2. Light Wavess

2.1 Harmonic Plane Waves, Phase Velocity

Many important aspects and phenomena of quantum mechanics can be visualized by means of *wave mechanics*, which was set up in close analogy to *wave optics*. Here the simplest building block is the harmonic plane wave of light in a vacuum describing a particularly simple configuration in space and time of the *electric field* **E** and the *magnetic induction field* **B**. If the x axis of a rectangular coordinate system has been oriented parallel to the direction of the wave propagation, the y axis can always be chosen to be parallel to the electric field strength so that the z axis is parallel to the magnetic field strength. With this choice the field strengths can be written as

$$\begin{aligned} E_y &= E_0 \cos(\omega t - kx) \quad , & B_z &= B_0 \cos(\omega t - kx) \quad , \\ E_x &= E_z = 0 \quad , & B_x &= B_y = 0 \quad . \end{aligned}$$

They are shown in Figures 2.1 and 2.2. The quantities E_0 and B_0 are the maximum values reached by the electric and magnetic fields, respectively. They are called *amplitudes*. The *angular frequency* ω is connected to the *wave number* k by the simple relation

$$\omega = ck \quad .$$

The points where the field strength is maximum, that is, has the value E_0, are given by the *phase* of the cosine function

$$\delta = \omega t - kx = 2\ell\pi \quad ,$$

where ℓ takes the integer values $\ell = 0, \pm 1, \pm 2, \ldots$. Therefore such a point moves with the velocity

$$c = \frac{x}{t} = \frac{\omega}{k} \quad .$$

Since this velocity describes the speed of a point with a given phase, c is called the *phase velocity* of the wave. For light waves in a vacuum, it is independent of the wavelength. For positive, or negative, k the propagation is in the direction of the positive, or negative, x axis, respectively.

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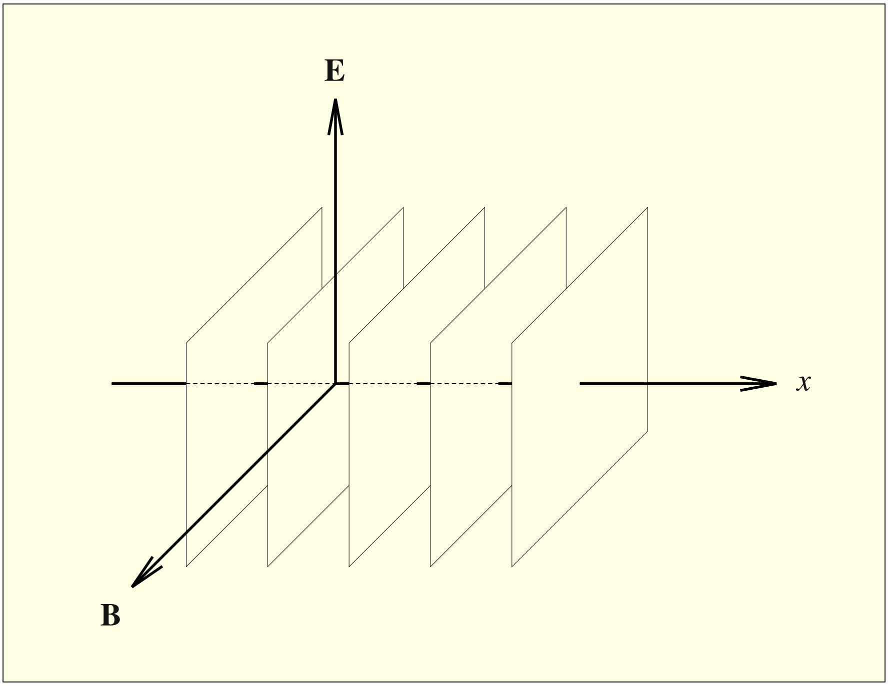

Fig. 2.1. In a plane wave the electric and magnetic field strengths are perpendicular to the direction of propagation. At any moment in time, the fields are constant within planes perpendicular to the direction of motion. As time advances, these planes move with constant velocity.

At a fixed point in space, the field strengths **E** and **B** oscillate in time with the angular frequency ω (Figures 2.3a and c). The *period* of the oscillation is

$$T = \frac{2\pi}{\omega} \quad .$$

For fixed time the field strengths exhibit a periodic pattern in space with a spatial period, the *wavelength*

$$\lambda = \frac{2\pi}{|k|} \quad .$$

The whole pattern moves with velocity c along the x direction. Figures 2.3b and 2.3d present the propagation of waves by a set of curves showing the field strength at a number of consecutive equidistant moments in time. Earlier moments in time are drawn in the background of the picture, later ones toward the foreground. We call such a representation a *time development*.

For our purpose it is sufficient to study only the electric field of a light wave,

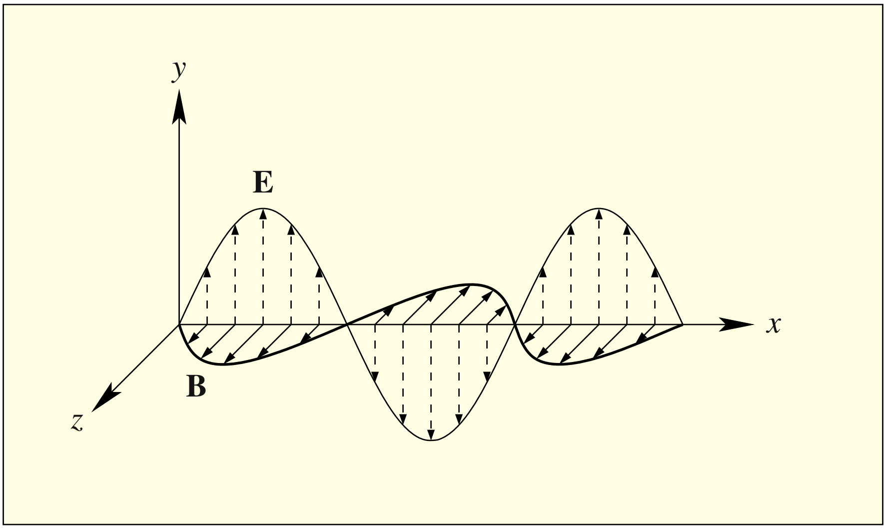

Fig. 2.2. For a given moment in time, the electric field strength E and the magnetic field strength B are shown along a line parallel to the direction of motion of the harmonic plane wave.

