_k + w^{\text{FP}}_k + w^{\text{C}}_k = 1$. Equation 2.7 is subject to the following constraints:

The passenger production process is given by

$$\sum_{j=1}^{J} z^{\text{PP}}_j x^{\text{PP}}_{ij} \leq (1 - \beta^{\text{PP}}_k) x^{\text{PP}}_{ik}, \quad i = 1, \ldots, I,$$  

(2.7.1)

$$\sum_{j=1}^{J} z^{\text{PP}}_j m^{(\text{PP}, \text{C})}_{nj} \geq m^{(\text{PP}, \text{C})}_{nk}, \quad n = 1, \ldots, N,$$  

(2.7.2)

$$\beta^{\text{PP}}_k \geq 0, \quad z^{\text{PP}}_j \geq 0, \quad j = 1, \ldots, J.$$  

(2.7.3)

The freight production process is given by

$$\sum_{j=1}^{J} z^{\text{FP}}_j x^{\text{FP}}_{lj} \leq (1 - \beta^{\text{FP}}_k) x^{\text{FP}}_{lk}, \quad l = 1, \ldots, L,$$  

(2.7.4)

$$\sum_{j=1}^{J} z^{\text{FP}}_j m^{(\text{FP}, \text{C})}_{dj} \geq m^{(\text{FP}, \text{C})}_{dk}, \quad q = 1, \ldots, Q,$$  

(2.7.5)

$$\beta^{\text{FP}}_k \geq 0, \quad z^{\text{FP}}_j \geq 0, \quad j = 1, \ldots, J.$$  

(2.7.6)
The consumption process consists of

\[
\sum_{j=1}^{J} z_{j}^{C} m_{nj}^{(PP, C)} \leq m_{nk}^{(PP, C)}, \; n = 1, \ldots, N, \tag{2.7.7}
\]

\[
\sum_{j=1}^{J} z_{j}^{C} m_{qj}^{(FP, C)} \leq m_{qk}^{(FP, C)}, \; q = 1, \ldots, Q, \tag{2.7.8}
\]

\[
\sum_{j=1}^{J} z_{j}^{C} y_{dj}^{C} \geq (1 + \beta_{k}^{C}) y_{dj}^{C}, \; d = 1, \ldots, D, \tag{2.7.9}
\]

\[
\beta_{k}^{C} \geq 0, \; z_{j}^{C} \geq 0, \; j = 1, \ldots, J, \tag{2.7.10}
\]

Equations 2.7.11 and 2.7.12 represent the allocation of shared inputs to the passenger and freight production processes:

\[
\sum_{j=1}^{J} \alpha_{PP} z_{j}^{PP} x_{mj}^{PFS} \leq (1 - \beta_{k}^{PP}) \alpha_{PP} x_{mk}^{PFS}, \; m = 1, \ldots, M \tag{2.7.11}
\]

\[
\sum_{j=1}^{J} (1 - \alpha_{PP}) z_{j}^{FP} x_{mj}^{PFS} \leq (1 - \beta_{k}^{FP})(1 - \alpha_{PP}) x_{mk}^{PFS}, \; m = 1, \ldots, M \tag{2.7.12}
\]

where \( z_{j}^{PP} \), \( z_{j}^{FP} \) and \( z_{j}^{C} \) represent intensity variables for passenger production, freight production and consumption processes of each DMU \( j \), respectively.

The objective function in (2.7) takes a value of zero if and only if the railway company’s PP is technically efficient, its FP is technically efficient, and its consumption process is simultaneously serviced effectively. However, its value is greater than zero if and only if the railway company is technically inefficient at least one of the two sub-processes or the service is ineffective.

### 2.5 Examples

In this section, we provide related three cases to illustrate applications in empirical studies. First, a route-based performance evaluation in a network DEA model will be described. Next, a case that incorporates environmental factors and multiple activities into a network DEA model will be explored. Finally, we will investigate a multi-activity DEA model with these undesirable outputs.
2.5.1 Route-Based Network DEA Model

To explore a route-based performance evaluation in a network DEA model, an example of 15 domestic air routes operated by a Taiwanese domestic airline in 2001 is applied. The performance of an air route also can be divided into production efficiency (PE), service effectiveness (SE) and operational effectiveness (OE).

