Preface

In 1998 Teresa Haynes, Stephen Hedetniemi, and Peter Slater [85, 86] wrote the two so-called "domination books" and in so doing provided the first comprehensive treatment of theoretical, algorithmic, and application aspects of domination in graphs. They did an outstanding job of unifying results scattered through some 1,200 domination papers at that time. The graph theory community is indeed much indebted to them.

Some 14 years have passed since these two domination books appeared in print and several hundred domination papers have subsequently been published. While these two books covered a variety of domination-type parameters and frameworks for domination, this book focuses on our favorite domination parameter, namely, total domination in graphs. We have tended to primarily emphasize the recent selected results on total domination in graphs that appeared subsequent to the two domination books by Haynes, Hedetniemi, and Slater.

In this book, we do assume that the reader is acquainted with the basic concepts of graph theory and has had some exposure to graph theory at an introductory level. However, since graph theory terminology sometimes varies, the book is self-contained, and we clarify the terminology that will be adopted in this book in the introductory chapter.

We have written this book primarily to reach the following audience. The first audience is the graduate students who are interested in exploring the field of total domination theory in graphs and wish to "familiarize themselves with the subject, the research techniques, and the major accomplishments in the field." It is our hope that such graduate students will find topics and problems that can be developed further. The second audience is the established researcher in domination theory who wishes to have easy access to known results and latest developments in the field of total domination theory in graphs.

We have supplied several proofs for the reader to acquaint themselves with a toolbox of proof techniques and ideas with which to attack open problems in the field. We have identified many unsolved problems in the area and provided a chapter devoted to conjectures and open problems.

Chapter 1 introduces graph theory and hypergraph theory concepts fundamental to the chapters that follow. Perhaps much of the recent interest in total domination in graphs arises from the fact that total domination in graphs can be translated to the problem of finding transversals in hypergraphs. In this introductory chapter, we discuss the transition from total domination in graphs to transversals in hypergraphs.

Chapter 2 establishes fundamental properties of total dominating sets in graphs. General bounds relating the total domination number to other parameters are presented in this chapter, and properties of minimal total dominating sets are listed.

Complexity and algorithmic results on total domination in graphs are discussed in Chap. 3. We outline a few of the best-known algorithms and state what is currently known in this field. A linear algorithm to compute the total domination of a tree is given. We discuss fixed parameter tractability problems for the total domination number. A simple heuristic that finds a total dominating set in a graph is presented.

In Chap. 4 we present results on total domination in trees, the simplest class of graphs. A constructive characterization of trees with largest possible total domination in terms of the order and number of leaves of the tree is provided. We explore the relationship between the total domination number and the ordinary domination number of a tree.

In Chap. 5, we determine upper bounds on the total domination number of a graph in terms of its minimum degree. A general bound involving the minimum degree is presented. We then focus in detail on upper bounds on the total domination number when the minimum degree is one, two, three, four, or five, respectively, and consider each case in turn. Known upper bounds on the total domination number of a graph in terms of its order and minimum degree are summarized in an appropriate table. We close this chapter with a discussion of bounds on the total domination number of a graph with certain connectivity requirements imposed on the graph.

In Chap. 6, we turn our attention to investigate upper bounds on the total domination number of a planar graph of small diameter.

Chapter 7 determines whether the absence of any specified cycle guarantees that the upper bound on the total domination number established in Chap. 5 can be lowered. Upper bounds on the total domination number of a graph with given girth are presented.

In Chap. 8 we relate the size and the total domination number of a graph of given order. A linear Vizing-like relation is established relating the size of a graph and its order, total domination number, and maximum degree.

In Chap. 9 we impose the structural restriction of claw-freeness on a graph and investigate upper bounds on the total domination number of such graphs, while in Chap. 10, we discuss a relationship between the total domination number and the matching number of a graph.

An in-depth study of criticality issues relating to the total domination is presented in Chap. 11. In this chapter, we discuss an important association with total domination edge critical graphs and diameter critical graphs.

In Chap. 12 we investigate the behavior of the total domination number on a graph product. In particular, we focus our attention on the Cartesian product and the direct product of graphs.

Chapter 13 considers the problem of partitioning the vertex set of a graph into a total dominating set and something else. In this chapter, we exhibit a surprising connection between disjoint total dominating sets in graphs, 2 coloring of hypergraphs, and even cycles in digraphs.

In Chap. 14 we determine an upper bound of $1+\sqrt{n\ln(n)}$ on the total domination number of a graph with diameter two and show that this bound is close to optimum.

In Chap. 15 we present Nordhaus–Gaddum-type bounds for the total domination number of a graph. In Chap. 16 we focus our attention on the upper total domination number of a graph. By imposing a regularity condition on a graph, we show using edge weighting functions on total dominating sets how to obtain a sharp upper bound on the upper total domination of the graph.

In Chap. 17 we present various generalizations of the total domination number. We select four such variations of a total dominating set in a graph and briefly discuss each variation.

We close with Chap. 18 which lists several conjectures and open problems which have yet to be solved.

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We have tried to eliminate errors, but surely several remain. We do welcome any comments the reader may have. A list of typographical errors, corrections, and suggestions can be e-mailed to the authors.

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