$$E_y = E = E_0 \cos(\omega t - kx - \alpha) \quad .$$

We have included an additional *phase* α to allow for the fact that the maximum of E need not be at $x = 0$ for $t = 0$. To simplify many calculations, we now make use of the fact that cosine and sine are equal to the real and imaginary parts of an exponential,

$$\cos\beta + \mathrm{i}\sin\beta = \mathrm{e}^{\mathrm{i}\beta} \quad ,$$

that is,

$$\cos\beta = \mathrm{Re}\,\mathrm{e}^{\mathrm{i}\beta} \quad , \qquad \sin\beta = \mathrm{Im}\,\mathrm{e}^{\mathrm{i}\beta} \quad .$$

The wave is then written as

$$E = \mathrm{Re}\,E_{\mathrm{c}} \quad ,$$

where E_{c} is the complex field strength:

$$E_{\mathrm{c}} = E_0\,\mathrm{e}^{-\mathrm{i}(\omega t - kx - \alpha)} = E_0\,\mathrm{e}^{\mathrm{i}\alpha}\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}kx} \quad .$$

It factors into a complex amplitude

$$A = E_0\,\mathrm{e}^{\mathrm{i}\alpha}$$

and two exponentials containing the time and space dependences, respectively. As mentioned earlier, the wave travels in the positive or negative x

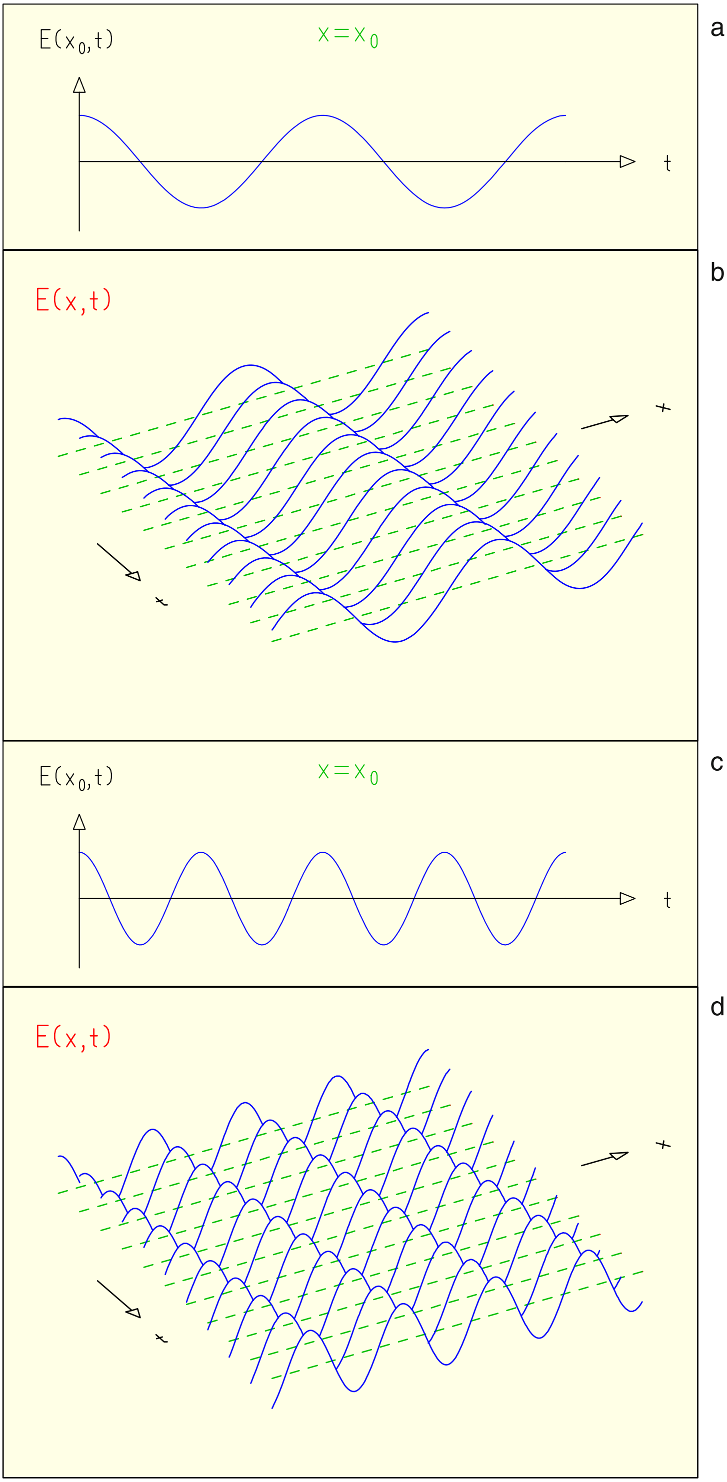

Fig. 2.3. (a) Time dependence of the electric field of a harmonic wave at a fixed point in space. (b) Time development of the electric field of a harmonic wave. The field distribution along the *x* direction is shown for several moments in time. Early moments are in the background, later moments in the foreground. (c, d) Here the wave has twice the frequency. We observe that the period *T* and the wavelength λ are halved, but that the phase velocity *c* stays the same. The time developments in parts b and d are drawn for the same interval of time.

direction, depending on the sign of k. Such waves with different amplitudes are

$$E_{c+} = A\,e^{-i\omega t}e^{ikx} \quad , \qquad E_{c-} = B\,e^{-i\omega t}e^{-ikx} \quad .$$

The factorization into a time- and a space-dependent factor is particularly convenient in solving Maxwell's equations. It allows the separation of time and space coordinates in these equations. If we divide by $\exp(-i\omega t)$, we arrive at the time-independent expressions

$$E_{s+} = A\,e^{ikx} \quad , \qquad E_{s-} = B\,e^{-ikx} \quad ,$$

which we call *stationary waves*.