2.5.1.1 The Data

The input–output framework on the network model is depicted in Fig. 2.4. Input–output variables of an air route are illustrated as follows:

1. Output: Number of passenger-miles.
2. Inputs: Personnel cost, fuel cost and aircraft cost.
3. Intermediate output: Number of seat-miles.

2.5.1.2 Empirical Results

Table 2.1 gives us a clear and complete picture of relative performance for the sample’s air routes in three performance dimensions. It follows that for an air route to be able to locate on the overall operational effectiveness frontier, it needs to achieve both full production efficiency and service effectiveness. Hence, it can be found that there is a possibility of improvement for all air routes since their operational effectiveness scores are all less than unity. Table 2.1 also indicates that the average air routes’ production efficiency, service effectiveness and operational effectiveness are 0.829, 0.833 and 0.689, with a standard deviation of 0.139, 0.099 and 0.135, respectively. It is worth mentioning that the scores of production efficiency and service effectiveness must be used together to identify which processes need to be improved. For example, the operational effectiveness of air routes TSA-KHH and TSA-MZG are about the same (their scores are 0.740 and 0.739,

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Fig. 2.4 Input–output variables in a network model

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1 Adapted from Yu and Chen (2011).
respectively). However, the activities they need to improve to achieve operational effectiveness frontier are different. Air route TSA-KHH, with production efficiency score = 1.000 and service effectiveness score = 0.740, only needs to expand its consumed output 35.1 % \((1/0.740)\) to the service effectiveness frontier and then it will achieve operational effectiveness frontier. On the other hand, air route TSA-MZG, with production efficiency score = 0.908 and service effectiveness score = 0.814, needs to contract its input 9.2 % \((1 – 0.908)\) and expand its consumed output 22.8 % \((1/0.814)\) simultaneously to achieve operational effectiveness.

<table>
<thead>
<tr>
<th>Table 2.1</th>
<th>Efficiency and effectiveness scores of the network model</th>
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<tbody>
<tr>
<td>PE</td>
<td>SE</td>
</tr>
<tr>
<td>TSA-KHH</td>
<td>1.000</td>
</tr>
<tr>
<td>TSA-TNN</td>
<td>0.975</td>
</tr>
<tr>
<td>TSA-TXG</td>
<td>0.625</td>
</tr>
<tr>
<td>TSA-CYI</td>
<td>0.780</td>
</tr>
<tr>
<td>TSA-TTT</td>
<td>0.895</td>
</tr>
<tr>
<td>TSA-MZG</td>
<td>0.908</td>
</tr>
<tr>
<td>TXG-MZG</td>
<td>0.778</td>
</tr>
<tr>
<td>CYI-MZG</td>
<td>0.588</td>
</tr>
<tr>
<td>TNN-MZG</td>
<td>0.636</td>
</tr>
<tr>
<td>KHH-MZG</td>
<td>0.696</td>
</tr>
<tr>
<td>TSA-KNH</td>
<td>0.910</td>
</tr>
<tr>
<td>TXG-KNH</td>
<td>0.819</td>
</tr>
<tr>
<td>CYI-KNH</td>
<td>0.894</td>
</tr>
<tr>
<td>TNN-KNH</td>
<td>0.932</td>
</tr>
<tr>
<td>KHH-KNH</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: Resources of the attributes of each air route are from Chiou and Chen (2006)
This is a possible indication that inferior production efficiency and/or service effectiveness cause operational ineffectiveness on air routes.

Next to the column of operational effectiveness score, Table 2.1 also shows some of the operational information including route length, aircraft type, operational market, and service areas for each air route. As indicated, there are five routes serving major inland cities and ten routes connecting these cities to two offshore cities. From the operational effectiveness point of view, offshore air routes are performing better on average in comparison to the inland air routes in the sample. In particular, the top five routes with higher operational effectiveness scores all belong to the offshore air routes. However, we should stress that the better performance of offshore air routes than inland air routes might not mainly come from the better management of the decision makers of those routes, but may be the result of limited substitution in transportation modes and the increasing demand from the tourism market offshore.

