Chapter 2
Modeling Vehicle Interactions and the Movement of Groups of Vehicles

The previous chapter focused on the movement of an individual vehicle and provided equations of motion assuming no interaction with adjacent vehicles. This chapter examines these interactions between vehicles, which is at the heart of traffic flow theory, as it is these which produce the observed traffic operational conditions. Important traffic operational characteristics we are interested in include the capacity of a facility (i.e., the maximum amount of traffic that can pass through a point or section, in vehicles or other units of traffic per unit of time) and its operating speed (i.e., the speed at which the facility operates under a given set of prevailing conditions, including the demand, the highway design, etc.). Those concepts are discussed in more detail in Part II.

Vehicle interactions can be defined in terms of three basic relationships: car-following, lane changing, and gap acceptance. Car-following is the process by which a vehicle follows another vehicle in close proximity. Generally, car-following occurs when the speed of the lead vehicle affects the speed of the following vehicle. Car-following algorithms provide the trajectory of the following vehicle as a function of the lead vehicle. Car-following affects both the capacity of a facility and its speed. The closer the vehicles follow each other, the higher the capacity of a facility (more on that in Part II). Also, the behavior of the vehicles following another vehicle affects the speed of the facility: more aggressive car-following generally leads to higher (although not necessarily safer) overall speeds.

Lane changing is the process by which a vehicle decides to change lanes, and it generally involves the requirement or decision to change lanes, the selection of a target lane (when it is relevant), and the selection of a suitable gap. Lane changing is thus also related to the third process, gap acceptance. Gap acceptance involves the selection of a suitable gap (usually defined as the time headway between the rear end of the lead vehicle and the front end of the following vehicle) to change lanes, or to cross a conflicting traffic stream, as in the case of a STOP-controlled approach. Similarly to car-following, lane changing and gap acceptance affect both the capacity and the operating speed of a facility. For example, more frequent lane changing to gain speed advantage typically leads to higher operating speeds.

L. Elefteriadou, *An Introduction to Traffic Flow Theory*, Springer Optimization and Its Applications 84, DOI 10.1007/978-1-4614-8435-6_2,

When drivers generally accept smaller gaps they are able to traverse the stop bar faster, leading to higher capacities.

These three processes and the respective algorithms provide the basic set of vehicle interactions and they are the key components of traffic microsimulation models, which have become increasingly popular as computer power has increased. These models replicate on a computer the movement of individual vehicles in highway networks in order to conduct experiments and evaluate various alternative improvements (see Chap. 7 for additional information on traffic simulation). This chapter first discusses car-following, presents a historical overview of car-following models, and summarizes the most important algorithms currently used. The second section summarizes lane-changing models, while the third one presents gap acceptance principles.

Car-Following

Let us consider two vehicles, one traveling behind the other in a single highway lane. The movement of the first vehicle is primarily defined by the principles discussed in Chap. 1. The movement and trajectory of the second vehicle, however, are based on additional factors. The following vehicle must take care not to collide with the vehicle in front. Thus it needs to maintain a suitable spacing and speed. Let us assume that the lead vehicle travels uninhibited from traffic ahead and thus its speed corresponds to the driver's desired speed considering the prevailing driving environment. If the following vehicle's desired speed is lower than that of the lead vehicle then their spacing will keep increasing, and the following vehicle will not be constrained by the position and speed of the lead vehicle. If however the following vehicle's desired speed is higher, then sooner or later it will enter a "car-following" state with the lead vehicle, where its spacing, speed, and acceleration will be dictated by those of the lead vehicle. The closer the following vehicle is to the lead vehicle, the more sensitive the reactions of the following vehicle would be to the actions of the lead vehicle. This sensitivity also increases with speeds.

Similarly to the discussion regarding the movement of a single vehicle (which would be the uninhibited lead vehicle in this case), the trajectory of the following vehicle depends to a significant degree on its driver and vehicle. More aggressive drivers tend to have higher desired speeds and to follow at shorter distances. With respect to vehicle characteristics, heavier trucks, because of their braking requirements, keep longer distances to the vehicles in front of them.

Figure 2.1 presents the time–space diagram and trajectories of vehicles A and B. As shown, the second vehicle (B) is initially traveling at a higher speed than the first vehicle (A). At time t_1, after it approached and decelerated, it continued to travel as dictated by the speed of vehicle A. In that figure, the horizontal distance between the two vehicles represents their time headway (h), while the vertical distance represents their space headway (or spacing, s). The time headway is defined as the time difference of successive vehicle crossings taken at a given location. The space

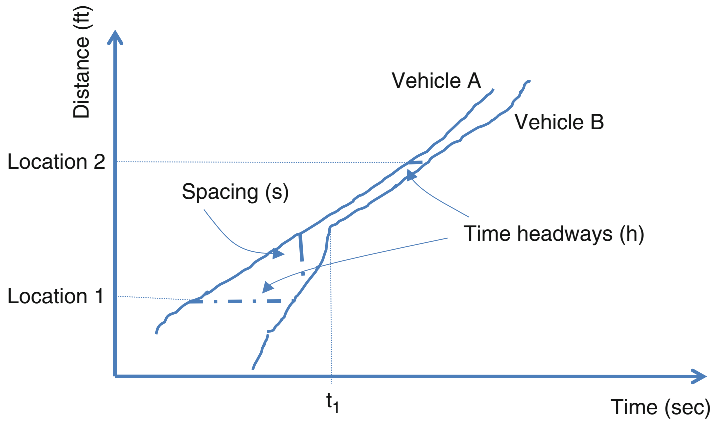


Fig. 2.1 Time–space diagram with two vehicle trajectories

headway is defined as the distance between two vehicles, measured usually from the front of the lead vehicle to the front of the following vehicle, at a given time. These are called microscopic traffic characteristics, because they consider the movement of individual vehicles and their relative time and space headways. Together with speed they are the three fundamental microscopic characteristics of traffic.

The time headway varies in space; thus if an observer is standing at Location 1, the time headway measured would be very different than that measured downstream at Location 2: the time headway at Location 2 is significantly smaller. Similarly, the spacing varies in time between these two vehicles.

Figure 2.2 presents a time–space diagram with several vehicles traveling along a single-lane roadway segment. Group X is separated by Group Y by a significant time and space gap. This occurs because the leading vehicle of Group Y desires to travel at a lower speed, and thus vehicles behind it must lower their speeds accordingly. As discussed earlier, the time headway between vehicles is represented by the horizontal distance between vehicles (h), while the space headway, or spacing, is represented by the vertical distance (s). Mathematically, the flow (F) at point P is

$$\boxed{F(\text{veh/h}) = 3{,}600/h_{\text{avg}}} \tag{2.1}$$

where h_{avg} is the average time headway in s.

In other words, Eq. (2.1) indicates that within an hour (or 3,600 s) the highway can process F vehicles with an average time headway h_{avg}. Similarly, within the analysis interval T of Fig. 2.2, the highway lane can process:

$$\text{Flow(vehicles)} = T\,\text{s}/h_{\text{avg}}$$

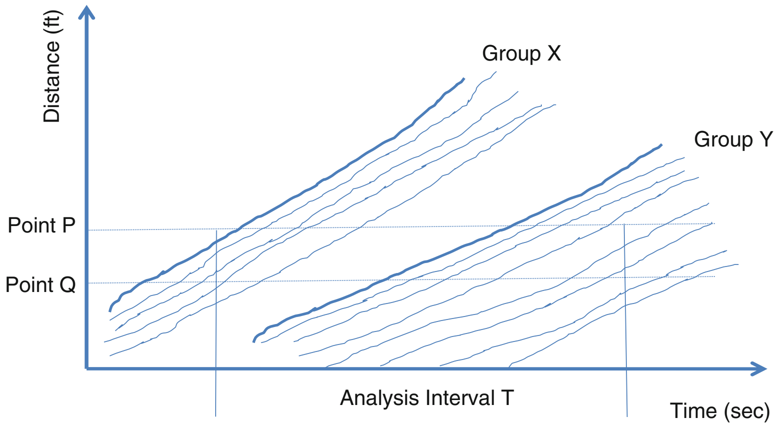


Fig. 2.2 Time–space diagram with groups of vehicles

This volume can be converted into flow by extrapolating the interval T into a full hour:

$$F(\text{veh/h}) = \text{Volume}\,(3,600/T) = \left(T\,\text{s}/h_{\text{avg}}\right)(3,600/T) = 3,600/h_{\text{avg}}$$

As h_{avg} is reduced, the total volume (or flow) increases. Conversely, as it increases, the total volume that can be processed decreases. Thus, the manner in which one vehicle follows another and their respective time headways are crucial in terms of the number of vehicles that a facility carries.

