Preface

Optimal control theory has become such an important field in aerospace engineering that no graduate student or practicing engineer can afford to be without a working knowledge of it. Unfortunately, there is no modern text which begins from scratch to teach the reader the basic principles of the calculus of variations, to develop the necessary conditions step-by-step, and to introduce the elementary computational techniques of optimal control.

Our book assumes that the reader has only the usual background of an undergraduate engineering, science, or mathematics program, namely, calculus, differential equations, and numerical integration.

We assume no other knowledge. We do not require the reader to know what calculus of variations is, what necessary conditions mean, nor what a two-point boundary-value problem entails. It does not matter if the reader has never heard of the Euler-Lagrange theorem, the Weierstrass condition, Pontryagin’s Minimum Principle, or Lawden’s primer vector.

Our goal is to provide the reader with sufficient knowledge so that he or she cannot only read the literature and study the next-level textbook (such as Bryson and Ho’s *Applied Optimal Control*) but also apply the theory to find optimal solutions in practice.

To accomplish the goals of this introductory text, we have incorporated a number of features as follows. Several theorems are presented along with “proof outlines” that favor a heuristic understanding over mathematical rigor. Numerous rigorous treatments are cited in the references and the book bibliography to support the reader’s advanced studies.

In presenting the Euler-Lagrange theorem, we treat two different versions which appear in the literature. In the first method (followed by Bryson and Ho [1975]), we adjoin terminal constraints to the cost functional through the use of additional Lagrange multipliers. We refer to this approach as the “adjoined method” and note that it has become a sort of gold standard in the literature since the revised printing of Bryson and Ho’s *Applied Optimal Control* in 1975. This approach leads to a form of the transversality condition which we refer to as the “algebraic form.” In publications prior to 1975, a number of authors use an approach which we refer to
as the “un-adjoined method” which does not adjoin the terminal constraints to the cost functional and hence does not introduce any additional multipliers to be solved. The un-adjoined method leads to a “differential form” of the transversality condition as given by Citron [1969], Hestenes [1966], Kenneth and McGill [1966], and Pierre [1969]. The un-adjoined method is particularly amenable to simple problems in which the terminal constraints are algebraically eliminated from the transversality condition. Each method has its strengths and weaknesses as observed by Citron [1969], who is one of the few authors who discuss both methods. Introducing the reader to both methods, with applications to current aerospace problems, is an important feature of the present text.

Throughout the book, we make use of the time-optimal launch of a satellite into orbit as an important case study, and we provide a detailed analysis of two examples: launch from the Moon and launch from the Earth. In the Moon-launch case, we assume constant acceleration (from thrusters), no drag, and uniform flat-Moon gravity. For Earth launch we include time-varying acceleration, drag from an exponential atmosphere, and uniform flat-Earth gravity. Appendices A and B provide MATLAB code to solve the resulting two-point boundary-value problems. In Appendix C, we also set up and provide MATLAB code to solve a geocentric low-thrust transfer problem.

A modern approach to Lawden’s primer vector theory is presented for optimal rocket trajectories. The important special cases of constant-specific impulse and variable-specific impulse are treated in detail.

An extensive annotated book bibliography lists the references we found most useful in the preparation of this text. These sources range from highly pragmatic application approaches (for engineers) to rigorous, theoretical treatments (for mathematicians). The second bibliography lists numerous papers and reports that demonstrate the vast range of related aerospace applications.

Finally, for the weary and the worried, we provide a few “Curious Quotations” (in Appendix D) to let the reader know that many great minds and renowned authors have expressed their own concerns, often in humble and humorous ways, about the vast challenges that the calculus of variations and optimal control present to all of us.

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