As for route length, long air routes perform better than short ones. This is intuitive, since one can easily realize that the longer the route is, the higher performance will be. First, bigger aircraft with more seats in general are used to serve longer distance travel. Secondly, shorter routes in general spend a longer proportion of their time in ground operations than long flights. The current results suggest that the sample airline needs to focus on improving performance of those short air routes. The above results show that the operational effectiveness of air routes is to a lesser extent due to the market types and to a greater extent due to the length and service area of air routes in the Taiwan domestic air transportation market.

Lastly, as it appears in Table 2.1, the use of different types of aircraft seems to show some effects on the air routes’ service effectiveness but not production efficiency measure, since air routes operating with mixed types of aircraft appear to be more service effective than those using a single type of aircraft, while mixed type air routes do not perform better in production efficiency. A possible explanation is that a higher loading factor can be achieved if different types of aircraft are alternatively dispatched to serve peak demand (MD-90) and off-peak demand (DH8-300), while the benefits from lower operating cost does not guarantee better production efficiency. This implies that air routes operations need to meet the obligation of providing a fixed timetable of flights. This result recommends that the sample airline alternatively dispatch different types of aircraft to serve varying-demand routes to increase its air routes’ service effectiveness.

### 2.5.2 Multi-activity DEA Model with Environmental Factors and Undesirable Output

We provide an example for 24 Taiwan’s multimode bus transit firms in 2001 that incorporate environmental factors (E) and undesirable output (U) into a multi-activity model.

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2 Adapted from Yu and Fan (2006).
DEA (MDEA) model to analyze the highway bus effectiveness (HBE), urban bus effectiveness (UBE) and operational effectiveness (OE) of each bus transit firm. All these firms operated both highway bus service (HB) and urban bus service (UB).

2.5.2.1 The Data

The input–output framework on the multi-activity model is portrayed in Fig. 2.5. Input–output variables of individual activities of a bus transit firm are illustrated as follows:

1. *Dedicated inputs of highway bus service*: Drivers, vehicles, fuel and network length in the highway bus sector.
3. *Dedicated inputs of urban bus service*: Drivers, vehicles, fuel and network length in the urban bus sector.

![Fig. 2.5 Input–output variables in a multi-activity model](image-url)
5. **Shared input for highway and urban bus services:** Management, operating and technical staff.

6. **Undesirable output for highway and urban bus services:** Accident cost.

7. **Environmental variable of highway bus service:** Long-haul transportation demand.

8. **Environmental variable of urban bus service:** Short-haul transportation demand.

### 2.5.2.2 Empirical Results

For each firm, four overall operational effectiveness measures have been calculated by different DEA models, as shown in Table 2.2. Note that all the operational effectiveness scores should be less than or equal to unity and that a higher score indicates a more effective status. The first column is the overall operational effectiveness obtained from a conventional DEA model. These conventional indices diverge from 0.523 to 1.0 with a mean level of 0.952. The number and percentage of the fully operationally effective units is 17 and 70.83% of the 24 bus firms. As the second column indicates, the overall operational effectiveness indices obtained from the multi-activity DEA model 1 have larger mean value ranges, from 0.421 to 1.0, with a mean overall operational effectiveness of 0.850. Moreover, only four out of 24 firms are operationally effective. Column 5 reports the overall operational effectiveness scores obtained from the multi-activity DEA model 2 which includes an environmental factor, but ignores undesirable output side effects. As can be noted, the estimated effectiveness diverges substantially from 0.570 to 1.0 with a mean value of 0.898. Of the 24 bus firms analyzed, only five are deemed effective. The results of column 8 are obtained from the multi-activity DEA model 3 in which the overall operational effectiveness of a firm is evaluated on the basis of its ability to increase desirable outputs and reduce inputs and undesirable output simultaneously. The overall operational effectiveness scores vary from 0.576 to 1.0 with a mean effectiveness score of 0.884. The number and percentage of the fully operationally effective units increases to 7 and 29.17% of the 24 bus firms as the undesirable output is included. If we concentrate on the highway bus service, ten of the bus firms exhibit operationally effective behavior that is superior to the rest. With regards to urban transit, a maximum level of effectiveness is achieved by nine firms, with bus firms that are operationally effective in each of the two services coinciding in only seven cases.