The *energy density* in an electromagnetic wave is equal to a constant, ε_0, times the square of the field strength,

$$w(x,t) = \varepsilon_0 E^2 \quad .$$

Because the plane wave has a cosine structure, the energy density varies twice as fast as the field strength. It remains always a positive quantity; therefore the variation occurs around a nonzero average value. This average taken over a period T of the wave can be written in terms of the complex field strength as

$$w = \frac{\varepsilon_0}{2} E_c E_c^* = \frac{\varepsilon_0}{2} |E_c|^2 \quad .$$

Here E_c^* stands for the complex conjugate,

$$E_c^* = \mathrm{Re}\,E_c - i\,\mathrm{Im}\,E_c \quad ,$$

of the complex field strength,

$$E_c = \mathrm{Re}\,E_c + i\,\mathrm{Im}\,E_c \quad .$$

For the average energy density in the plane wave, we obtain

$$w = \frac{\varepsilon_0}{2} |A|^2 = \frac{\varepsilon_0}{2} E_0^2 \quad .$$

2.2 Light Wave Incident on a Glass Surface

The effect of glass on light is to reduce the phase velocity by a factor n called the *refractive index*,

$$c' = \frac{c}{n} \quad .$$

Although the frequency ω stays constant, wave number and wavelength are changed according to

$$k' = nk \quad , \qquad \lambda' = \frac{\lambda}{n} \quad .$$

The Maxwell equations, which govern all electromagnetic phenomena, demand the continuity of the electric field strength and its first derivative at the boundaries of the regions with different refractive indices. We consider a wave traveling in the x direction and encountering at position $x = x_1$ the surface of a glass block filling half of space (Figure 2.4a). The surface is oriented perpendicular to the direction of the light. The complex expression

$$E_{1+} = A_1 \, e^{ik_1 x}$$

describes the incident stationary wave to the left of the glass surface, that is, for $x < x_1$, where A_1 is the known amplitude of the incident light wave. At the surface only a part of the light wave enters the glass block; the other part will be reflected. Thus, in the region to the left of the glass block, $x < x_1$, we find in addition to the incident wave the reflected stationary wave

$$E_{1-} = B_1 \, e^{-ik_1 x}$$

propagating in the opposite direction. Within the glass the transmitted wave

$$E_2 = A_2 \, e^{ik_2 x}$$

propagates with the wave number

$$k_2 = n_2 k_1$$

altered by the refractive index $n = n_2$ of the glass. The waves E_{1+}, E_{1-}, and E_2 are called *incoming*, *reflected*, and *transmitted constituent waves*, respectively. The continuity for the field strength E and its derivative E' at $x = x_1$ means that

$$E_1(x_1) = E_{1+}(x_1) + E_{1-}(x_1) = E_2(x_1)$$

and

$$E_1'(x_1) = ik_1 \left[E_{1+}(x_1) - E_{1-}(x_1) \right] = ik_2 E_2(x_1) = E_2'(x_1) \quad .$$

The two unknown amplitudes, B_1 of the reflected wave, and A_2 of the transmitted, can now be calculated from these two continuity equations. The electric field in the whole space is determined by two expressions incorporating these amplitudes,

$$E_s = \begin{cases} A_1 \, e^{ik_1 x} + B_1 \, e^{-ik_1 x} & \text{for } x < x_1 \\ A_2 \, e^{ik_2 x} & \text{for } x > x_1 \end{cases} \quad .$$

The electric field in the whole space is obtained as a superposition of constituent waves physically existing in regions 1 and 2. By multiplication with the time-dependent phase $\exp(-i\omega t)$, we obtain the complex field strength E_c, the real part of which is the physical electric field strength.

