Preface

Mathematics is both an art and a science. It is a science because it provides a panoply of analytic tools for understanding the world around us. It is an art because it is elegant and beautiful.

It is a shame that, because of its ostensibly austere nature, mathematics does not appeal to many people. It is true that not only is mathematics is demanding and unforgiving, but it also offers many rewards and insights. The world would be a better place if we could get more people to appreciate the joys of mathematics.

One of the main purposes of this book is to provide the uninitiated reader with some insights into modern mathematics. The past few years have seen some astonishing advances that could offer charms to even the most jaded reader. This book in fact explores several hot-button topics that cannot be found anywhere else in the nontechnical literature. These include the Black–Scholes option pricing scheme, dynamical systems, relativity theory, wavelets, RSA encryption, the **P/NP** problem, primality testing, Fermat's last theorem, and the Poincaré conjecture.

The Black–Scholes theory is Nobel-Prize-winning work that has dramatically changed the financial world. Now finance is greatly influenced by mathematics, and many new mathematics Ph.D.s end up plying their trade on Wall Street.

Dynamical systems, popularly known in the context of fractals, is a new way of looking at the world around us and understanding its structure. Many colorful figures, including Benoît Mandelbrot, have contributed to our growing understanding of dynamical systems.

Of course relativity theory dates back to the days of Albert Einstein. But new studies, especially regarding cosmology and the "grand unified theory," have brought relativity theory into focus. The new theory of strings provides a detailed structure for physics both in the large and in the small.

Wavelets is a new branch of Fourier analysis—the theory of breaking up complicated waves into simple, component waves. Recently a mathematician in New Hampshire won a Grammy Award for using wavelets to clean up the only known live recording of Woody Guthrie. Wavelets have been used to compress images of fingerprints in order to aid the FBI in its data storage activities. Wavelets are used daily in image compression, signal processing, and many other aspects of modern technology. They are a genuine revolution in modern mathematics.

RSA encryption is a new type of coding theory based on ideas from elementary number theory. It makes possible public key encryption and other new types of code paradigms. Much of modern security depends on RSA encryption. And it all hinges on the intractability of factoring large integers. This is a very practical application that depends directly on rather abstract theoretical mathematics.

The **P/NP** problem is considered by many to be the most important unsolved problem in the mathematical sciences. It has to do with the computational complexity of solving certain problems. The prototype problem in this subject is the factoring of large integers (with 150 digits or more). It takes quite a long time to actually *factor* such a number. But only a few moments to verify that a given factorization is correct (by multiplying the factors together). This dialectic is the key to the **P/NP** problem.

Very closely related to RSA encryption is the question of primality testing. In 2004, M. Agrawal and his students in India found a polynomial-time algorithm for testing whether any given integer is prime. It does *not* tell us what the prime factorizaton is, but it tells us whether there is one. This is a dramatic breakthrough and is based on only very elementary mathematical ideas.

Fermat's last theorem was one of the more dramatic mathematical events of the past 20 years. Princeton's Andrew Wiles in 1993 announced that he could solve the 350-year-old enigma of Fermat. He subsequently appeared on the front page of every newspaper in the world. Princeton was overrun with journalists and news cameras. And then Wiles had to announce that there was an error in his proof. It took over a year and the aid of Wiles's student Richard Taylor to finally fix the error and nail down the proof. This is one of the most dramatic stories of modern mathematics.

Finally, there is the saga of the Poincaré conjecture. A hundred-year-old problem about the shape of the universe, the Poincaré conjecture captured the imaginations of scores of mathematicians. Many proofs have been offered,

and many have failed. Finally a rather eccentric Russian mathematician, Grigori Perelman from St. Petersburg, put three preprints on an Internet preprint server which claimed to solve this age-old problem. These preprints were problematical because they were so sketchy and enigmatic. But teams of mathematicians jumped into the breach and spent literally years filling in the gaps in Perelman's arguments. And it all worked. The Poincaré conjecture is proved. And now there are new mysteries to fathom.

This book demonstrates that mathematics is an exciting and ongoing enterprise. It shows the neophyte what mathematicians think about, what they care about, and what their goals are. It shares the excitement, the sorrow, and the frustrations of being a modern mathematician. It shows what we are after, what we can achieve, and what we can appreciate in the process.

The book has few prerequisites beyond a good grounding in basic mathematics. And it will put the reader right into the guts of the problems being discussed. We are not afraid to analyze diagrams, create graphs, and manipulate equations. But we do so in a fashion that readers can appreciate and understand. We hope that the reader will continually be tickled into action and hasten to move on to the next idea. This is likely to be a hard book to put down.

Mathematics is a growing and changing enterprise, full of some of the most important and daring ideas of modern times. Our goal here is to help non-mathematicians appreciate this part of the intellectual pie and perhaps to develop some taste for the saga and journey that is mathematics.

It is a pleasure to thank the many and varied referees who offered their wisdom in the development of this book. Our Editor Ann Kostant provided her own edits and contributed a good deal. Finally we thank Lynn Apfel, whose insights and criticisms have proved to be invaluable.

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