These above results imply that the conventional DEA operational effectiveness measure may be seriously misleading if it ignores the operational effectiveness of firms, which carry out various activities whilst sharing common resources. In addition, for those bus firms where environmental factors and undesirable output are important, the illustration shows that different multi-activity DEA models lead to different results. The multi-activity DEA model 3 provides a deep structure that more fully takes the shared inputs, environmental factors and undesirable output into consideration.
<table>
<thead>
<tr>
<th></th>
<th>Conventional (DEA)</th>
<th>MDEA model 1 (excluding $E$ and $U$)</th>
<th>MDEA model 2 (including $E$ only)</th>
<th>MDEA model 3 (including $E$ and $U$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OE</td>
<td>OE</td>
<td>HBE</td>
<td>HBE</td>
</tr>
<tr>
<td>Max</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Min</td>
<td>0.523</td>
<td>0.421</td>
<td>0.405</td>
<td>0.167</td>
</tr>
<tr>
<td>Mean</td>
<td>0.952</td>
<td>0.850</td>
<td>0.846</td>
<td>0.854</td>
</tr>
<tr>
<td>SD</td>
<td>0.118</td>
<td>0.171</td>
<td>0.160</td>
<td>0.230</td>
</tr>
<tr>
<td>No. of fully effective units</td>
<td>17</td>
<td>4</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>% of fully effective units</td>
<td>70.83</td>
<td>16.67</td>
<td>29.17</td>
<td>20.83</td>
</tr>
</tbody>
</table>
2.5.3 Multi-activity Network DEA Model with Environmental Factors

Furthermore, the operational process of a multimode bus transit firms can be divided into two sub-processes: production and consumption processes. In addition, the production process includes two activities: highway bus service (HB) and urban bus service (UB). Hence, we also apply this example for multimode bus transit firms to illustrate the performance obtained from multi-activity network DEA model, but, in this section, the used model incorporates multiple activities, multiple processes and environmental factors to analyze the highway bus efficiency, urban bus efficiency, production efficiency, service effectiveness and operational effectiveness of each bus transit firm. The data set used in the measurement of performance in Taiwan’s bus transit system comprised a sample of 23 firms located all over the island in 2001 and 2002. All these firms operated both highway bus service and urban bus service.

2.5.3.1 The Data

The input–output framework on the multi-activity network model is represented in Fig. 2.6. Input–output variables and environmental variables of individual activities and processes of a bus transit firm are illustrated as follows:

1. **Dedicated inputs of highway bus production service**: Drivers, vehicles, fuel and network length in the highway bus sector.
2. **Intermediate output of highway bus production service**: Vehicle-kms in the highway bus sector.
3. **Dedicated inputs of urban bus production service**: Drivers, vehicles, fuel and network length in the urban bus sector.
4. **Intermediate output of urban bus production service**: Vehicle-kms in the urban bus sector.
5. **Dedicated input in consumption process**: Sales staff.
6. **Output in consumption process**: Passenger-kms and passengers.\(^4\)
7. **Shared input for highway and urban bus production services**: Mechanics.
8. **Shared input for highway bus production service, urban bus production service and consumption process**: Management employees.
9. **Environmental variables**: Population density and car ownership.

---

\(^3\) Adapted From Yu and Fan (2009).

\(^4\) The passenger-km are not available for UB service, so the number of passengers is used as a proxy variable in this paper. It is more appropriate to use passenger-kms as final output variables.
In this section, we present estimates of performance measures based on the all-in-one multi-activity network DEA model, and three separate conventional DEA models. It is worth noting that the production efficiency, service effectiveness and environmental performance measures.

**Environmental variable:** Population density

**Dedicated inputs:**
1. Drivers
2. Vehicles
3. Fuel
4. Network length

**Shared input:** Mechanics

**Dedicated input:** Sales staff

**Shared input:** Management employees

**Dedicated inputs:**
1. Drivers
2. Vehicles
3. Fuel
4. Network length

**Intermediate output:** Vehicle-kms

**Consumption process**

**Outputs:**
1. Passenger-kms
2. Passengers

**Intermediate output:** Vehicle-kms

**Environmental variable:** Car ownership

**Environmental variable:** Population density

**Fig. 2.6** Input–output variables in a multi-activity network model

### 2.5.3.2 Empirical Results

In this section, we present estimates of performance measures based on the all-in-one multi-activity network DEA model, and three separate conventional DEA models. It is worth noting that the production efficiency, service effectiveness and
operational effectiveness estimated by the multi-activity network DEA model imply that those performance measures are not independent. The results of multi-activity network DEA are summarized in Table 2.3. If the value of the production efficiency is equal to unity, this denotes that it is “efficient”, whereas values less than 1 indicate that it is “inefficient”. On the other hand, if the value of the service effectiveness or operational effectiveness is equal to unity, this denotes that it is “effective”, whereas values greater than 1 denote that it is “ineffective”.