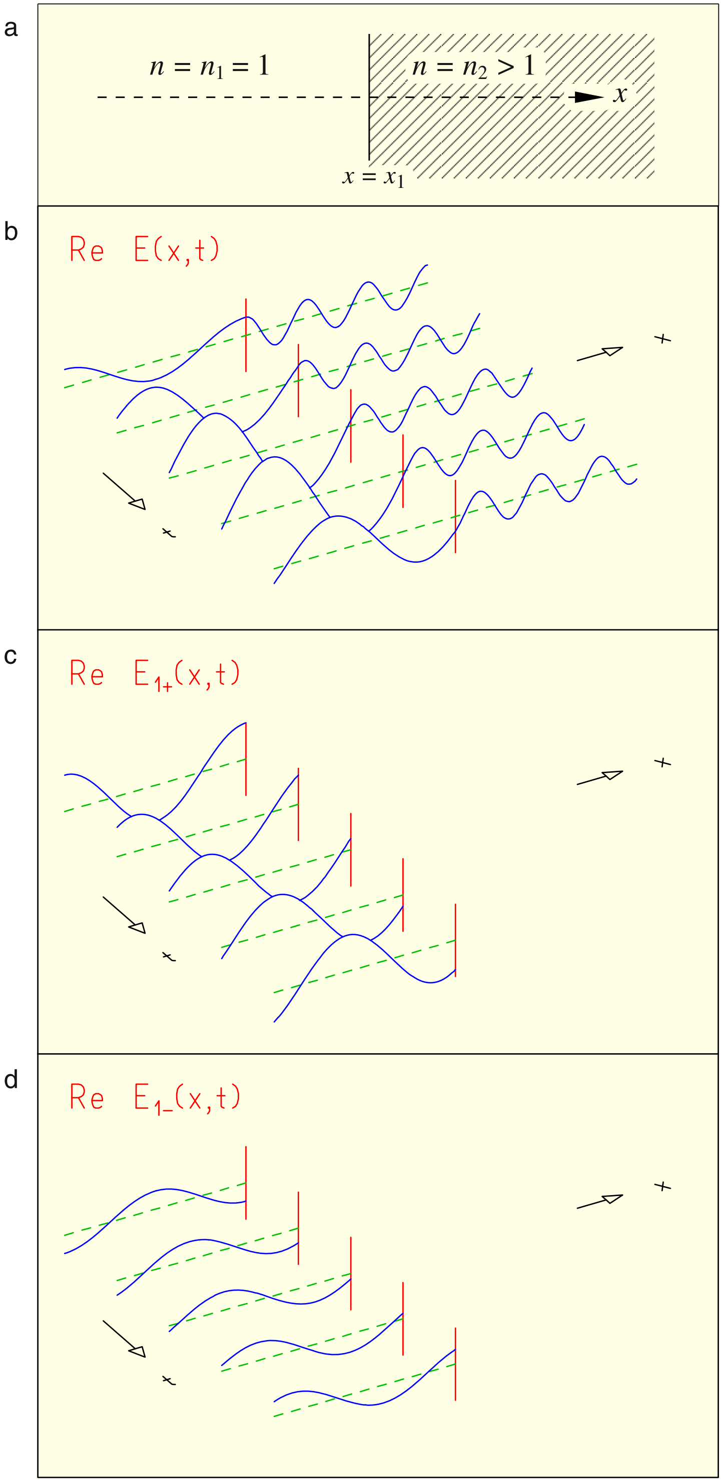

Fig. 2.4. (a) To the right of the plane $x = x_1$, a glass block extends with refractive index $n = n_2$; to the left there is empty space, $n = 1$. **(b)** Time development of the electric field strength of a harmonic wave which falls from the left onto a glass surface, represented by the vertical line, and is partly reflected by and partly transmitted into the glass. **(c)** Time development of the incoming wave alone. **(d)** Time development of the reflected wave alone.

Figure 2.4b gives the time development of this electric field strength. It is easy to see that in the glass there is a harmonic wave moving to the right. The picture in front of the glass is less clear. Figures 2.4c and d therefore show separately the time developments of the incoming and the reflected waves which add up to the total wave to the left of x_1, observed in Figure 2.4b.

2.3 Light Wave Traveling through a Glass Plate

It is now easy to see what happens when light falls on a glass plate of finite thickness. When the light wave penetrates the front surface at $x = x_1$, again reflection occurs so that we have as before the superposition of two stationary waves in the region $x < x_1$:

$$E_1 = A_1 \mathrm{e}^{\mathrm{i}k_1 x} + B_1 \mathrm{e}^{-\mathrm{i}k_1 x} \quad .$$

The wave moving within the glass plate suffers reflection at the rear surface at $x = x_2$, so that the second region, $x_1 < x < x_2$, also contains a superposition of two waves,

$$E_2 = A_2 \mathrm{e}^{\mathrm{i}k_2 x} + B_2 \mathrm{e}^{-\mathrm{i}k_2 x} \quad ,$$

which now have the refracted wave number

$$k_2 = n_2 k_1 \quad .$$

Only in the third region, $x_2 < x$, do we observe a single stationary wave

$$E_3 = A_3 \mathrm{e}^{\mathrm{i}k_1 x}$$

with the original wave number k_1.

As a consequence of the reflection on both the front and the rear surface of the glass plate, the reflected wave in region 1 consists of two parts which interfere with each other. The most prominent phenomenon observed under appropriate circumstances is the destructive interference between these two reflected waves, so that no reflection remains in region 1. The light wave is completely transmitted into region 3. This phenomenon is called a *resonance of transmission*. It can be illustrated by looking at the *frequency dependence* of the stationary waves. The upper plot of Figure 2.5 shows the stationary waves for different fixed values of the angular frequency ω, with its magnitude rising from the background to the foreground. A resonance of transmission is recognized through a maximum in the amplitude of the transmitted wave, that is, in the wave to the right of the glass plate.

The signature of a resonance becomes even more prominent in the frequency dependence of the average energy density in the wave. As discussed in Section 2.1, in a vacuum the average energy density has the form

