

Contents

Part I Abstract theory

1	The pullback attractor	3
1.1	Processes and semigroups	3
1.2	Global attractors for semigroups	5
1.3	Some motivating non-autonomous examples	8
1.4	Pullback attractors	12
1.5	Attractors for semigroups and pullback attractors for processes	15
1.6	Example: a non-autonomous logistic ODE	17
1.7	Pullback attractors and random attractors	19
2	Existence results for pullback attractors	23
2.1	Omega-limit sets	24
2.2	First result: from the existence of a compact attracting set	28
2.2.1	Example: a saddle-node bifurcation	30
2.3	Second result: from the existence of a bounded attracting set	32
2.4	Third result: from the pullback flattening property	35
2.5	Pullback point dissipativity	38
2.5.1	An abstract application: gradient semigroups	42
2.5.2	Example: the Chafee–Infante equation	46
2.6	Pullback attractors with more general basins of attraction	48
3	Continuity of attractors	55
3.1	Standing assumptions	56
3.2	Upper semicontinuity	59
3.3	Lower semicontinuity	61

- 3.4 Equi-attraction and continuity of attractors 65
 - 3.4.1 Equi-attraction implies continuity of attractors 65
 - 3.4.2 Continuity of attractors implies equi-attraction 66
 - 3.4.3 Rate of convergence of pullback attractors 68
- 4 Finite-dimensional attractors 71**
 - 4.1 Upper box-counting dimension 71
 - 4.2 A general result bounding the box-counting dimension of invariant sets 74
 - 4.3 Covering linear images of balls in Banach spaces 76
 - 4.4 Bounding the dimension of attractors in Hilbert spaces 80
 - 4.4.1 Linear images of balls and ellipsoids in Hilbert spaces 80
 - 4.4.2 Multilinear algebra in Hilbert spaces 81
 - 4.4.3 Covering linear images of balls in Hilbert spaces 86
 - 4.4.4 The trace formula 90
 - 4.5 Embedding sets with finite box-counting dimension in Euclidean spaces 94
 - 4.6 Embedding finite-dimensional non-autonomous sets 100
- 5 Gradient semigroups and their dynamical properties 103**
 - 5.1 Dynamical properties of gradient semigroups 103
 - 5.2 The Morse decomposition and attractor–repeller pairs 105
 - 5.3 A Morse decomposition of the attractor of a dynamically \mathcal{S} -gradient system 109
 - 5.4 Constructing a Lyapunov function from the Morse decomposition 112
 - 5.5 Perturbations of gradient systems 118
 - 5.5.1 Homoclinic structures and chain recurrence 118
 - 5.5.2 Stability of gradient semigroups under perturbation 121
 - 5.5.3 Hyperbolicity and the continuity of equilibria 124
 - 5.6 Non-autonomous perturbations of gradient systems 128
 - 5.7 Exponential attraction 130
 - 5.8 Dynamically gradient processes, non-autonomous Morse decompositions, and non-autonomous Lyapunov functions 136

Part II Invariant manifolds of hyperbolic solutions

- 6 Semilinear differential equations 143**
 - 6.1 Linear operators and their adjoints 143
 - 6.2 Strongly continuous linear semigroups and their generators 146
 - 6.3 Dissipative operators 150
 - 6.4 Fractional powers of linear operators 151
 - 6.4.1 Fractional powers of self-adjoint operators 151
 - 6.4.2 Fractional powers of sectorial operators 153

- 6.5 Examples of generators of semigroups 157
 - 6.5.1 Self-adjoint operators 157
 - 6.5.2 The Laplace operator on L^2 158
 - 6.5.3 The Stokes operator 159
 - 6.5.4 Wave operators 161
 - 6.5.5 The Laplace operator on L^p 163
- 6.6 Gronwall’s inequalities 167
- 6.7 Well-posedness for abstract semilinear equations 169
- 6.8 Energy estimates and global existence 175
- 6.9 Differentiability with respect to initial conditions 177
- 6.10 Monotonicity and comparison results 179
 - 6.10.1 Some operators with positive resolvent 183
- 6.11 Finite-dimensional pullback attractors for semilinear evolution equations 184
- 7 Exponential dichotomies 187**
 - 7.1 Discrete dichotomies for discrete processes 188
 - 7.1.1 Definition and basic properties 188
 - 7.1.2 Characterisation of a discrete dichotomy 190
 - 7.1.3 Robustness of discrete dichotomies 198
 - 7.2 Exponential dichotomies for continuous processes 203
 - 7.2.1 Definition and basic properties 203
 - 7.2.2 Moving between discrete and continuous exponential dichotomies 206
 - 7.2.3 Robustness of exponential dichotomies under perturbation 211
 - 7.2.4 Characterisation of an exponential dichotomy 217
- 8 Hyperbolic solutions and their stable and unstable manifolds 223**
 - 8.1 Hyperbolic global solutions 224
 - 8.2 Persistence of hyperbolic global solutions under perturbation 226
 - 8.3 Existence of unstable manifolds as a graph 229
 - 8.4 Existence of stable manifolds as a graph 238
 - 8.5 Perturbation of unstable manifolds 241
 - 8.6 Applications to semilinear evolution equations 245
 - 8.6.1 Continuity and characterisation of attractors under non-autonomous perturbation 246
 - 8.6.2 Asymptotically autonomous differential equations 247