In Figure 2.2, the total number of vehicles an observer at Point P would count in time T is 9. During the same interval T, an observer at Point Q would count eight vehicles. Because of this variability in the manner in which vehicles follow one another (both between vehicles and for a specific vehicle in space) there is variability in our measurements of volume and flow. Considering the equations developed thus far, we can now examine the maximum amount of traffic a facility can carry, i.e., capacity. In theory the capacity of a particular highway can be obtained as follows:

$$\text{Capacity} = 3,600/(h_{\text{min}})$$

In practice however this minimum varies widely in time and space, as shown in Fig. 2.2. The minimum time headway at point P is not the same as that at point Q. Furthermore, this minimum is not representative of the time headways present during a particular time interval, and thus the actual maximum flow would be lower than that estimated using the above equation. Lastly, the minimum headways vary on a daily basis as a function of various factors including driver characteristics and behavior as well as vehicle capabilities for the traffic stream. Thus, a better

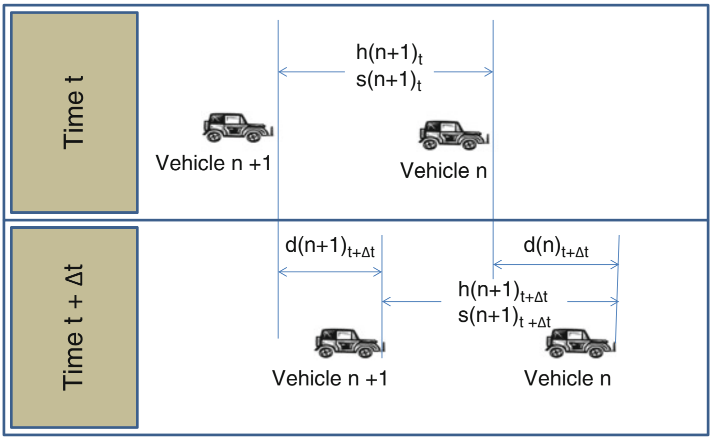


Fig. 2.3 Conceptual diagram and notation for car-following

understanding of the car-following process can help us appreciate the extent and impact of variability on traffic operations and the manner in which car-following affects our observations at the macroscopic level.

Let us now turn our attention to modeling the car-following process. Car-following algorithms determine the movement of the following vehicle at time $t + \Delta t$, as a function of its relationship to the lead vehicle at time t. Conceptually, the movement of the following vehicle (Vehicle $n + 1$) relative to the lead vehicle (Vehicle n) can be described as indicated in Fig. 2.3. At time t, vehicle $n + 1$ follows vehicle n at a time headway $h(n + 1)_t$ and a spacing $s(n + 1)_t$. Within time Δt, vehicle n has traveled a distance $d(n)_{t+\Delta t}$, while vehicle $n + 1$ has traveled a distance $d(n + 1)_{t+\Delta t}$. If the lead vehicle is unconstrained by traffic or other conditions ahead, its speed will be dictated by the geometry of the highway as well as its respective driver and vehicle characteristics, and its trajectory will follow the rules discussed in Chap. 1. The movement of the following vehicle however will largely depend on the trajectory of the lead vehicle, in addition to the characteristics of the follower driver and vehicle. At time $t + \Delta t$ its time headway from the lead vehicle will be $h(n + 1)_{t+\Delta t}$, and its spacing will be $s(n + 1)_{t+\Delta t}$.

Various car-following algorithms have been developed and are reported in the literature, and a variety of performance measures have been used to predict the trajectory of the following vehicle. A car-following algorithm might be based on predicting the acceleration of the following vehicle, another might be based on its speed, and another might be based on its space headway from the lead vehicle. The next subsection provides an historical overview of car-following algorithm development and implementation. Next, there is a discussion on some of the most widely cited algorithms, followed by an overview of efforts to compare car-following models to field data. The last section provides come concluding comments on car-following algorithms.

A Historical Overview of Car-Following Algorithms

Car-following models were initially developed in the 1950s [1], as traffic was increasing and researchers sought to understand what affects highway capacity. Since then, several models have been developed to replicate the time and space headways in car-following mode and generally to replicate car-following behavior. These models generally describe the trajectory of the following vehicle as a function of the trajectory of the first vehicle and the vehicles' distance/time headway. This section describes a few representative car-following models in chronological order starting from the earliest ones and concluding with some of the currently used models. The discussion in this section is not exhaustive in any way, and it only aims to familiarize the reader with some of the basic relationships developed to describe car-following.

Early Models

In one of the first papers to address car-following, Pipes [2] examined mathematically the dynamics of a line of traffic composed of n vehicles. His model assumed that the distance of the following vehicle to the lead vehicle is equal to one vehicle length for every 10 miles per hour of speed, plus a minimum distance between vehicles at standstill. This assumption was based on the California Vehicle Code which stated that "A good rule for following another vehicle at a safe distance is to allow yourself the length of a car (about 15 ft) for every 10 miles per hour you are travelling." For this rule to be satisfied, the coordinates $x(n)$ and $x(n + 1)$ of two successive vehicles, n and $n + 1$, must satisfy the following equation:

$$\boxed{x(n) = x(n+1) + b + L(n) + s_{\mathrm{MIN}}(n+1)} \tag{2.2}$$

where

$x(n)$, $x(n + 1)$ are the coordinates of vehicles n and $n + 1$, measured from the front of each vehicle
b is the distance between vehicles when they are at a standstill, in ft
$L(n)$ is the length of vehicle n, in ft
$s_{\mathrm{MIN}}(n + 1)$ is the minimum distance between vehicles based on the California Vehicle Code, in ft

The quantity $s_{\mathrm{MIN}}(n + 1)$ can also be written as follows:

$$\boxed{s_{\mathrm{MIN}}(n+1) = Tv(n+1)} \tag{2.3}$$

where

$v(n + 1)$ is the speed of vehicle $n + 1$, in ft/s
T is the time it takes to travel this minimum distance, in s

For example, for a vehicle traveling at 10 mph (≈14.67 ft/s), and assuming a vehicle length of 15 ft, T is

$$T = s_{\mathrm{MIN}}(n+1)/v(n+1) = (15\,\mathrm{ft})/(14.67\,\mathrm{ft/s}) = 1.023s$$

If a vehicle travels at a speed of 50 mph, its length is 15 ft, and the distance between vehicles at standstill is 6 ft, then it is expected that the vehicle will keep a distance of 5 × 15 + 6 = 81 ft from the lead vehicle.

Combining Eqs. (2.2) and (2.3) results in

$$x(n) = x(n+1) + b + L(n) + Tv(n+1)$$

Differentiating with respect to time:

$$\dot{x}(n) = \dot{x}(n+1) + T\dot{v}(n+1) \Rightarrow$$

$$\boxed{v(n) = v(n+1) + Ta(n+1), \quad n = 1,2,3\ldots k \text{ vehicles in the line of traffic}} \tag{2.4}$$

Equation (2.4) are the dynamical equations that describe the movement of k vehicles. Using these, Pipes developed differential equations for providing the velocity and acceleration of a line of vehicles, as a function of the movement of the lead vehicle. He solved these equations for specific initial conditions of the lead vehicle and recognized that the complexity of vehicle movement in terms of velocity and acceleration functions of the lead vehicle would require the use of a computer in order to solve them.

Aside from the difficulty in solving those equations analytically, the Pipes model assumes that the target spacing is achieved instantaneously and does not consider the reaction time of the following vehicle [3]. Also, the model requires the use of a theoretical spacing which cannot be violated, as it does not allow for deviations from the "following distance" law assumed. In reality, drivers have a time lag in reacting to changes in the lead vehicle's trajectory and position themselves with considerable deviation from this law, with significant variability in their spacing, both between vehicles, and longitudinally for a particular driver.

Regardless of these limitations, the Pipes model opened the door to significant development in the area of car-following. Starting from the system of Eq. (2.4), one can obtain:

$$v(n) = v(n+1) + Ta(n+1) \Rightarrow$$

$$\boxed{a(n+1) = \frac{v(n) - v(n+1)}{T}} \tag{2.5}$$

In this form, the Pipes model is similar to the stimulus-response type models developed by Gazis, Herman, Rothery, and others (also known as GHR models or GM models, because they were developed and tested at the General Motors laboratory). These models [1, 3–6] considered the sensitivity of the reaction to the lead vehicle, and they were of the form:

$$\boxed{a(n+1)_{t+\Delta t} = \lambda_{m,l}\left[v(n+1)_t\right]^m \frac{v(n)_t - v(n+1)_t}{\left[x(n)_t - x(n+1)_t\right]^1}} \tag{2.6}$$

where Δt is the next time interval (it can also be interpreted as the reaction time), $\lambda_{m,l}$ represented a sensitivity parameter, and m and l are calibration exponents. As shown in Eq. (2.6) a major difference between the GHR model and the Pipes model is that the GHR model incorporates time into the equation and provides the acceleration of the following vehicle at time $t + \Delta t$ as a function of the vehicles' status at time t. The general form of Eq. (2.6) is

$$\text{Response}\,(t + \Delta t) = \text{Function}\{\text{Sensitivity},\ \ \text{Stimulus}\,(t)\}$$

where response is the acceleration of the following vehicle at time $t + \Delta t$, stimulus is the speed difference between the two vehicles at time t, and the remaining factors represent the *s*ensitivity of the model.

For $m = 0$ and $l = 0$, the model in Eq. (2.6) reduces to

$$a(n+1)_{t+\Delta t} = \lambda_{0,0}\left[v(n)_t - v(n+1)_t\right]$$

This was the first form of the GHR model which was tested and calibrated by conducting car-following experiments. According to [1] in those initial tests the parameter value for λ ranged between 0.17 and 0.74.