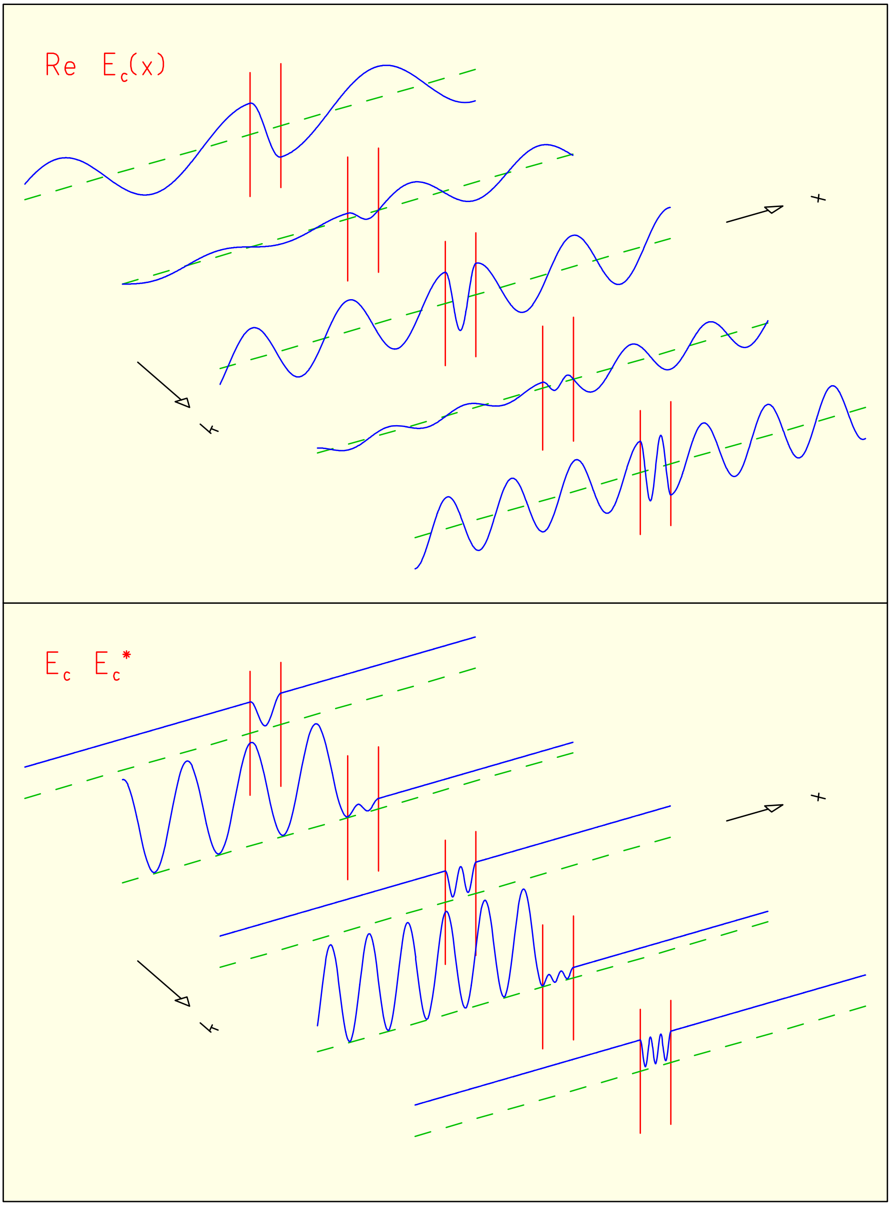

Fig. 2.5. Top: Frequency dependence of stationary waves when a harmonic wave is incident from the left on a glass plate. The two vertical lines indicate the thickness of the plate. Small values of the angular frequency ω are given in the background, large values in the foreground of the picture. Bottom: Frequency dependence of the quantity $E_c E_c^*$ (which except for a factor n_2 is proportional to the average energy density) of a harmonic wave incident from the left on a glass plate. The parameters are the same as in part a. At a resonance of transmission, the average energy density is constant in the left region, indicating through the absence of interference wiggles that there is no reflection.

$$w = \frac{\varepsilon_0}{2} E_c E_c^* \quad .$$

In glass, where the refractive index n has to be taken into account, we have

$$w = \frac{\varepsilon\varepsilon_0}{2} E_c E_c^* = n^2 \frac{\varepsilon_0}{2} E_c E_c^* \quad ,$$

where $\varepsilon = n^2$ is the dielectric constant of glass. Thus, although E_c is continuous at the glass surface, w is not. It reflects the discontinuity of n^2. Therefore we prefer plotting the continuous quantity

$$\frac{2}{n^2 \varepsilon_0} w = E_c E_c^* \quad .$$

This plot, shown in the lower plot of Figure 2.5, indicates a resonance of transmission by the maximum in the average energy density of the transmitted wave. Moreover, since there is no reflected wave at the resonance of transmission, the energy density is constant in region 1.

In the glass plate we observe the typical pattern of a resonance.

(i) The amplitude of the average energy density is maximum.

(ii) The energy density vanishes in a number of places called *nodes* because for a resonance a multiple of half a wavelength fits into the glass plate. Therefore different resonances can be distinguished by the number of nodes.

The ratio of the amplitudes of the transmitted and incident waves is called the *transmission coefficient* of the glass plate,

$$T = \frac{A_3}{A_1} \quad .$$

2.4 Free Wave Packet

The plane wave extends into all space, in contrast to any realistic physical situation in which the wave is localized in a finite domain of space. We therefore introduce the concept of a wave packet. It can be understood as a *superposition*, that is, a sum of plane waves of different frequencies and amplitudes. As a first step we concentrate the wave only in the x direction. It still extends through all space in the y and the z direction. For simplicity we start with the sum of two plane waves with equal amplitudes, E_0:

$$E = E_1 + E_2 = E_0 \cos(\omega_1 t - k_1 x) + E_0 \cos(\omega_2 t - k_2 x) \quad .$$

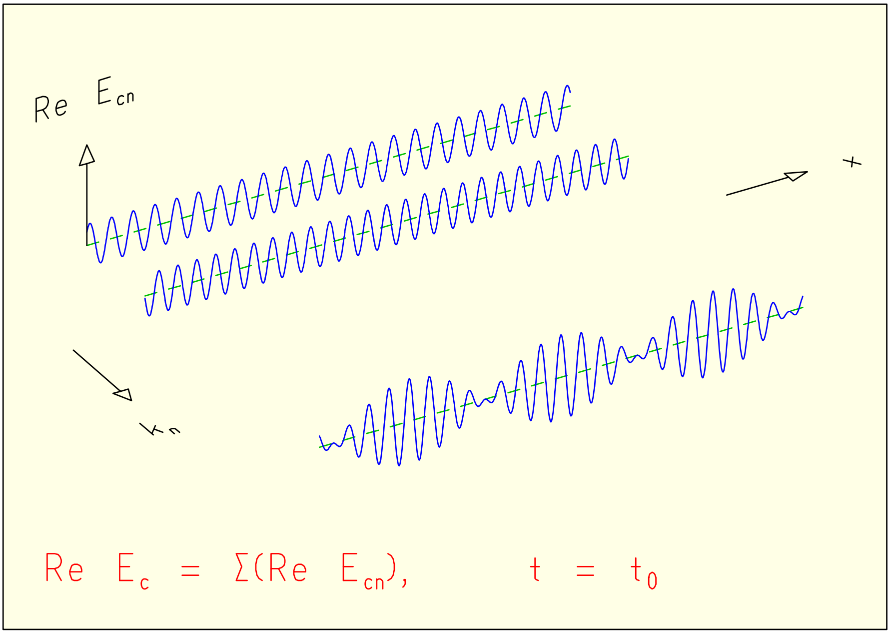

Fig. 2.6. Superposition of two harmonic waves of slightly different angular frequencies ω_1 and ω_2 at a fixed moment in time.

