Preface

In recent years, there has been a noticeable increase in the number of college courses designed to introduce mathematics to non-math students. I am not talking about the sort of mathematics studied at school, a glorified version of arithmetic with sole letters thrown in to represent numbers, but real mathematics—the study of patterns and structures (and yes, sometimes numbers are involved) that arise in our everyday lives.

This book is designed as a text for such a course, a one-semester course designed for students with a minimal background in mathematics.

Outline of Topics

The first two chapters discuss the basics of numbers and set theory. Chapter 1 (*Numbers and Sets*) introduces the idea of a set and a subset (a part of a set); the sets of numbers that we use are discussed. Much of this material will be familiar to the reader, but you may see these topics differently after you see how the ideas are connected. We then introduce the idea of a base for numbers; we normally use base 10, but other bases are useful for dealing with computers. The rest of the chapter is about representing and using numerical data: summation, some notation for summations, and some ways of combining sets are introduced, Venn diagrams are discussed, and finally we look at arrays of numbers and define matrices.

Counting is covered in Chap. 2 (*Counting*). The concept of an event, a collection of possible outcomes, is introduced. Summations, which were a topic in Chap. 1, are studied further. We also distinguish between counting the number of selections or combinations—subsets of a given size—and counting arrangements or permutations—subsets where the order of the elements is important.

Counting leads naturally to probability theory. In Chap. 3 (*Probability*) we introduce the basic ideas of probability. Randomness and random events are defined. A number of examples involving dice and playing cards are included. In order to discuss compound events, tree diagrams (similar to family trees, which we'll mention later) are introduced. We go through two examples where probabilities are different from what many people expect—the birthday problem and the Monty Hall problem. Finally, the more mathematical concept of a probability model is introduced.

The next two chapters deal with statistics, and in particular those aspects (sampling, polls, predictions) that we often see in everyday life. As a first step toward describing statistics, Chap. 4 (*Data: Distributions*) examines the way in which numerical data about our world is collected and described. The ideas of a population and taking a sample from that population are introduced, together with ways of displaying this information—the dotplot, histogram, and boxplot—and parameters that describe the information, such as means, medians, and quartiles. Probability distributions are introduced, as a way of representing the probability model of a phenomenon; in particular, we look at the normal distribution.

Chapter 5 (*Sampling: Polls, Experiments*) looks at how statistics impinges on our world: estimating numerical properties of our society and deciding how reliable are those estimates. A sample is the group we examine to study a property (such as family income and height). The obvious question is: how reliable are our estimates in describing the whole population? We discuss the reliability of methods of sampling and ways of obtaining data, by observation or controlled experiments. Experimental designs are discussed briefly, and in particular Latin squares are studied. The relation of these designs to sudoku puzzles is mentioned.

We next look at graph theory. This is the study of (linear) graphs. A graph consists of a set of objects (called *vertices*) and a set of connections between pairs of vertices (called *edges*). For example, the vertices might represent cities and an edge joining two of them might mean there is a nonstop airline flight between those two cities; another example is the family tree, mentioned earlier, where vertices are people and edges represent parent–child connections.

In Chap. 6 (*Graphs: Traversing Roads*), we start by modelling roads as edges and look at the problem of traversing all roads in an area. Vertices might represent cities, parts of a town, or intersections. The Königsberg bridge problem asks whether one can traverse all roads in an area without any repeats; each road is to be traveled exactly once. If this is not possible, one can try to cover each road at least once, with the minimum possible number of repeats. Some basic ideas about graphs (adjacency, multiple edges, simplicity, connectedness) are discussed.

Chapter 7 (*Graphs: Visiting Vertices*) starts by defining some special types of graphs, such as paths, cycles, and bipartite graphs. The problem of visiting all towns in an area exactly once each—Hamilton's problem—and the problem of doing so as cheaply as possible—the traveling salesman problem—are introduced.

In Chap. 8 (*More About Graphs*) we discuss two further aspects of graph theory, trees and graph coloring. Trees are graphs reminiscent of the tree diagrams introduced in Chap. 3. In particular, spanning trees are defined and algorithms for finding minimum cost spanning trees are outlined. Coloring and chromatic number are introduced, as are some applications of graph coloring. The famous four-color theorem is discussed briefly. We use this chapter to illustrate a method of mathematical proof, the *proof by contradiction*, which some readers may choose to skip over on a first reading.