Subsequent work produced several different calibrations of those factors. References [1, 6] provide a very thorough discussion of the development and calibration of the GHR models along with optimal values for the m and l parameters. An important finding of those investigations was that different conditions resulted in different combinations of m and l. Thus, for example, it was suggested that non-congested and congested conditions should use different calibration parameters [6, 7].

From Eq. (2.6), one of the shortcomings of the GHR models is that when the vehicles travel at the same speed (speed difference equals zero and thus the stimulus becomes zero), the model cannot estimate the reaction of the following vehicle. Also, the speed difference determines whether the following vehicle will accelerate or decelerate, regardless of the vehicles' spacing: even when the vehicles are far apart, the following vehicle will always decelerate as a response of the lead vehicle's deceleration. Finally, when the following vehicle's speed increases, the sensitivity of the following vehicle also increases: thus, at high speeds, the model becomes too sensitive to the stimuli from the lead vehicle. In practical applications, minor changes in speed differences result in large acceleration values. These issues, along with the difficulty in reaching consensus as to the appropriate set of l and m factors that should be used under different conditions, have inhibited the use of those models. For a detailed discussion on the GM models along with applications, consult [1, 2].

Early Microsimulator Implementation: The PITT Model

One of the first car-following models to be developed and implemented in a microsimulator environment was the PITT model (developed at the University of Pittsburgh). The model was initially implemented into INTRAS, a microsimulator developed in the late 1970s by the Federal Highway Administration (FHWA) to evaluate freeway control and management strategies [8]. Subsequently INTRAS was replaced by FRESIM and then CORSIM both of which have been widely used and evaluated (additional information on CORSIM is provided in Chap. 7).

Similarly to the Pipes model, the PITT model is also based on the assumption that the follower vehicle attempts to maintain a fixed time headway between its front bumper and the rear bumper of the lead vehicle [8]. It is assumed that this distance is proportional to the speed of the follower and the speed difference between the leader and the follower. Mathematically:

$$\boxed{s(n+1)_t = L(n) + B(n) + kv(n+1)_t + bk\left[v(n)_t - v(n+1)_t\right]^2} \quad (2.7)$$

where

$L(n)$ is the length of the lead vehicle (n)
$B(n)$ is the buffer between vehicles at a standstill
k and b are car-following sensitivity parameters
$v(n)_t$ and $v(n+1)_t$ are the speeds of the lead and following vehicles, respectively, at time t

Similarly, at time $t + \Delta t$, the equation becomes

$$\boxed{s(n+1)_{t+\Delta t} = L(n) + B(n) + kv(n+1)_{t+\Delta t} + bk\left[v(n)_{t+\Delta t} - v(n+1)_{t+\Delta t}\right]^2} \quad (2.8)$$

Equations (2.7) and (2.8) provide the required equilibrium conditions for two vehicles in car-following. However, in a car-following model we need to estimate the acceleration of the following vehicle at time $t + \Delta t$, as a function of the positions and speeds of the two vehicles at time t. To introduce the following vehicle's acceleration at time $t + \Delta t$ into the equations, we express the spacing as a function of the respective coordinates as follows:

$$s(n+1)_{t+\Delta t} = x(n)_{t+\Delta t} - x(n+1)_{t+\Delta t}$$

The last term of the equation, which provides the distance the following vehicle travels during Δt, can be estimated assuming that the vehicle travels with a constant acceleration, and the equation becomes

$$\boxed{s(n+1)_{t+\Delta t} = x(n)_{t+\Delta t} - \left[x(n+1)_t + v(n+1)_t\Delta t + a(n+1)_{t+\Delta t}\frac{\Delta t^2}{2}\right]} \quad (2.9)$$

We also express $v(n + 1)_{t+\Delta t}$ as a function of acceleration as follows:

$$\boxed{v(n+1)_{t+\Delta t} = v(n+1)_t + a(n+1)_{t+\Delta t}\Delta t} \quad (2.10)$$

Next, we equate the right parts of Eqs. (2.8) and (2.9), replacing $v(n + 1)_{t+\Delta t}$ from Eq. (2.10), and we solve for $a(n + 1)_{t+\Delta t}$. Also, because the last part of Eq. (2.8) is very small, we replace $v(n + 1)_{t+\Delta t}$ with $v(n + 1)_t$. The final equation is

$$\boxed{\begin{aligned} &a(n+1)_{t+\Delta t} \\ &= 2\frac{\left[x(n)_{t+\Delta t} - x(n+1)_t - L(n) + B(n) - v(n+1)_t(k+\Delta t) - bk\left(v(n)_{t+\Delta t} - v(n+1)_t\right)^2\right]}{\Delta t^2 + 2k\Delta t} \end{aligned}} \quad (2.11)$$

Equation (2.11) represents the basic car-following equation for the PITT model. According to [8], the parameter b is a constant which takes the value 0.1 when $v_n < v_{n+1}$, or 0 otherwise; in essence, when the lead vehicle is traveling faster than the follower their speed difference does not affect the acceleration of the following vehicle.

Contrary to the GM models, the PITT car-following model considers the final speed of the lead vehicle, i.e., the speed at the end of Δt, rather than the respective conditions at time t. In other words, the acceleration during Δt is estimated as a function of where the lead vehicle will be at the *end* of Δt. In implementing this model the perception and reaction time of the driver need to be considered separately and after the acceleration is calculated [8, 9]. According to [9], the PITT car-following model does not replicate traffic oscillations very well, and any traffic disturbances diminish quickly. This is important in that breakdown does not occur randomly, as is the case in the field, but only after demand exceeds capacity (see Chap. 4 for a discussion of capacity and breakdown). Finally, to implement the model it is necessary to introduce several constraints to ensure that the trajectory of the following vehicle is reasonable; application of the model has shown that those constraints determine the acceleration most of the time [9].

Currently Used Models: The Gipps Model

The Gipps model [10] was developed in Australia and is one of the most widely used and cited car-following models. It is currently used in the AIMSUN microsimulator [11]. This model can be categorized as a multi-regime model because it considers the desired speed of the following vehicle, as well as whether the following vehicle is in breaking mode or car-following mode. The model calculates two speeds: the speed of the following vehicle under non-constrained conditions and the speed that would result if the following vehicle is constrained by the lead vehicle. The minimum of these two speeds is selected as the follower vehicle speed. The model is the following:

$$v(n+1)_{t+\Delta t} = \min\begin{cases} v(n+1)_t + 2.5a(n+1)_{\text{MAX}}\Delta t\left(1 - \frac{v(n+1)_t}{v(n+1)_{\text{DES}}}\right)\left(0.025 + \frac{v(n+1)_t}{v(n+1)_{\text{DES}}}\right)^{1/2} \\ b(n+1)\Delta t + \sqrt{\left(b(n+1)^2\Delta t^2 - b(n+1)\left[2\left[x(n)_t - L(n) - x(n+1)_t\right] - v(n+1)_t\Delta t - \frac{v(n)_t^2}{\widehat{b}_n}\right]\right)} \end{cases} \tag{2.12}$$

where

$v(n+1)_{t+\Delta t}$ is the speed of vehicle $n+1$ at time $t+\Delta t$
Δt is the apparent reaction time, a constant for all vehicles
$v(n+1)_t$ is the speed of vehicle $n+1$ at time t
$a(n+1)_{\text{MAX}}$ is the maximum acceleration which the driver of vehicle $n+1$ wishes to undertake
$v(n+1)_{\text{DES}}$ is the speed at which the driver of vehicle $n+1$ wishes to travel
$b(n+1)$ is the actual most severe deceleration rate that the driver of vehicle $n+1$ wishes to undertake ($b(n+1) < 0$)
$x(n)_t$ is the location of the front of vehicle n at time t
$x(n+1)_t$ is the location of the front of vehicle $n+1$ at time t
$L(n)$ is the effective size of vehicle n; that is the physical length plus a margin into which the following vehicle is not willing to intrude even when at rest
$v(n)_t$ is the speed of vehicle n at time t
$\widehat{b}_n$ is the most severe deceleration rate that vehicle $n+1$ estimates for vehicle n

The first term of Eq. (2.12) estimates the following vehicle's speed under non-constrained conditions and was developed based on field data. The equation ensures that the speed of vehicle $n+1$ will not exceed the driver's desired speed. It also ensures that the acceleration of vehicle $n+1$ will reach zero when the desired speed is reached. Finally, it modifies the acceleration of vehicle $n+1$ when it is not constrained by vehicle n, so that it increases with decreasing speed and vice versa.

The second term in Eq. (2.12) was developed to allow the following vehicle to be able to stop safely even for the most severe braking of the lead vehicle. The equation is based on the most severe braking of the lead vehicle and estimates the stopping distance of the following vehicle considering the driver's reaction time plus a safety margin.

One of Gipps' goals in developing this model was to be able to relate each of its parameters to vehicle or driver characteristics that the driver of the following vehicle could estimate or perceive. Thus, instead of using the most severe deceleration of vehicle n, he used the respective estimate of the following driver for that quantity $\left(\widehat{b}_n\right)$. Calibrating this model is relatively easy, as one can measure or estimate its parameters based on vehicle and driver characteristics.

