

Contents

1	Numerical Approximation of Exact Controls for Waves	1
1.1	Introduction	1
1.1.1	An Abstract Functional Setting	1
1.1.2	Contents of Chap. 1	4
1.2	Main Results	6
1.2.1	An “Algorithm” in an Infinite-Dimensional Setting	6
1.2.2	The Continuous Approach	9
1.2.3	The Discrete Approach	11
1.2.4	Outline of Chap. 1	13
1.3	Proof of the Main Result on the Continuous Setting	13
1.3.1	Classical Convergence Results	13
1.3.2	Convergence Rates in X_s	14
1.4	The Continuous Approach	16
1.4.1	Proof of Theorem 1.2	17
1.4.2	Proof of Theorem 1.3	18
1.5	Improved Convergence Rates: The Discrete Approach	18
1.5.1	Proof of Theorem 1.4	19
1.5.2	Proof of Theorem 1.5	20
1.6	Advantages of the Discrete Approach	20
1.6.1	The Number of Iterations	20
1.6.2	Controlling Non-smooth Data	21
1.6.3	Other Minimization Algorithms	23
1.7	Application to the Wave Equation	24
1.7.1	Boundary Control	24
1.7.2	Distributed Control	40
1.8	A Data Assimilation Problem	44
1.8.1	The Setting	44
1.8.2	Numerical Approximation Methods	46

2	Observability for the $1 - d$ Finite-Difference Wave Equation	49
2.1	Objectives	49
2.2	Spectral Decomposition of the Discrete Laplacian	50
2.3	Uniform Admissibility of Discrete Waves	51
2.3.1	The Multiplier Identity	51
2.3.2	Proof of the Uniform Hidden Regularity Result	52
2.4	An Observability Result	53
2.4.1	Equipartition of the Energy	53
2.4.2	The Multiplier Identity Revisited	54
2.4.3	Uniform Observability for Filtered Solutions	55
2.4.4	Proof of Theorem 2.3	58
3	Convergence of the Finite-Difference Method for the $1 - d$ Wave Equation with Homogeneous Dirichlet Boundary Conditions	59
3.1	Objectives	59
3.2	Extension Operators	59
3.2.1	The Fourier Extension	60
3.2.2	Other Extension Operators	60
3.3	Orders of Convergence for Smooth Initial Data	64
3.4	Further Convergence Results	70
3.4.1	Strongly Convergent Initial Data	70
3.4.2	Smooth Initial Data	71
3.4.3	General Initial Data	73
3.4.4	Convergence Rates in Weaker Norms	74
3.5	Numerics	75
4	Convergence with Nonhomogeneous Boundary Conditions	79
4.1	The Setting	79
4.2	The Laplace Operator	81
4.2.1	Natural Functional Spaces	81
4.2.2	Stronger Norms	86
4.2.3	Numerical Results	88
4.3	Uniform Bounds on y_h	89
4.3.1	Estimates in $C([0, T]; L^2(0, 1))$	90
4.3.2	Estimates on $\partial_t y_h$	93
4.4	Convergence Rates for Smooth Data	98
4.4.1	Main Convergence Result	98
4.4.2	Convergence of y_h	100
4.4.3	Convergence of $\partial_t y_h$	104
4.4.4	More Regular Data	106
4.5	Further Convergence Results	110
4.6	Numerical Results	111

- 5 Further Comments and Open Problems** 115
 - 5.1 Discrete Versus Continuous Approaches 115
 - 5.2 Comparison with Russell’s Approach 116
 - 5.3 Uniform Discrete Observability Estimates 117
 - 5.4 Optimal Control Theory 117
 - 5.5 Fully Discrete Approximations 118

- References** 119



<http://www.springer.com/978-1-4614-5807-4>

Numerical Approximation of Exact Controls for Waves

Ervedoza, S.; Zuazua, E.

2013, XVII, 122 p. 17 illus., 3 illus. in color., Softcover

ISBN: 978-1-4614-5807-4