For a fixed time this sum represents a plane wave with two periodic structures. The slowly varying structure is governed by a spatial period,

$$\lambda_- = \frac{4\pi}{|k_2 - k_1|} \quad ,$$

the rapidly varying structure by a wavelength,

$$\lambda_+ = \frac{4\pi}{|k_2 + k_1|} \quad .$$

The resulting wave can be described as the product of a “carrier wave” with the short wavelength λ_+ and a factor modulating its amplitude with the wavelength λ_-:

$$E = 2E_0 \cos(\omega_- t - k_- x) \cos(\omega_+ t - k_+ x) \quad ,$$

$$k_\pm = |k_2 \pm k_1|/2 \quad , \qquad \omega_\pm = ck_\pm \quad .$$

Figure 2.6 plots for a fixed moment in time the two waves E_1 and E_2, and the resulting wave E. Obviously, the field strength is now concentrated for the most part in certain regions of space. These regions of great field strength propagate through space with the velocity

$$\frac{\Delta x}{\Delta t} = \frac{\omega_-}{k_-} = c \quad .$$

Now we again use complex field strengths. The superposition is written as

$$E_c = E_0 \, e^{-i(\omega_1 t - k_1 x)} + E_0 \, e^{-i(\omega_2 t - k_2 x)} \quad .$$

For the sake of simplicity, we have chosen in this example a superposition of two harmonic waves with equal amplitudes. By constructing a more complicated "sum" of plane waves, we can concentrate the field in a single region of space. To this end we superimpose a continuum of waves with different frequencies $\omega = ck$ and amplitudes:

$$E_c(x,t) = E_0 \int_{-\infty}^{+\infty} dk \, f(k) e^{-i(\omega t - kx)} \quad .$$

Such a configuration is called a *wave packet*. The *spectral function* $f(k)$ specifies the amplitude of the harmonic wave with wave number k and circular frequency $\omega = ck$. We now consider a particularly simple spectral function which is significantly different from zero in the neighborhood of the wave number k_0. We choose the *Gaussian* function

$$f(k) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left[-\frac{(k-k_0)^2}{2\sigma_k^2}\right] \quad .$$

It describes a bell-shaped spectral function which has its maximum value at $k = k_0$; we assume the value of k_0 to be positive, $k_0 > 0$. The width of the region in which the function $f(k)$ is different from zero is characterized by the parameter σ_k. In short, one speaks of a Gaussian with *width* σ_k. The Gaussian function $f(k)$ is shown in Figure 2.7a. The factors in front of the exponential are chosen so that the area under the curve equals one. We illustrate the construction of a wave packet by replacing the integration over k by a sum over a finite number of terms,

$$\begin{aligned} E_c(x,t) &\approx \sum_{n=-N}^{N} E_n(x,t) \quad , \\ E_n(x,t) &= E_0 \, \Delta k \, f(k_n) \, e^{-i(\omega_n t - k_n x)} \quad , \end{aligned}$$

where

$$k_n = k_0 + n\,\Delta k \quad , \qquad \omega_n = ck_n \quad .$$

In Figure 2.7b the different terms of this sum are shown for time $t = 0$, together with their sum, which is depicted in the foreground. The term with the lowest wave number, that is, the longest wavelength, is in the background of the picture. The variation in the amplitudes of the different terms reflects the Gaussian form of the spectral function $f(k)$, which has its maximum, for $k = k_0$, at the center of the picture. On the different terms, the partial waves, the point $x = 0$ is marked by a circle. We observe that the sum over all terms is concentrated around a rather small region near $x = 0$.

Figure 2.7c shows the same wave packet, similarly made up of its partial waves, for later time $t_1 > 0$. The wave packet as well as all partial waves have moved to the right by the distance ct_1. The partial waves still carry marks at the phases that were at $x = 0$ at time $t = 0$. The picture makes it clear that all partial waves have the same velocity as the wave packet, which maintains the same shape for all moments in time.

If we perform the integral explicitly, the wave packet takes the simple form

$$\begin{aligned} E_{\mathrm{c}}(x,t) &= E_{\mathrm{c}}(ct-x) \\ &= E_0 \exp\left[-\frac{\sigma_k^2}{2}(ct-x)^2\right] \exp\left[-\mathrm{i}(\omega_0 t - k_0 x)\right] \quad , \end{aligned}$$

that is,

$$E(x,t) = \mathrm{Re}\, E_{\mathrm{c}} = E_0 \exp\left[-\frac{\sigma_k^2}{2}(ct-x)^2\right] \cos(\omega_0 t - k_0 x) \quad .$$

It represents a plane wave propagating in the positive x direction, with a field strength concentrated in a region of the spatial extension $1/\sigma_k$ around point $x = ct$. The time development of the field strength is shown in Figure 2.8b. Obviously, the maximum of the field strength is located at $x = ct$; thus the wave packet moves with the velocity c of light. We call this configuration a *Gaussian wave packet* of spatial width

$$\Delta x = \frac{1}{\sigma_k} \quad ,$$

and of wave-number width

$$\Delta k = \sigma_k \quad .$$

We observe that a spatial concentration of the wave in the region Δx necessarily requires a spectrum of different wave numbers in the interval Δk so that

$$\Delta x\, \Delta k = 1 \quad .$$

This is tantamount to saying that the sharper the localization of the wave packet in x space, the wider is its spectrum in k space. The original harmonic wave $E = E_0 \cos(\omega t - kx)$ was perfectly sharp in k space ($\Delta k = 0$) and therefore not localized in x space. The time development of the average energy density w shown in Figure 2.8c appears even simpler than that of the field