Chapters 9–11 deal with numbers we actually use in everyday life: credit card numbers, PINs, and so on. We also look at encoding and decoding, both for transmission of data and for secrecy.

In Chap. 9 (*Identification Numbers*) we look at the numbers we all use nowadays—account numbers, social security numbers, etc. We look at how these numbers are made up, and how they are used. In particular, the formulas for drivers' license numbers in Illinois and Florida are explained, as well as the check digits used in credit cards, postal money orders, and book identification numbers (ISBNs) to make sure the numbers are legitimate and have been transmitted correctly.

In our electronic world, much data are transmitted electronically. Computers basically transmit strings of digits, so it is necessary to encode and decode messages. Moreover, errors can occur, so check digits are required, just as they are for identification numbers. Chapter 10 (*Transmitting Data*) deals with this topic. One typical method, Venn diagram encoding/decoding, is examined in detail. This is an example of nearest neighbor decoding and is an example of a family of codes called Hamming codes. Variable-length codes, including Morse code and the genetic code, are also introduced. A surprising application of Hamming codes, the hat game, is described.

In Chap. 11 (*Cryptography*) we explore another reason for encoding material: secrecy. The history of secret writing, including the scytale and the Caesar cipher, is outlined. More modern techniques include the Vigenère method and substitution ciphers. Modular arithmetic is defined, and the RSA scheme of cryptography is studied.

The next two chapters are devoted to voting. In Chap. 12 (*Voting Systems*) some simpler voting systems and methods of deciding elections are discussed, starting with majority and plurality systems, then sequential voting and runoff elections. Preference profiles are defined. The Hare method for simple elections is described, together with the generalizations of the Hare method called instant runoff elections. Condorcet winners are defined, along with Condorcet's method of dealing with the case when there is no Condorcet winner. Sequential pairwise elections and pointscore methods are outlined.

Then in Chap. 13 (*More on Voting*) we describe two methods for elections when more than one candidate is to be elected: the generalized Hare method and approval voting. Then two methods of manipulating the vote are discussed. First is strategic voting, where voters might vote for their second favorite candidate to ensure that their least favorite candidate is not elected (called an insincere ballot); second is the introduction of amendments to change the final outcome. We close with an example of how different methods, even though they are fair, may give different results.

We finish by discussing various aspects of finance and related topics. Most readers have some idea of the mathematics of finance, but will be surprised by what they do not know. Chapter 14 (*The Mathematics of Finance*) covers simple and compound interest, the mathematics of compounding, and defines the annual percentage rate (the one lenders tell you about) and annual percentage yield (the one you actually pay). Geometric growth is introduced.

Chapter 15 (*Investments: Loans*) studies the mathematics of investments (regular savings) and (compound interest) loans. These two are similar: when you borrow, it is as if the lender is making an investment. Your equity when you borrow to cover a purchase is discussed. Two special types of loan, the add-on loan and the discounted loan, are explained.

The last chapter, Chap. 16 (*Growth and Decay*), looks at the growth of human and animal populations and at radioactive decay. These two are closely related. Both involve the limiting case of compounding, where the process is continuous. Exponential functions, exponential growth, and natural logarithms are discussed. The change in the cost of living is covered, as an extension of exponential growth.

Problems, Exercises, and Further Reading

A number of worked examples, called Sample Problems, are included in the body of each section. Many of these are accompanied by a practice exercise labeled "Your Turn," designed primarily to test the reader's comprehension of the ideas being discussed. It is recommended that students work all of these exercises; complete solutions are provided at the end of the book.

The book contains a large selection of exercises, collected at the end of the chapters. They should be enough for students to practice the concepts involved; most of the problems are quite easy. Answers are provided. There is also a set of multiple-choice questions in each chapter.

Books of this kind often contain historical sketches, biographies, and the like. In many years of teaching, I have observed that students hardly ever read these items. Moreover, students are quite capable of looking up this information for themselves. For example, Googling "Samuel Morse" or "Morse code" will give as much information on the gentleman and his inventions as you could possibly need. So I chose to opt for a concise text; the Internet will provide all needed background and the student will not need to carry a huge book around.

Carbondale, IL W.D. Wallis

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