The results this model produces have generally been found reasonable [6, 12]. One of the criticisms of this model is that it is based on a "safe" headway, which may not necessarily be valid in real traffic [6]. Drivers may in practice be willing to accept shorter headways. Also, researchers have suggested that considering the traffic conditions downstream may result in a more realistic model.

Example 2.1 A vehicle is in car-following mode, following a lead vehicle with the trajectory shown in Table 2.1. Assuming that the desired maximum speed of the following vehicle, $v(n+1)_{\text{DES}}$, is 75 mph, the maximum acceleration which the following vehicle wishes to undertake, $a(n+1)_{\text{MAX}}$, is 6.5 ft/s^2, the actual most severe deceleration that the follower wishes to undertake, $b(n+1)$, is -9.5 ft/s^2, the most severe deceleration rate that vehicle $n+1$ estimates for vehicle n, $\widehat{b}_n$, is -11.5 ft/s^2, and the effective vehicle length, $L(n)$, is 25 ft, calculate the trajectory of the following vehicle using the Gipps car-following model.

Solution to Example 2.1
Table 2.2 calculates the detailed trajectories of both vehicles. The left side of the table provides the data related to the movement of the lead vehicle. The acceleration of the lead vehicle for each time interval is calculated using the speeds at time t and $t+1$. For example, the average acceleration during the interval 2–3 s is (70.40–74.73)/1 s = -4.33 ft/s^2. The location of the lead vehicle is estimated based on the equations of motion assuming constant acceleration during each 1-s interval.

The speed of the following vehicle is estimated using Eq. (2.12). The column labeled "Speed 1" estimates the first part of the Gipps car-following equation, while the one labeled "Speed 2" estimates the second part. The minimum of these two speeds is the estimated speed of the following vehicle. The acceleration, location, and spacing of the following vehicle are estimated using the equations of motion similarly to those for the lead vehicle. Figure 2.4 provides the speed vs. time plot for both the lead and following vehicles, while Fig. 2.5 provides the trajectories of the two vehicles.

Table 2.1 Lead vehicle trajectory and follower starting position

Lead vehicle		Following vehicle	
Time (s)	Speed (mph)	Speed (mph)	Spacing (ft)
1.00	52.37	54.3	120
2.00	50.95		
3.00	48.00		
4.00	46.00		
5.00	44.00		
6.00	42.00		
7.00	41.00		
8.00	39.00		
9.00	37.00		
10.00	35.00		
11.00	33.00		
12.00	31.00		
13.00	29.00		
14.00	25.00		
15.00	22.00		
16.00	19.00		
17.00	21.00		
18.00	24.00		
19.00	26.00		
20.00	29.00		
21.00	31.00		
22.00	33.00		
23.00	36.00		
24.00	39.00		
25.00	42.00		
26.00	45.00		
27.00	49.00		
28.00	50.91		
29.00	51.50		
30.00	50.67		

Other Currently Used Models

The MITSIM Model

The car-following model used by the MIT Simulator (MITSIM [13]) is a multi-regime version of the GHR model. Multi-regime models are those which consider various conditions (or regimes) for the car-following vehicle and specify different models for each condition.

The MITSIM model estimates the acceleration of the following vehicle at the end of $t + \Delta t$ as a function of the headway and the relative speed between the lead and the following vehicle. Depending on the size of the headway between the lead and the

Table 2.2 Lead and following vehicle trajectory data for Example 2.1

Lead vehicle					Following vehicle						
Time (s)	Speed (mph)	Speed (ft/s)	Acceleration (ft/s^2)	Location (ft)	Speed 1 (mph)	Speed 2 (mph)	Min of speeds 1 and 2	Speed (ft/s)	Location (ft)	Spacing (ft)	Acceleration (ft/s^2)
1.00	52.37	76.81	Not known	0.00	54.30			79.64	−120.00	120.00	Not known
2.00	50.95	74.73	−2.08	75.77	56.95	46.39	46.39	68.04	−46.16	121.93	−11.60
3.00	48.00	70.40	−4.33	148.33	49.78	45.89	45.89	67.30	21.51	126.82	−0.74
4.00	46.00	67.47	−2.93	217.26	49.32	43.99	43.99	64.53	87.42	129.84	−2.78
5.00	44.00	64.53	−2.93	283.26	47.58	42.83	42.83	62.81	151.09	132.17	−1.71
6.00	42.00	61.60	−2.93	346.33	46.50	41.59	41.59	61.00	213.00	133.33	−1.82
7.00	41.00	60.13	−1.47	407.20	45.35	40.28	40.28	59.08	273.04	134.16	−1.92
8.00	39.00	57.20	−2.93	465.86	44.13	39.72	39.72	58.25	331.71	134.16	−0.83
9.00	37.00	54.27	−2.93	521.60	43.60	38.30	38.30	56.18	388.92	132.68	−2.07
10.00	35.00	51.33	−2.93	574.40	42.27	36.83	36.83	54.02	444.02	130.38	−2.16
11.00	33.00	48.40	−2.93	624.26	40.88	35.31	35.31	51.79	496.93	127.34	−2.24
12.00	31.00	45.47	−2.93	671.20	39.44	33.73	33.73	49.47	547.56	123.64	−2.31
13.00	29.00	42.53	−2.93	715.20	37.93	32.11	32.11	47.09	595.84	119.36	−2.39
14.00	25.00	36.67	−5.87	754.80	36.37	30.43	30.43	44.63	641.70	113.10	−2.46
15.00	22.00	32.27	−4.40	789.26	34.75	27.28	27.28	40.01	684.02	105.25	−4.63
16.00	19.00	27.87	−4.40	819.33	31.67	24.73	24.73	36.27	722.15	97.18	−3.73
17.00	21.00	30.80	2.93	848.66	29.15	22.11	22.11	32.44	756.51	92.16	−3.84
18.00	24.00	35.20	4.40	881.66	26.53	22.78	22.78	33.42	789.43	92.23	0.98
19.00	26.00	38.13	2.93	918.33	27.21	24.57	24.57	36.04	824.16	94.17	2.62
20.00	29.00	42.53	4.40	958.66	29.00	25.96	25.96	38.08	861.22	97.45	2.04
21.00	31.00	45.47	2.93	1,002.66	30.37	28.29	28.29	41.49	901.00	101.67	3.41
22.00	33.00	48.40	2.93	1,049.60	32.66	29.99	29.99	43.98	943.73	105.87	2.50
23.00	36.00	52.80	4.40	1,100.20	34.32	31.75	31.75	46.57	989.01	111.19	2.59
24.00	39.00	57.20	4.40	1,155.20	36.03	34.37	34.37	50.40	1,037.50	117.70	3.83
25.00	42.00	61.60	4.40	1,214.60	38.54	37.05	37.05	54.34	1,089.87	124.73	3.94
26.00	45.00	66.00	4.40	1,278.40	41.09	39.78	39.78	58.35	1,146.21	132.18	4.01
27.00	49.00	71.87	5.87	1,347.33	43.66	42.55	42.55	62.41	1,206.59	140.74	4.06
28.00	50.91	74.67	2.80	1,420.60	46.24	46.17	46.17	67.72	1,271.66	148.94	5.31

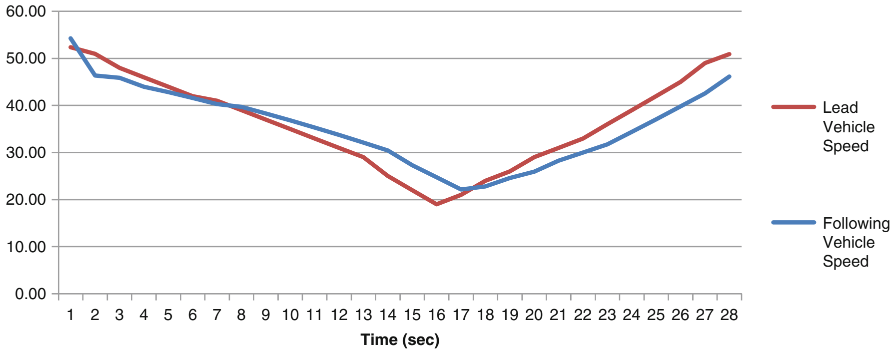


Fig. 2.4 Lead and following vehicle speeds for Example 2.1

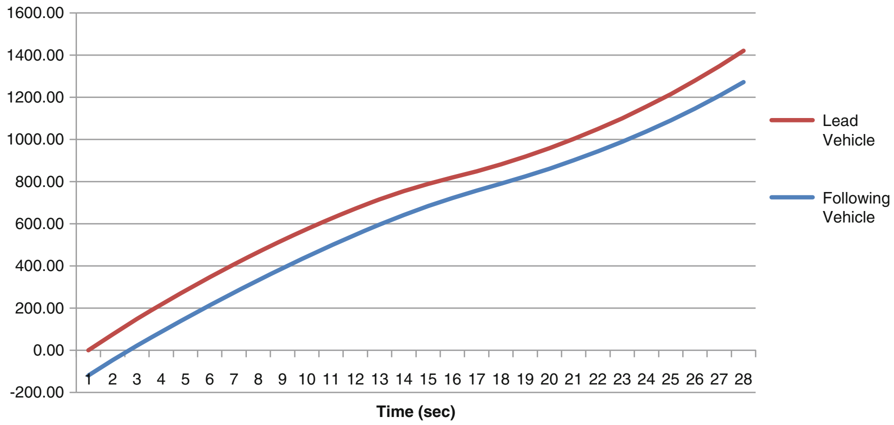


Fig. 2.5 Lead and following vehicle trajectories

following vehicles, the following vehicle is classified to be into one of three regimes: free following, emergency decelerating, and car-following.