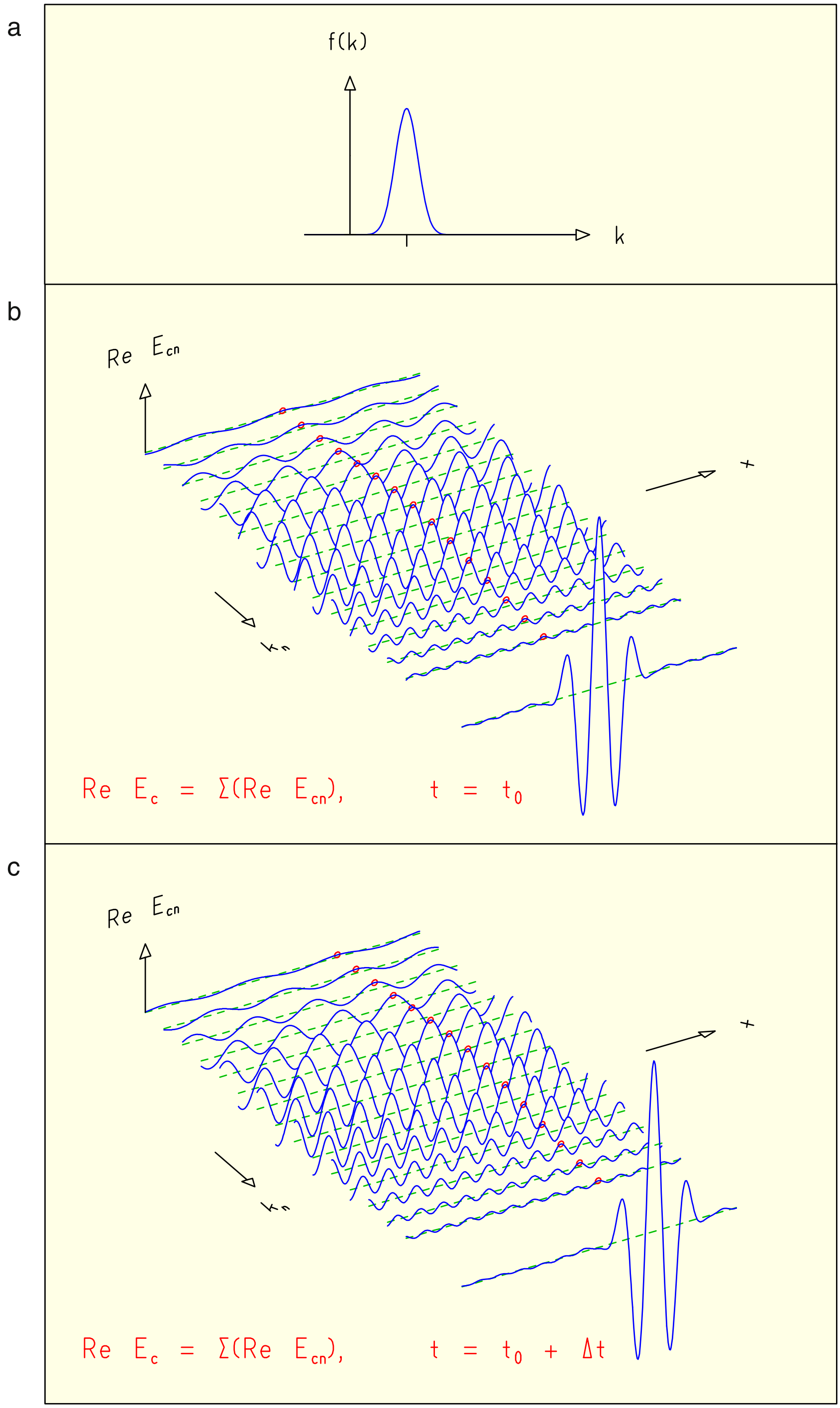
a
f(k)
k
b
Re E_cn
x
k_n
Re E_c = Σ(Re E_cn), t = t_0
c
Re E_cn
x
k_n
Re E_c = Σ(Re E_cn), t = t_0 + Δt

Fig. 2.7.

strength. It is merely a Gaussian traveling with the velocity of light along the x direction. The Gaussian form is easily explained if we remember that

$$w = \frac{\varepsilon_0}{2} E_c E_c^* = \frac{\varepsilon_0}{2} E_0^2 e^{-\sigma_k^2 (ct-x)^2} \quad .$$

We demonstrate the influence of the spectral function on the wave packet by showing in Figure 2.8 spectral functions with two different widths σ_k. For both we show the time development of the field strength and of the average energy density.

2.5 Wave Packet Incident on a Glass Surface

The wave packet, like the plane waves of which it is composed, undergoes reflection and transmission at the glass surface. The upper plot of Figure 2.9 shows the time development of the average energy density in a wave packet moving in from the left. As soon as it hits the glass surface, the already reflected part interferes with the incident wave packet, causing the wiggly structure at the top of the packet. Part of the packet enters the glass, moving with a velocity reduced by the refractive index. For this reason it is compressed in space. The remainder is reflected and moves to the left as a regularly shaped wave packet as soon as it has left the region in front of the glass where interference with the incident packet occurs.

We now demonstrate that the wiggly structure in the interference region is caused by the fast spatial variation of the carrier wave characterized by its wavelength. To this end let us examine the time development of the field strength in the packet, shown in the lower plot of Figure 2.9. Indeed, the spatial variation of the field strength has twice the wavelength of the average energy density in the interference region.

Another way of studying the reflection and transmission of the packet is to look separately at the average energy densities of the constituent waves, namely the incoming, transmitted, and reflected waves. We show these constituent waves in both regions 1, a vacuum, and 2, the glass, although they contribute physically only in either the one or the other. Figure 2.10 gives

Fig. 2.7. (a) Gaussian spectral function describing the amplitudes of harmonic waves of different wave numbers k. (b) Construction of a light wave packet as a sum of harmonic waves of different wavelengths and amplitudes. For time $t = 0$ the different terms of the sum are plotted, starting with the contribution of the longest wavelength in the background. Points $x = 0$ are indicated as circles on the partial waves. The resulting wave packet is shown in the foreground. (c) The same as part b, but for time $t_1 > 0$. The phases that were at $x = 0$ for $t = 0$ have moved to $x_1 = ct_1$ for all partial waves. The wave packet has consequently moved by the same distance and retained its shape.

