Preface

The Boltzmann equation is a milestone of classical and quantum kinetic theory of gases [1–6]. It describes the evolution of a dilute gas, initially prepared in a nonequilibrium state, by means of a statistical approach which allows to disregard the detailed knowledge of the motion of the single particles. In the absence of external forces, the gas undergoes a process of relaxation to equilibrium, whose mathematical essence is captured by the celebrated H-Theorem, which also marked the first clear onset of irreversibility in classical particle systems. On the other hand, a different perspective in the behaviour of a fluid is offered by the laws of continuum physics, which found a self-contained settlement in the equations of hydrodynamics. This work is concerned with the ambitious and long-standing task of linking the two different levels of description, the kinetic and the hydrodynamic. The question arises as to whether this is actually an original ambition. The Grad's moment method, for instance, which still spreads its influence on the modern theory of Extended Irreversible Thermodynamics [7], dates back to the late 1940s of the last century. Earlier attempts are represented by Hilbert's procedure and by the Chapman-Enskog expansion, which constitute an important success in kinetic theory, as they allowed to derive the hydrodynamic laws from the Boltzmann equation and provided consistent expressions for the transport coefficients. However, almost a century of effort to extend the hydrodynamic description beyond the Navier–Stokes–Fourier approximation failed even in the case of small deviations around the equilibrium, due to the onset of instabilities which prevent the use of the hydrodynamic solutions [8, 9]. A different route, in kinetic theory, is represented by the recent Invariant Manifold method [10]. This technique, based on the computation of a slow invariant manifold in the space of distribution functions, provides hydrodynamic equations which are stable and remain valid also at short length scales, provided that the condition of local equilibrium holds. Thus, the purpose of this work is to offer a short survey over a field of active research, which aims at bridging time and length scales, from the particle-like description inherent in the Boltzmann theory up to the hydrodynamic setting. Our plan is to perform a bottom-up approach, which steps from the statistical foundations of the Boltzmann equation to the spectral properties of hydrodynamic fluctuations. Our natural inclination is to shape things as they were rooted and naturally emerging each from the other: i.e., by showing, for instance, how typical kinetic equilibration rates affect the normal modes of hydrodynamic fluctuations and, also, how the hydrodynamic setting may be successfully extended to length scales comparable with the mean free path. A great scientist of our times, J. L. Lebowitz, expressed, in [11], his own surprise in considering that, in spite of the hierarchical structure of Nature, characterised by a variety of time and length scales, it is still possible, to some extent, to discuss the various levels of description independently of one another. "*Thus, arrows of explanations between different levels always point from smaller to larger scales, although the origin of higher level phenomena in the more fundamental lower level laws is often very far from transparent*". Our overall impression, which we would like to share with the reader, is that a unifying approach is finally starting to take shape.

References

1. S. Chapman and T. G. Cowling, *The Mathematical Theory of Nonuniform Gases*, (Cambridge University Press, New York, 1970)
2. C. Cercignani, R. Illner and M. Pulvirenti, *The Mathematical Theory of Dilute Gases* (Springer, Berlin, 1994)
3. C. Cercignani, *Theory and Application of the Boltzmann Equation* (Scottish Academic Press, Edinburgh, 1975)
4. D. Benedetto, F. Castella, R. Esposito and M. Pulvirenti, A short review on the derivation of the nonlinear quantum Boltzmann equations, *Commun. Math. Sci.* **5**, 55 (2007)
5. D. Benedetto, F. Castella, R. Esposito and M. Pulvirenti, From the N-body Schrödinger equation to the quantum Boltzmann equation: a term-by-term convergence result in the weak coupling regime, *Comm. Math. Phys.* **277**, 1 (2008)
6. N. Bellomo Ed., *Lecture Notes on the Mathematical Theory of the Boltzmann Equation* (World Scientific, 1995)
7. D. Jou, J. Casas-Vázquez and G. Lebon, *Extended Irreversible Thermodynamics* (Springer, 2010)
8. M. Colangeli, I. V. Karlin and M. Kröger, From hyperbolic regularization to exact hydrodynamics for linearized Grad's equations, *Phys. Rev. E* **75**, 051204 (2007)
9. M. Colangeli, I. V. Karlin and M. Kröger, Hyperbolicity of exact hydrodynamics for three-dimensional linearized Grad's equations, *Phys. Rev. E* **76**, 022201 (2007)
10. A. N. Gorban and I. V. Karlin, *Invariant Manifolds for Physical and Chemical Kinetics*, Lect. Notes Phys. **660** (Springer, Berlin, 2005)
11. J. L. Lebowitz, Statistical mechanics: A selective review of two central issues, *Rev. Mod. Phys.* **71**, S346–S357 (1999)

Springer
http://www.springer.com/978-1-4614-6305-4

From Kinetic Models to Hydrodynamics
Some Novel Results
Colangeli, M.

2013, X, 96 p. 21 illus., 9 illus. in color., Softcover
ISBN: 978-1-4614-6305-4