Free flowing regime: If the time headway is larger than a predetermined threshold h^{upper}, the vehicle acceleration is not estimated as a function of the lead vehicle trajectory. If the vehicle's current speed is lower than its maximum speed, it accelerates at the maximum acceleration rate to achieve its desired speed as quickly as possible. If its current speed is higher than the maximum speed, the vehicle decelerates with the normal deceleration rate.

Emergency regime: If a vehicle has headway smaller than a predetermined threshold h^{lower}, it is in emergency regime. In this case, the vehicle uses an appropriate deceleration rate in order to avoid colliding with the lead vehicle.

Car-following regime: If a vehicle has headway between h^{lower} and h^{upper}, it is in car-following. In this case the acceleration rate is calculated using the GHR model [13, 14]:

$$\alpha(n+1)_{t+\Delta t} = \alpha^{\pm} \frac{v(n+1)_t^{\beta\pm}}{s(n+1)_t^{\gamma\pm}} \left[v(n)_t - v(n+1)_t \right] \tag{2.13}$$

where

$a(n+1)_{t+\Delta t}$ is the acceleration of vehicle $n + 1$ at time $t + \Delta t$
$v(n)_t$ is the speed of vehicle n at time t
$v(n+1)_t$ is the speed of vehicle $n + 1$ at time t
$s(n+1)_t$ is the spacing between the follower and the lead vehicle
α, β and γ are model parameters that can be calibrated as a function of driver behavior. Positive values of the parameters correspond to acceleration, while negative values to deceleration.

Equation (2.13) is identical to Eq. (2.6), only with slight changes in notation (α, β and γ instead of $\lambda_{m,l}$, m, l). The major difference between the MITSIM and the GHR models is that MITSIM explicitly considers the different regimes the following vehicle may belong in and sets appropriate limits in the application of the car-following equation.

The Wiedemann Model

The Wiedemann model is also a multi-regime model, which uses specified thresholds for anticipated changes in driver behavior. The model is also defined as a psychophysical model, because the thresholds used to define different regimes are based on driver perceptions and actions. For example, instead of using deterministic thresholds for spacing in order to apply a particular model, the Wiedemann approach considers the distance at which drivers are able to perceive relative velocity between their vehicle and the lead vehicle; when they cannot perceive it, the following vehicle is not in car-following any more.

Figure 2.6 displays the thresholds and regimes of Wiedemann model. Four driving modes (or regimes) can be distinguished [14]:

1. Free driving: no influence of leading vehicles. In this mode the follower vehicle seeks to reach and maintain her/his individually desired speed.
2. Approaching: when passing the approaching point (SDV) threshold. This regime consists of the process of adapting the driver's own speed to the lower speed of the lead vehicle. While approaching, a driver applies a deceleration so that the speed difference of the two vehicles is zero in the moment she/he reaches her/his desired safety distance.
3. Following: the thresholds SDV, ABX (desired minimum following distance at low speed differences), SDX (the maximum following distance), and OPDV (the

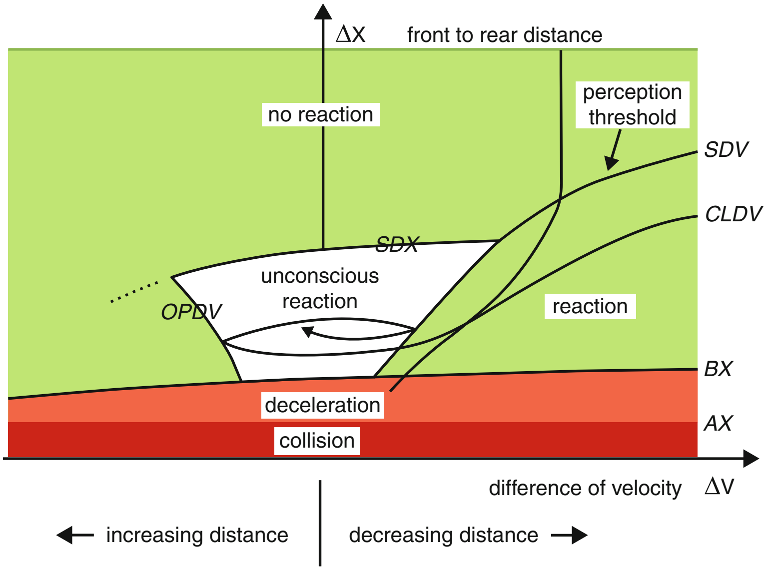


Fig. 2.6 Thresholds and regimes in the Wiedemann car-following model (From PTV VISSIM Users Manual; Reproduced with permission of PTV AG)

increasing speed difference) constitute this regime. The driver follows the preceding car without any conscious acceleration or deceleration. She/he keeps the safety distance more or less constant.

4. Braking or emergency regime: when the front to rear distance is smaller than ABX the follower adopts the emergency regime. The driver applies medium to high deceleration rates if the distance falls below the desired safety distance. This can occur if the preceding car changes speed abruptly or if a third car changes lanes in front of the lead vehicle.

For each regime, the acceleration is described as a result of speed, speed difference, distance, and the individual characteristics of driver and vehicle. The driver switches from one mode to another as soon as she/he reaches a certain threshold that can be expressed as a combination of speed difference and distance.

Evaluations of Car-Following Algorithms Using Field Data

Various techniques have been used to compare car-following models to field data. Some of the earlier attempts [15, 16] compared the GM model results to data collected by wire-linked vehicles. More recently [17], researchers used an aerial data collection technique to collect car-following data. Recent advances in GPS and sensor technology have led to an increasing number of studies that compare car-following models to field data. A few studies have used GPS [18], while others

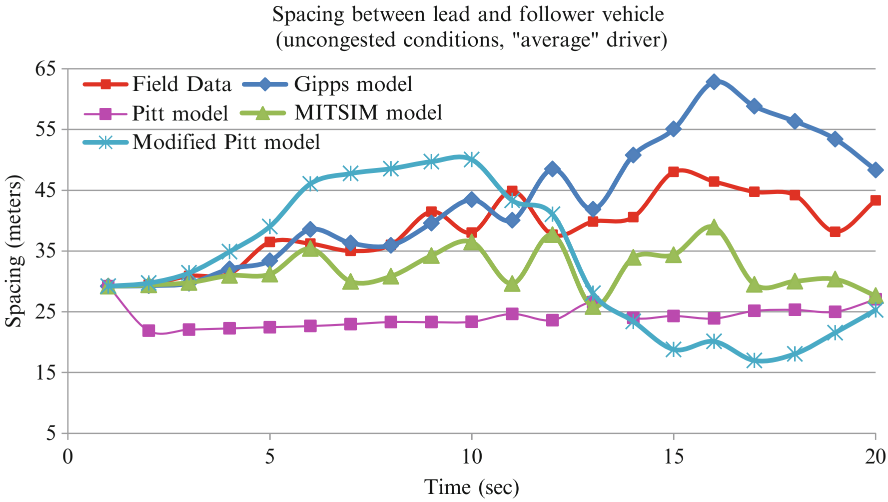


Fig. 2.7 Comparison of selected car-following models to field data [12]

[12, 19] have used an instrumented vehicle to record the lead and following vehicle trajectories as well as driver information. One of the often cited conclusions is the significance of accurate measurements in obtaining speeds and distances of the subject vehicles.

Figure 2.7 provides an illustration of a car-following trajectory compared to estimated trajectories obtained using an instrumented vehicle [12] The graph compares car-following models to a trajectory by an "average" driver (i.e., neither too aggressive nor too conservative) and for non-congested conditions. The red line indicates the field-measured spacing. As shown, each model predicts a slightly different spacing throughout the car-following event. The authors of the research reported that the models generally predict speed more accurately than spacing. However, the best variable to be used in calibration is spacing: calibrating by spacing minimizes the errors that can be accumulated and can distort the final trajectory. Generally, car-following for congested scenarios has been found to be most accurately predicted because there is much less freedom and variability in driver behavior during congested conditions [12]. For non-congested conditions the driver type variability is much more significant, and calibration needs to take into consideration driver aggressiveness.

Concluding Remarks on Car-Following Models

Car-following is a complex process, which depends significantly on driver decisions and actions. There are several recent advances both to develop more accurate models for replicating car-following and to control car-following behavior through the development of advanced vehicle technologies.