Part III Applications

- 9 A non-autonomous competitive Lotka–Volterra system 255**
 - 9.1 Autonomous case 255
 - 9.2 A non-autonomous logistic equation 256

9.3	Order-preserving properties	257
9.3.1	Some general theory	257
9.3.2	Competitive Lotka–Volterra systems are order preserving	258
9.4	Cases (i) and (iii): a single attracting fixed point	259
9.5	Case (ii): a unique positive attracting trajectory	261
10	Delay differential equations	265
10.1	Delay differential equations as dynamical systems	265
10.2	Attractors for non-autonomous delay differential equations	266
10.2.1	Strong dissipativity	267
10.2.2	A more general nonlinear term	269
10.2.3	Weak dissipativity	272
10.3	Pullback attractors for periodic equations	274
10.4	Perturbation by small delays	275
11	The Navier–Stokes equations with non-autonomous forcing	281
11.1	Technical preliminaries	281
11.2	Existence and uniqueness for $u_0 \in H$ and $f \in D(A^{-1/2})$	283
11.3	Existence of a pullback attractor in H	285
11.4	Existence of pullback attractor in $D(A^{1/2})$	287
11.5	Finite-dimensional pullback attractor	290
11.6	When the pullback attractor is a single trajectory	293
11.7	Parametrisation of the attractor by point values	295
11.8	Semigroup approach to existence and uniqueness	298
11.8.1	Local well-posedness for $n = 2, 3$	299
11.8.2	Differentiability	299
11.8.3	Global well-posedness for $n=2$	300
12	Applications to parabolic problems	301
12.1	Local well-posedness, regularity, and differentiability with respect to initial conditions	302
12.2	Comparison results for parabolic equations	309
12.3	Global well-posedness and pullback attractors	309
12.4	Gradient structure for autonomous parabolic problems	313
13	A non-autonomous Chafee–Infante equation	317
13.1	The autonomous Chafee–Infante equation	317
13.2	Preliminaries: lap number and monotonicity properties	318
13.3	Trivial dynamics	321
13.4	Extremal solutions and the pullback attractor	321
13.5	Upper and lower bounds on the attractor dimension	323
13.6	Pullback dynamics of positive solutions	324
13.7	Forwards dynamics of positive solutions	329
13.8	Hyperbolic equilibria in the pullback attractor	331
13.9	Non-autonomous equilibria within the pullback attractor	332
13.10	The pullback attractor when $b(t)$ is close to a constant	335
13.11	The pullback attractor when $b(t)$ is slowly varying	337

14 Perturbation of diffusion and continuity of global attractors with rate of convergence 339

14.1 Perturbation of diffusion and continuity of global exponential attractors with rate of convergence 339

14.2 Convergence of A_ε^{-1} to A_0^{-1} 344

14.3 Convergence of eigenvalues and eigenfunctions 345

14.4 Rate of convergence of the linear and nonlinear semigroups 349

14.5 Uniform bounds on resolvents of linearised operators 351

14.6 Rate of convergence of equilibria and of linearisations 352

14.7 Rate of convergence and uniform attraction of local unstable manifolds 356

14.8 Proof of Theorem 14.4..... 358

15 A non-autonomous damped wave equation..... 361

15.1 Local and global existence 361

15.2 Local well-posedness 362

15.3 Differentiability 363

15.4 Global well-posedness and strong bounded dissipativity 365

15.5 Existence of pullback attractors 367

15.6 Regularity of the pullback attractor 369

15.7 Gradient-like structure of the pullback attractor..... 371

16 Appendix: Skew-product flows and the uniform attractor..... 377

16.1 Skew-product flows 378

16.2 Generation of skew-product flows by non-autonomous equations 378

16.3 Pullback attractors for skew-product flows 381

16.4 Skew-product flows as semigroups 382

16.5 Uniform attractors 385

16.6 Uniform attractors for processes and pullback attractors 388

References..... 393

Index..... 405



<http://www.springer.com/978-1-4614-4580-7>

Attractors for infinite-dimensional non-autonomous
dynamical systems

Nolasco de Carvalho, A.; Langa Rosado, J.A.; Robinson,
J.

2013, XXXVI, 412 p., Hardcover

ISBN: 978-1-4614-4580-7