In the first two columns, the highway bus efficiency and urban bus efficiency, and in the fourth column, the service effectiveness, are evaluated on the basis of their ability to share common inputs among different activities, and to determine simultaneously their efficiency and effectiveness. With regard to the average production efficiency, the means of highway and urban bus efficiencies are lower than 1, indicating that there was inefficient in the production process for the sample as a whole. When the mean of service effectiveness score is greater than 1, in this case 1.160, this denotes an “ineffective” score for the sample as a whole. This service effectiveness may be explained by the inability of firms to expand ridership, as the vehicle-km provision cannot be reduced under the same environment. The average operational effectiveness was also greater than 1 (1.141), indicating that the sample as a whole was “ineffective”. For efficient firms that are efficient in regard to their production but not consumption processes, it is implied that they operate ineffectively, and hence there is further improvement in terms of service effectiveness. The managers could pay more attention to increasing the utilization of the produced service to improve their service effectiveness. For firms that are inefficient in their production processes but effective in their consumption processes, it implies that they are not production efficient. This could mean that firms should reduce their input proportions with respect to their frontiers in order to determine the improvement needed in each activity to catch up with the frontier firms.

Based on the comparison, efficiency and effectiveness measurements are examined, and are depicted in Table 2.4. The production efficiency index in the multi-activity network model has slightly lower efficiency score, and only 3 of the 23 firms are operating on the production frontier, while 9 of the 23 are operating efficiently on the production frontier under the conventional model. With respect to service effectiveness, the results reveal a relatively lower effectiveness score (lower

<table>
<thead>
<tr>
<th></th>
<th>Highway bus efficiency (1 – β[H]k)</th>
<th>Urban bus efficiency (1 – β[U]k)</th>
<th>Production efficiency (β[P]k)</th>
<th>Service effectiveness (1 + β[C]k)</th>
<th>Operational effectiveness (1 + β[k])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>2.502</td>
<td>1.837</td>
</tr>
<tr>
<td>Min</td>
<td>0.613</td>
<td>0.514</td>
<td>0.738</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.894</td>
<td>0.864</td>
<td>0.879</td>
<td>1.160</td>
<td>1.141</td>
</tr>
<tr>
<td>SD</td>
<td>0.103</td>
<td>0.141</td>
<td>0.073</td>
<td>0.329</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Notes: (1) Each of the efficiency or effectiveness scores is the mean of the estimated values of 2 years’ observations; (2) \( β^P_k = 1 - \left( \frac{β^H_k + β^U_k}{2} \right) \); and (3) \( β_k = 0.25β^H_k + 0.25β^U_k + 0.5β^C_k \)
effectiveness score represents more effective) than the conventional DEA model. As to operational effectiveness, the results also indicate that the average effectiveness score is relatively lower (representing more effective).

In order to provide statistically robust findings about these transit firms’ respective performances, paired difference experiments are applied. This experiment is conducted to verify whether the sample firms for the two kinds of models were drawn from the same performance populations for the three measures, respectively. The significance of paired comparisons is that it is based on a two-tailed test at the 0.05 acceptance level. As shown in Table 2.4, the test of significance yielded a \( p \)-value of 0.000 of production efficiency, which shows a statistically significant difference in terms of production efficiency. However, the statistical test confirmed that the service effectiveness and operational effectiveness measures were not significantly different, having \( p \)-values of 0.097 and 0.885, respectively. On the other hand, the statistical test for the entire sample, which pooled the three measures in a set, yielded a \( p \)-value of 0.003 which reveals a significant difference between the two models at the 5 % acceptance level. The results of the statistical tests for the two models may imply that the significant difference in production efficiency estimated by the mixed structure network and conventional models gave rise to the significant differences in the overall samples for these three measures, even though the differences between the service effectiveness and operational