Various car-following models have been developed and continue to be developed in light of tremendous advances in traffic simulation. Multi-regime models appear to provide the most logical approach to car-following, particularly those that consider driver behavior and its variability. With respect to future developments on the topic, one of the most recent research efforts which appears promising used fuzzy logic to describe driver behavior in car-following [20]. Such research is based on the recognition that the reactions of the following vehicle to the lead vehicle are not based on a deterministic relationship, but rather on a set of random or approximate driving behaviors. Such models are at their infancy and have not been extensively calibrated or tested.

More generally, the following factors may affect car-following behavior, but have not yet been thoroughly evaluated to quantify their impact: highway design elements such as grade or lane width; vehicle characteristics such as braking ability; driver characteristics such as reaction time; and adverse weather and other environmental conditions. Thus, there is still quite a bit of research that needs to be completed to be able to replicate car-following behavior reasonably well for a variety of conditions and cases and considering variability in driver behavior.

An essential part of car-following model development is the ability to obtain good quality field data and evaluate or calibrate following vehicle trajectories as estimated by various types of models. Existing data collection capabilities (video data collection, sensors, etc.) afford much greater opportunities than previously available to accomplish this. It is important to be able to accurately observe the trajectories of both the lead and the following vehicle and also to measure and evaluate driver behavior and characteristics, so that car-following behavior can be modeled.

An important concept that has been discussed since the infancy of car-following models is the stability of the traffic stream, i.e., its ability to absorb a perturbation [3, 21]. A traffic system is referred to as stable when the fluctuation in acceleration of the lead vehicle does not cause an increasingly fluctuating acceleration and spacing in the following vehicle or vehicles. Local stability refers to the acceleration and spacing of only one car-following vehicle; asymptotic stability refers to the acceleration and spacing of a series of vehicles traveling one behind another. Although a series of vehicles may be stable locally, the system may not be stable when each vehicle amplifies the perturbation which travels upstream with increasing magnitude [3]. It has been shown [16] that when a car-following vehicle considers information related to the car downstream from the lead vehicle the traffic stream stability increases. It is speculated [3] that when such information is lacking in the real world, as for example when there is reduced visibility, the traffic stream becomes highly unstable and thus there is a high probability of rear-end collisions. With respect to car-following models, it is thus important that factors related to two or more vehicles downstream of the car-following vehicle be incorporated, so that the resulting traffic stream is as stable as a similar one in the field.

As researchers understood the importance of car-following, research has also sought to develop vehicle-based technologies that can control it in order to improve safety and increase capacity. The concept of automated highways where vehicles follow one another under specified rules has been around for quite some time. More recent efforts by vehicle manufacturers to develop adaptive cruise control (ACC) and cooperative adaptive cruise control (CACC) mechanisms for use in passenger vehicles have the potential not only to significantly increase safety, but also to result in significant changes in traffic flow and car-following. Studies have shown [22] that ACC may reduce congestion even at a market penetration of 20 % of vehicles in the traffic stream. Thus it is possible that future work in car-following focuses on the development of appropriate algorithms that can be used within vehicles such that they will be able to maintain stable headways, thus increasing safety and reducing congestion.

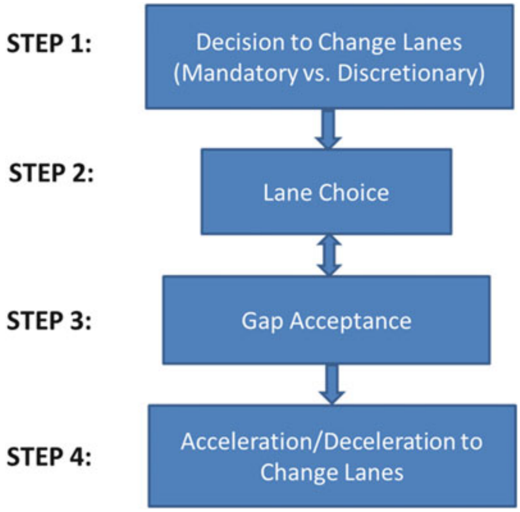
Lane Changing

Compared to car-following models, in which the behavior of the lead vehicle is relatively unaffected by the lag one, the lane-changing process depends on many parameters, and hence it is more complex. Drivers have many different reasons for changing lanes, and their lane change maneuver is likely to be affected by its urgency (e.g., it may be imperative to change lanes in order to get positioned for an upcoming turn, while it may not be as pressing to pass a slightly slower vehicle ahead). Also, drivers may change lanes with or without cooperation from the vehicles in the target lane. Therefore the characteristics of lane changing are several and diverse. However, it has not been studied as extensively as car-following. Its examination has intensified relatively recently and particularly with the increased use of microsimulation.

There are no analytical relationships that encompass the entire lane-changing process. Instead, it is typically modeled as a sequence of several decision-making steps, such as those shown in Fig. 2.8 [23]. In Step 1 the driver considers whether a lane change is necessary or desirable. When a lane change is necessary (e.g., because of an upcoming turn) the lane change is called mandatory; otherwise it is called discretionary. Discretionary lane changes are performed to gain some advantage, such as to increase the vehicle's speed, or improve its position in the queue. Different drivers have different criteria on whether to attempt a discretionary lane change, and thus driver characteristics and behavior become very important in the lane-changing process. In modeling this step, one may develop criteria or thresholds that, once met, would trigger an attempt for a lane change. These criteria, or thresholds, may differ for different types of drivers and under different conditions, and they could take into consideration various factors related to the traffic conditions and the driving environment.

Next, the driver determines the target lane for the lane change (Step 2). This decision is more complex for discretionary lane changes, where the driver needs to select a lane based on a set of criteria (e.g., queue length or operating speed in the

Fig. 2.8 The four steps typically involved in lane changing [23]



target lane). Again, those criteria differ widely for different types of drivers. An approach that has been used in target lane determination is discrete choice (or discrete outcome) models [24, 25]. Such models are based on the concept of utility maximization, i.e., they assume that the driver will select the lane that will provide the most utility, and they estimate the utility provided by each alternative. The probability that driver x will select lane i is

$$\boxed{P_{x,i} = \text{prob}[V_{x,i} > V_{x,n}] \quad \text{for all} \quad n \neq i} \tag{2.14}$$

where

$V_{x,i}$ is the total utility of lane i to traveler x
$V_{x,n}$ is the total utility of all other lanes to traveler x

The total utility can be written as follows:

$$V_{x,i} = U_{x,i} + \varepsilon_{x,i} = \sum_j \alpha_{i,j} y_{x,i,j} + \varepsilon_{x,i}$$

where

$U_{x,i}$ is the specifiable nonrandom component of utility for lane i and traveler x
$\varepsilon_{x,i}$ is the unspecifiable component of utility for lane i and traveler x, assumed to be random
$\alpha_{i,j}$ is the coefficient estimated using field data for lane i and roadway/environment characteristic j
$y_{x,i,j}$ is the roadway/environment characteristic for lane i and traveler x

Assuming that the random unspecified component of utility $\varepsilon_{x,i}$, generalized extreme value distributed, we obtain the logit model formulation [24, 25]:

$$\boxed{P_{x,i} = \frac{e^{U_{x,i}}}{\sum_n e^{U_{x,n}}}} \tag{2.15}$$

where e is the base of the natural logarithm ($e = 2.718$).

In Step 3 the driver evaluates the size of possible gaps in the target lane(s). The driver may accept a gap, reject a given gap, and attempt to find an acceptable one or reevaluate whether to perform a lane change. This step involves the gap acceptance process, which is discussed in more detail in the next section. Steps 2 and 3 may occur simultaneously, as for example when a driver is attempting to find a suitable gap in either of two adjacent lanes during a discretionary lane change. In Step 4, the vehicle moves into the target lane by adjusting its speed as necessary (accelerating or decelerating).

Many previous studies have focused on lane-changing behavior along freeways [26, 27], while fewer have studied lane changes in urban arterials, where the possible lane-changing reasons and occasions are much more numerous [23]. Gipps [28] developed one of the first lane-changing models for microsimulation tools, focusing on urban streets. In his model Gipps considered (a) whether it is physically possible and safe to change lanes; (b) the location of permanent obstructions; (c) the presence of transit lanes; (d) the driver's intended turning movement, (e) the presence of heavy vehicles; and (f) speed. The model consists of a flowchart to replicate decision making related to changing lanes and uses mathematical expressions to answer specific questions in the decision process. It was developed to satisfy the conflicting objectives of being positioned in the correct lane for an upcoming turn (long-term objective) while taking advantage of opportunities to gain speed (short-term objective). It assumes that the closer the vehicle is to the upcoming turn, the lower the probability of making lane changes for speed advantage.

In subsequent work, Hidas [29] developed a lane-changing algorithm which incorporated forced and cooperative lane-changing maneuvers. A forced lane change is defined as one where the lag vehicle in the target lane is forced to decelerate in order to create a suitable gap for the lane-changing vehicle. A cooperative lane-changing maneuver is one where the lag vehicle willingly decelerates to accommodate the lane-changing vehicle. These maneuvers are more likely to occur during congested conditions.

