Preface

There have been dramatic developments in the areas of quadratic and higher degree forms in recent years, and so the time seemed opportune to convene meetings devoted to these topics. During March 2009 there were two major conferences in the area of quadratic forms. One was a research conference at the University of Florida in Gainesville, on “Quadratic forms, sums of squares, and integral lattices” where the latest advances were presented. Immediately after this was the Arizona Winter School on “Quadratic Forms” at the University of Arizona in Tucson, which was an instructional workshop for graduate students with the goal of preparing them for research in this important area. These two conferences were followed by the Conference on Higher Degree Forms at the University of Florida in May 2009.

This volume is an outgrowth of these three conferences, all of which were completely funded by the National Science Foundation. We gratefully acknowledge this support from the NSF. The Tucson conference was the twelfth Arizona Winter School, a longstanding series of NSF-supported workshops on topics in arithmetic geometry. The two Gainesville conferences were in keeping with the tradition there of having annual conferences on various aspects of number theory; they were followed by two Focused Weeks (one on quadratic forms and another on the related topic of integral lattices) at the University of Florida during the Spring of 2010, also fully supported by the NSF. The PIs for the 2009 Florida NSF grant DMS-0753080 were Krishnaswami Alladi and Pham Tiep (then at the University of Florida), with Manjul Bhargava (Princeton) as a consultant. The PIs for the Arizona Winter School NSF grant DMS-0602287 were Matthew Papanikolas, Fernando Rodriguez-Villegas, David Savitt, William Stein, and Dinesh Thakur.

The Arizona Winter School featured instructional lectures by Manjul Bhargava, John Conway, Noam Elkies, Jonathan Hanke, and R. Parimala on various aspects of quadratic forms. The informal (but comprehensive) notes of these lectures are available at the website of the 2009 Arizona Winter School (http://swc.math.arizona.edu). Parimala and Hanke have polished their articles and submitted excellent surveys to this volume.

Even though the Florida conference on quadratic forms was a research conference focusing on the latest developments, there was significant participation by graduate and undergraduate students to help them enter this exciting domain of research. In order to prepare them for the advanced conference lectures, an instructional workshop preceded this conference for which Jonathan Hanke was the main lecturer. Some aspects of his Florida talks are covered in his survey paper in this book.

In his survey, Hanke discusses fundamental connections between the classical theory of quadratic forms over number fields and their rings of integers, and the theory of modular and automorphic forms. In doing so he provides a treatment of theta functions and some aspects of Clifford algebras as well. Hanke's survey is nicely complemented by that of Parimala who provides a lucid introduction to the algebraic theory of quadratic forms, the invariants associated with quadratic forms, and connections with Galois cohomology. She also states some open problems and discusses recent progress. These two surveys are augmented by the survey and research paper of Voight on quaternion algebras and quadratic forms.

The classical theorems of Lagrange that every integer is a sum of four squares and Gauss that every integer is a sum of three triangular numbers motivate the study of "universal forms", namely those that represent all integers, as well as the investigation of ternary forms in general. The papers of Jagy on integral positive ternary quadratic forms, of Berkovich on sums of three squares, and of Chan and Haensch on certain almost universal ternary forms, show that there still are fundamental questions worthy of investigation on very classical topics.

Whereas the study of universal quadratic forms addresses the question of representing all integers, one could consider the question of representing quadratic forms by integral quadratic forms. In 2008 Ellenberg and Venkatesh introduced ergodic theory as a new tool in this study and made dramatic progress going beyond what Eichler and Kneser had achieved using an arithmetic approach. In his survey of such representation problems, Schulze-Pillot sketches three approaches—arithmetic, algebraic and ergodic—and gives a comparative study of them.

The theory of integral lattices has important links with quadratic forms. Bannai and Miezaki discuss a famous conjecture of D. H. Lehmer on the Fourier coefficients of weighted theta series of certain integral lattices and describe recent progress on this classical question. Integral lattices and quadratic forms have links with binary linear codes, and this is investigated by Elkies and Kominers. In doing so, they provide a new structural development of harmonic polynomials on Hamming space analogous to the treatment of harmonic polynomials on Euclidean space, and present several applications.

Finally, the paper of Reznick discusses certain fundamental questions on the length of binary forms of higher degree starting from the seminal work of Sylvester in the mid-nineteenth century. After discussing some current research, he concludes with a list of important open questions.

We hope that this volume, which comprises both introductory survey articles and research papers reporting the latest developments, will be of interest to students and senior mathematicians alike. In conducting the conferences in Florida, we owe a special debt to Frank Garvan as a conference organizer and to Margaret Somers for taking care of all local arrangements. Similarly, we wish to acknowledge Annette

Horn for handling the local arrangements for the 2009 Arizona Winter School. We thank Elizabeth Loew of Springer for her support and interest in including this book in the series Developments in Mathematics.

Gainesville, FL, USA — Krishnaswami Alladi
Princeton, NJ, USA — Manjul Bhargava
Tucson, AZ, USA — David Savitt
Tucson, AZ, USA — Pham Huu Tiep

http://www.springer.com/978-1-4614-7487-6

Quadratic and Higher Degree Forms
Alladi, K.; Bhargava, M.; Savitt, D.; Tiep, P.H. (Eds.)
2013, IX, 298 p., Hardcover
ISBN: 978-1-4614-7487-6