| Table 2.4 Descriptive statistics of the conventional and multi-activity network models’ performance scores and the results of the test of significance |
|----------------------------------------|--------|--------|--------|--------|
| All samples                           |        | Production efficiency | Service effectiveness | Operational effectiveness |
| **Multi-activity network model**      |        |                    |                     |                      |
| Number of firms                       | 69     | 23                 | 23                 | 23                 |
| Number of efficient or effective scores | 12     | 3                  | 8                  | 1                  |
| Number of inefficient or ineffective scores | 57     | 20                 | 15                 | 22                 |
| Mean of efficiency or effectiveness scores | –      | 0.879              | 1.160              | 1.141              |
| **Conventional model**                |        |                    |                     |                      |
| Number of firms                       | 69     | 23                 | 23                 | 23                 |
| Number of efficient or effective scores | 21     | 9                  | 6                  | 6                  |
| Number of inefficient or ineffective scores | 48     | 14                 | 17                 | 17                 |
| Mean of efficiency or effectiveness scores | –      | 0.965              | 1.237              | 1.144              |
| **Correlations**                      |        |                    |                     |                      |
| Network vs. conventional              | 0.901  | 0.471              | 0.935              | 0.858              |
| **Test of significance**             |        |                    |                     |                      |
| \( p \)-value                         | 0.003**| 0.000**            | 0.097*             | 0.885              |

Notes: ‘*’ and ‘**’ mean significant at the 10 % and 5 % level of significance, respectively.
effectiveness measures estimated by these two models are generally insignificant. Therefore some more means are applied for further comparison.

The results obtained from the multi-activity network and the conventional models are quite different in terms of efficient or effective units. In general, the multi-activity network model is more demanding than the conventional one. This is explained by the following two facts. First, the achievement of a better degree of efficiency or effectiveness in the multi-activity network model requires that good productive and consumption matching behaviors are demonstrated on the part of the two services (HB and UB) as well as between the production and consumption processes, respectively. However, with the conventional model, it is possible that there are compensations between the two production activities and one consumption process in such a way that one firm will always achieve the production frontier provided that, in global terms, it demonstrates behavior which is superior to the rest, even if such superiority is not demonstrated in all the activities (services) it carries out. Second, a representation of both production and consumption processes in a unified framework is allowed in the multi-activity network model, and hence the three measures interact to determine the performance, while with the conventional model the three measures are calculated independently, even though there is a high degree of correlation between individual scores (service and operational effectiveness) obtained from the multi-activity network DEA model and those derived from the conventional DEA model. This indicates that the multi-activity network DEA model provides a nearly coincident result in terms of service and operational effectiveness, while it is worth noting that production efficiency is quite different. It is more reasonable to use the results of the multi-activity network DEA model for gauging the transit firms’ performance, since the potential benefit of this model is that it provides the possibility of looking deeply into the production and consumption processes. This shows that by considering the multiple activities and unstorable characteristics of transit services in the network model, firms may not only compare their performances with those of peer groups under practical and realistic conditions, but the inter-related effects caused by the various activities and processes may also be considered.

2.6 Conclusions

In this chapter, we describe a network graph of operational structure in the transportation sector to represent the operational characteristics of transportation services, and apply this concept to construct a network DEA model that illustrates the operational behavior in the sense of maximization of consumed outputs and minimization of initial inputs. To document its practicality, the network DEA model provides a deeper structure that takes unstorable characteristics of transportation services into consideration. Since the focus of the chapter is on providing a more reasonable performance measurement in the transportation sector and how the DEA model can be applied practically, we further incorporate route-based performance
evaluation, environmental factors, undesirable outputs and multi-activity framework into the network DEA model, respectively. These models can provide the sources of inefficiency within a transportation organization. Identification of such sources can help managers to design the implementation of operational policies and management strategies to improve performance. In addition, we have provided three relative applications in transportation organizations to illustrate the selection of inputs and outputs as well as the results.

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Data Envelopment Analysis
A Handbook of Empirical Studies and Applications
Zhu, J. (Ed.)
2016, XIII, 587 p. 97 illus., 56 illus. in color., Hardcover
ISBN: 978-1-4899-7682-6