Example 2.2 The following utility functions have been developed using data from three-lane freeways to indicate driver preference for aggressive drivers for each of the three lanes:

$$U_{x,\mathrm{L}} = 23.5 - 1.65\,F_{\mathrm{TL}} + 0.18v_{\mathrm{L}}$$
$$U_{x,\mathrm{M}} = 71.87 - 1.85F_{\mathrm{TM}} + 0.21v_{\mathrm{M}}$$
$$U_{x,\mathrm{R}} = 259.14 - 2.27F_{\mathrm{TR}} + 0.25v_{\mathrm{R}}$$

where

$U_{x,\mathrm{L}}$, $U_{x,\mathrm{M}}$, $U_{x,\mathrm{R}}$ are the utilities of the left, middle, and right lanes, respectively
F_{TL}, F_{TM}, and F_{TR} are the truck flows of the left, middle, and right lanes, respectively
v_{L}, v_{M}, and v_{R} are the operating speeds of the left, middle, and right lanes, respectively

If during the peak hour the truck flows are $F_{\mathrm{TL}} = 20$ vph, $F_{\mathrm{TM}} = 45$ vph, and $F_{\mathrm{TR}} = 120$ vph and the speeds are $v_{\mathrm{L}} = 60$ mph, $v_{\mathrm{M}} = 58$ mph, and $v_{\mathrm{R}} = 53$ mph, estimate the probabilities that aggressive drivers will select each of the three lanes.

Solution to Example 2.2
The nonrandom component of utility for each of the three lanes is as follows:

$$U_{x,\mathrm{L}} = 23.5 - 1.65F_{\mathrm{TL}} + 0.18v_{\mathrm{L}} = 23.5 - 1.65 \times 20 + 0.18 \times 60 = 1.3$$
$$U_{x,\mathrm{M}} = 71.87 - 1.85F_{\mathrm{TM}} + 0.21v_{\mathrm{M}} = 0.8$$
$$U_{x,\mathrm{R}} = 259.14 - 2.27F_{\mathrm{TR}} + 0.25v_{\mathrm{R}} = -0.01$$

From Eq. (2.15), the probabilities that drivers will select each of the three lanes are as follows:

$$P_{x,\mathrm{L}} = \frac{e^{U_{x,\mathrm{L}}}}{\sum_n e^{U_{x,n}}} = \frac{e^{1.5}}{e^{1.5} + e^{0.8} + e^{-0.01}} = \frac{3.699}{3.699 + 2.23 + 0.09} = \frac{3.699}{6.919} = 0.535$$
$$P_{x,\mathrm{M}} = \frac{e^{U_{x,\mathrm{M}}}}{\sum_n e^{U_{x,n}}} = \frac{e^{0.8}}{e^{1.5} + e^{0.8} + e^{-0.01}} = \frac{2.23}{3.699 + 2.23 + 0.99} = \frac{2.23}{6.919} = 0.322$$
$$P_{x,\mathrm{R}} = \frac{e^{U_{x,\mathrm{R}}}}{\sum_n e^{U_{x,n}}} = \frac{e^{-0.01}}{e^{1.5} + e^{0.8} + e^{-0.01}} = \frac{0.99}{3.699 + 2.23 + 0.99} = \frac{0.99}{6.919} = 0.143$$

Gap Acceptance

Gap acceptance models are used to determine the number of vehicles or units of traffic that can pass through a conflicting traffic stream, when the conflicting traffic has to evaluate the size of the gap and make decisions regarding its acceptance. A gap is usually defined as the time headway between the rear end of the lead vehicle and the front end of the following vehicle; however, some references also define it as the time headway between successive vehicle arrivals. We will use here the former definition, and we will refer to the second one simply as the time headway. We also refer to the lead gap and the lag gap, as illustrated in Fig. 2.9. The lead gap is the gap to the lead vehicle in the target lane, while the lag gap is the gap to the lag vehicle in the target lane.

Gap acceptance modeling seeks to predict whether a particular size gap will be accepted or rejected by a driver or group of drivers under a given set of prevailing conditions. The driver arriving from a conflicting traffic stream, or changing lanes

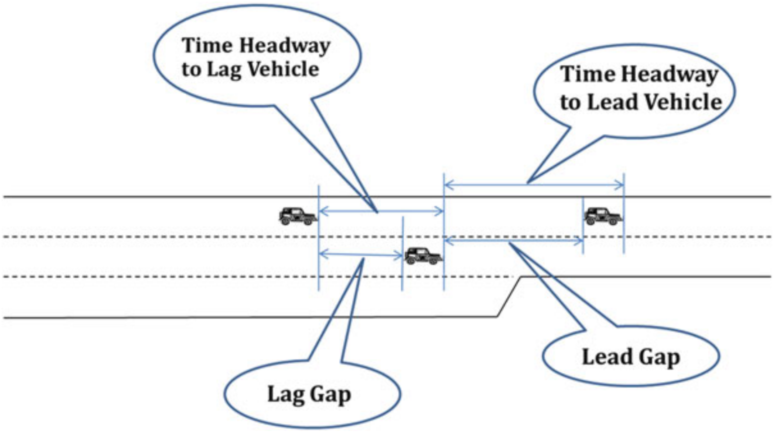


Fig. 2.9 Lane changing with consideration of lead and lag gaps

into the target lane, must evaluate the size of the gap and decide whether to accept it or not. Critical gap is defined as the minimum gap that a driver is willing to accept. The size of the critical gap varies significantly depending on the type of maneuver (lane changing along a freeway segment vs. making a left turn out of a stop-controlled approach) and the driver characteristics. Also, the size of the gap a driver would accept varies based on the amount of time a driver has already been waiting; the longer this interval is, the more likely that the driver would accept a shorter gap.

A major difficulty in gap acceptance modeling is that this critical gap cannot be directly measured. It must be inferred based on the gaps drivers accept and those they reject. Critical gap estimation can be accomplished using a variety of methods. Reference [30] provides an excellent overview of critical gap estimation methods for unsignalized intersections. Even though critical gap may be estimated for a variety of maneuvers, the first gap acceptance models focused on unsignalized intersection operations and thus the majority of the literature to date discusses gap acceptance for that movement.

One of the most popular methods for critical gap estimation is the maximum likelihood estimation. This method uses the maximum rejected gap and the accepted gap for each driver (there is only one accepted gap we can observe) to estimate the probability that the critical gap (t_c) is between those two values. If the cumulative distribution of accepted gaps a_i for a particular driver i is $F(a_i)$, and the cumulative distribution of the maximum rejected gaps r_i is $F(r_i)$, then the probability that this driver's critical gap t_c is between r_i and a_i is $F(a_i) - F(r_i)$. Figure 2.10a provides an example of the probability density function (pdf) of the accepted and maximum rejected gaps for a particular driver. As expected, as the gap size increases, the probability that the driver will accept the gap increases, while the probability that it will reject it decreases. Figure 2.10b shows the cumulative density function (cdf) for the two distributions along with the

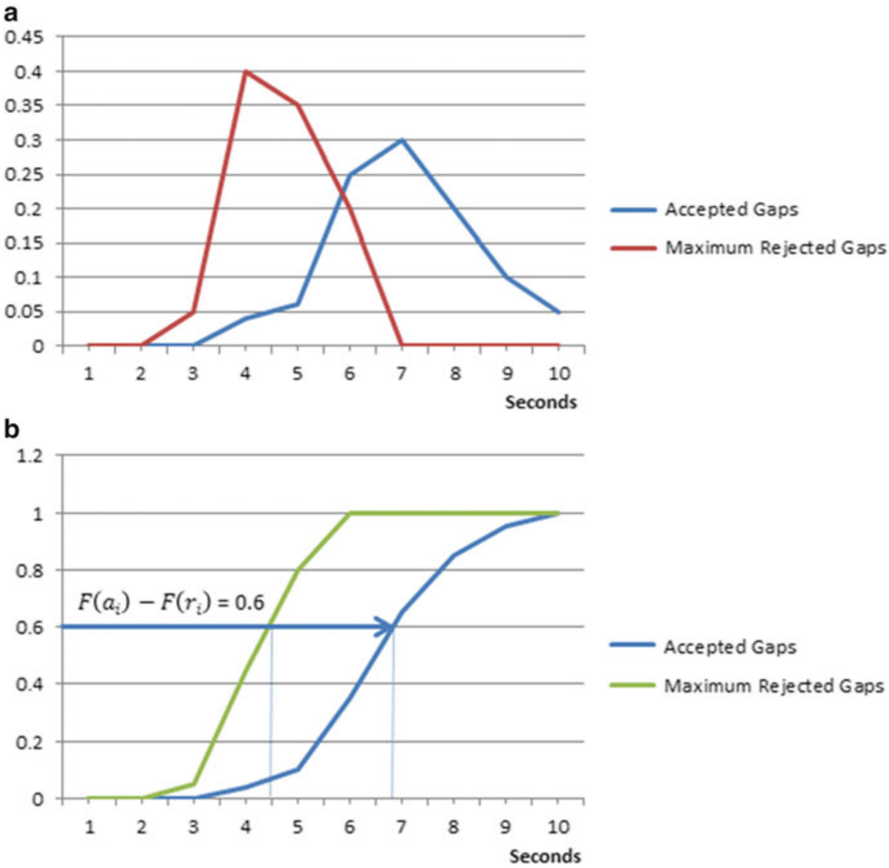


Fig. 2.10 Pdf and Cdf of the accepted and maximum rejected gaps for driver *i*

probability that the driver's critical gap t_c is between r_i and a_i. For the example of Fig. 2.10 the probability that the critical gap is between 4.5 and 6.8 s is 0.6.

The likelihood that a sample of n drivers having an accepted gap and a largest rejected gap of (a_i, r_i) is [30]:

$$\boxed{L^* = \prod_{i=1}^{n} [F(a_i) - F(r_i)]} \tag{2.16}$$

The logarithm, L, of the likelihood is

$$\boxed{L = \sum_{i=1}^{n} \text{In}[F(a_i) - F(r_i)]} \tag{2.17}$$

The likelihood L^* is maximized when its logarithm L is maximized. Next, the user must assume a given type of distribution of critical gaps $F(t_c)$ for the prevailing driver population; it is also assumed that all drivers are consistent (they will always accept a gap of size x and reject a gap of size y). The parameters of the critical gap distribution (mean and variance) are estimated by setting the partial derivatives of L with respect to these parameters to zero:

$$\boxed{\frac{\partial L}{\partial \mu} = \sum_{i=1}^{n} \frac{\frac{\partial F(a_i)}{\partial \mu} - \frac{\partial F(r_i)}{\partial \mu}}{F(a_i) - F(r_i)} = 0} \tag{2.18}$$

$$\boxed{\frac{\partial L}{\partial \sigma^2} = \sum_{i=1}^{n} \frac{\frac{\partial F(a_i)}{\partial \sigma^2} - \frac{\partial F(r_i)}{\partial \sigma^2}}{F(a_i) - F(r_i)} = 0} \tag{2.19}$$

These two equations have to be solved using iterative procedures.

Most gap acceptance models have assumed that drivers are both consistent and homogeneous (all drivers behave identically and will accept the same size gaps). These assumptions are not realistic, but they have proven to be acceptable from a model accuracy perspective [31].

References

1. May AD (1990) Traffic flow fundamentals. Prentice Hall, Englewood Cliffs, NJ
2. Pipes LA (1953) An operational analysis of traffic dynamics. J Appl Phys 24(3):274–281
3. Gazis DC (2002) Traffic Theory. Kluwer's international series. Kluwer academic publishers, ISBN: 0-306-48217-7, p259
4. Gazis D, Herman R, Rothery RW (1961) Non-linear follow-the Leader models of traffic flow. Oper Res 9:545–567
5. Edie LC (1960) Car following and steady state theory for non-congested traffic. Oper Res 9:66–76
6. Brackstone MA, McDonald M (1999) Car-following: a historical review. Transp Res Part F 2:181–196
7. Ceder A, May AD (1976) Further evaluation of single and two regime traffic flow models. Transp Res Rec 567:1–30
8. Roess RP, Ulerio JM (1997) NCHRP Report 385, Comparison of the 1994 highway capacity manual's Ramp analysis procedures and the FRESIM model, Transportation Research Board, Washington, DC
9. Cohen SL (2002) Application of car-following systems in microscopic time-scan simulation models, Transportation Research Record 1802, Transportation Research Board, Washington, DC, pp 239–247
10. Gipps PG (1981) A behavioural car-following model for computer simulation. Transp Res Part B 15B:105–111
11. AIMSUN 5.0 User's Guide (2010) Transport simulation systems - TSS, Barcelona, Spain
12. Soria I (2010) Assessment of car-following models using field data. MS thesis, University of Florida, May 2010

13. Yang Q, Koutsopoulos HN (1996) A microscopic traffic simulator for evaluation of dynamic traffic management systems. Transp Res Part C 4(3):113–129
14. VISSIM 5.10 User's Guide (2010) PTV Planung Transport Verkehr AG, Germany
15. Chandler RE, Herman R, Montroll EW (1958) Traffic dynamics: studies in car-following. Oper Res 6(2):165–184
16. Herman R, Montroll EW, Potts RB, Rothery RW (1959) Traffic dynamics: analysis of stability in car-following. Oper Res 7(1):86–106
17. Ozaki H (1993) Reaction and anticipation in the car-following behavior. In: Proceedings of the 13th international symposium on traffic and transportation theory, p 366
18. Chundury S, Wolshon B (2000) Evaluation of CORSIM car-following model by using Global Positioning System field data. Transp Res Rec 1710(-1):114–121
19. Panwai S, Dia H (2005) Comparative evaluation of microscopic car-following behavior. IEEE Trans Intell Transp Syst 6(3):314–325
20. Chakroborty P, Kikuchi S (1999) Evaluation of the general motors based car-following models and a proposed fuzzy inference model. Transp Res Part C Emerg Technol 7(4):209–235
21. Traffic flow theory, a monograph (Web Document: http://www.tfhrc.gov/its/tft/tft.htm), Chapter 4: Car-following
22. Martin B (2010) Evaluating the impacts of advanced driver assistance systems using a driving simulator: an exploratory analysis. MS Thesis, University of Florida, Dec 2010
23. Sun D, Elefteriadou L (2010) Research and implementation of lane changing model based on driver behavior, Transportation Research Record: Journal of the Transportation Research Board, No 2161, Transportation Research Board of the National Academies, Washington, DC, pp 1–10
24. Mannering FL, Washburn SS, Kilareski WP (2009) Principles of highway engineering and traffic analysis, 4th edn. Wiley, Hoboken, NJ
25. Washington SP, Karlaftis MG, Mannering FL (2003) Statistical and econometric methods for transportation data analysis, 2nd edn. CRC, Boca Raton, FL
26. Ahmed KI (1999) Modeling divers' acceleration and lane changing behavior. Sc.D. Dissertation, Massachusetts Institute of Technology, Department of Civil and Environmental Engineering, Cambridge, MA
27. Laval JA, Daganzo CF (2006) Lane-changing in traffic streams. Transp Res Part B 40 (3):251–264
28. Gipps PG (1986) A model for the structure of lane changing decisions. Transp Res 20B:403–414
29. Hidas P (2002) Modeling lane changing and merging in microscopic traffic simulation. Transp Res Part C 10:351–371
30. Brilon W, Koenig R, Troutbeck RJ (1999) Useful estimation procedures for critical gaps. Transp Res Part A 33:161–186, Pergamon
31. (1992) Revised monograph on traffic flow theory, Federal Highway Administration. http://www.fhwa.dot.gov/publications/research/operations/tft/

Problems

1. Solve Example 2.1 assuming the following: desired maximum speed of the following vehicle, $v(n+1)_{DES}$, is 80 mph, the maximum acceleration which the following vehicle wishes to undertake, $a(n+1)_{MAX}$, is 8 ft/s^2, the actual most severe deceleration that the follower wishes to undertake, $b(n+1)$, is -10.5 ft/s^2, the most severe deceleration rate that vehicle $n+1$ estimates for vehicle n, $\widehat{b}_n$, is -12.5 ft/s^2, and the effective vehicle length, $L(n)$, is 25 ft. How do the results differ from those of Example 2.1. Explain any differences observed.

2. Solve Example 2.1 assuming that the car-following model is GHR with parameters $m = 1$, $l = 1$, and $\lambda_{m,l} = 2.6$. How do the results compare to those of Example 2.1?
3. Conduct a literature review of car-following models. What parameters are used in addition to the ones described in this chapter?
4. Conduct a literature review of lane-changing models for freeways and arterials.
5. The following utility functions have been developed using data from four-lane freeways to indicate driver preference for truck drivers for each of the four lanes:

$$U_{x,L1} = 181.25 - 0.18F_{L1} + 1.27v_{L1}$$

$$U_{x,L2} = 341.3 - 0.27F_{L2} + 1.35v_{L2}$$

$$U_{x,L3} = 40.53 - 0.05F_{L3} + 0.35v_{L3}$$

$$U_{x,L4} = 18.26 - 0.02\ \mathrm{F}_{\mathrm{L4}} + 0.07\ \mathrm{v}_{\mathrm{L4}}$$

   where $U_{x,L1}$, $U_{x,L2}$, $U_{x,L3}$, $U_{x,\mathrm{L4}}$ are the utilities of each of the four lanes.
   F_{L1}, F_{L2}, F_{L3}, and F_{L4} are the passenger vehicle flows of each of the four lanes.
   v_{L1}, v_{L2}, v_{L3}, and v_{L4} are the operating speeds of each of the four lanes.
   If during the peak hour the passenger vehicle flows are $F_{L1} = 1{,}380$ vph, $F_{L2} = 1{,}560$ vph, $F_{L3} = 1{,}250$ vph, and $F_{L4} = 1{,}320$ vph and the speeds are $v_{L1} = 55$ mph, $v_{L2} = 60$ mph, $v_{L3} = 62$ mph, and $v_{L4} = 62$ mph, estimate the probabilities that truck drivers will select each of the four lanes.
6. Conduct a literature review of gap acceptance models for unsignalized intersections.
7. Conduct a literature review of gap acceptance models for merging at freeway junctions.

http://www.springer.com/978-1-4614-8434-9

An Introduction to Traffic Flow Theory
Elefteriadou, L.
2014, XX, 251 p. 97 illus., 79 illus. in color., Hardcover
ISBN: 978-1-4614-8434-9