a d b e c f

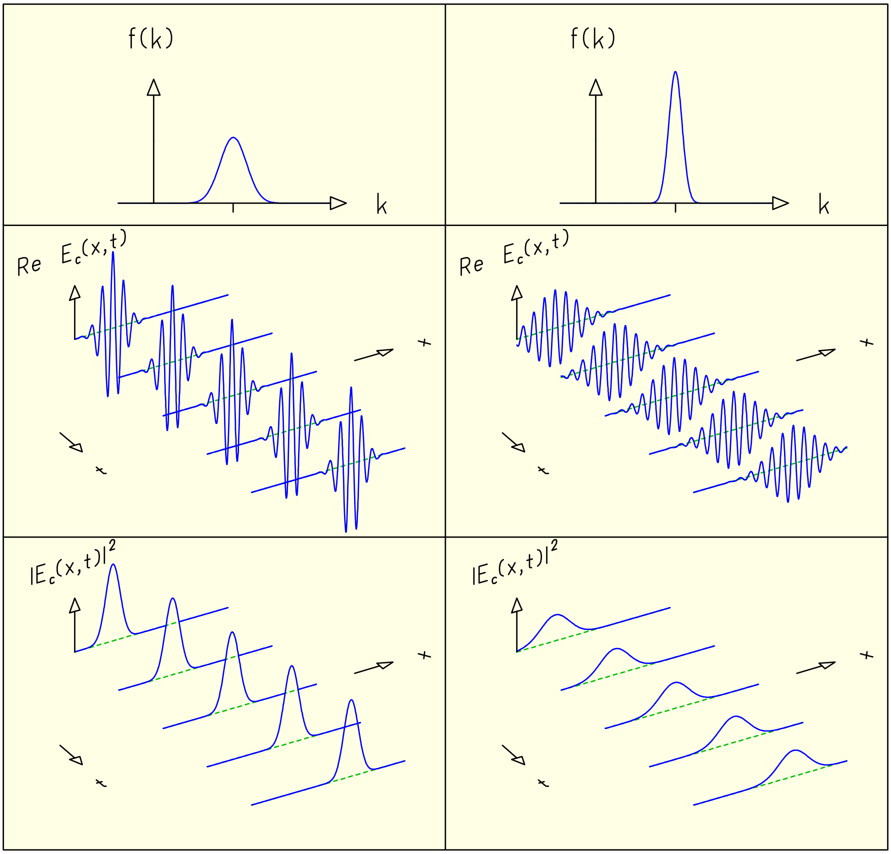

Fig. 2.8. (a, d) Spectral functions, (b, e) time developments of the field strength, and (c, f) time developments of the average energy density for two different Gaussian wave packets.

their time developments. All three have a smooth bell-shaped form and no wiggles, even in the interference region. The time developments of the field strengths of the constituent waves are shown in Figure 2.11. The observed average energy density of Figure 2.9 corresponds to the absolute square of the sum of the incoming and reflected field strengths in the region in front of the glass and, of course, not to the sum of the average energy densities of these two constituent fields. Their interference pattern shows half the wavelength of the carrier waves.

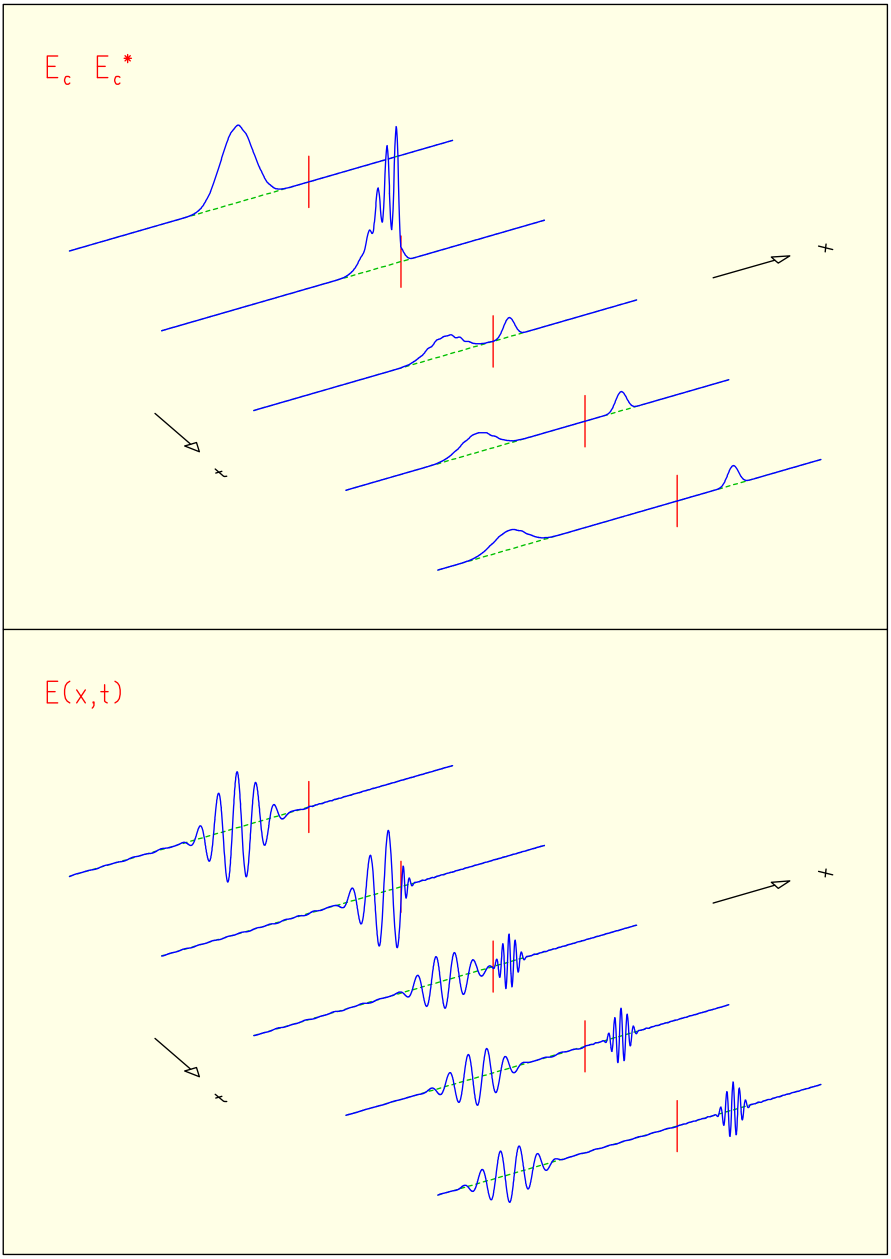

Fig. 2.9. Time developments of the quantity $E_c E_c^*$ (which except for a factor n^2 is proportional to the average energy density) and of the field strength in a wave packet of light falling onto a glass surface where it is partly reflected and partly transmitted through the surface. The glass surface is indicated by the vertical line.

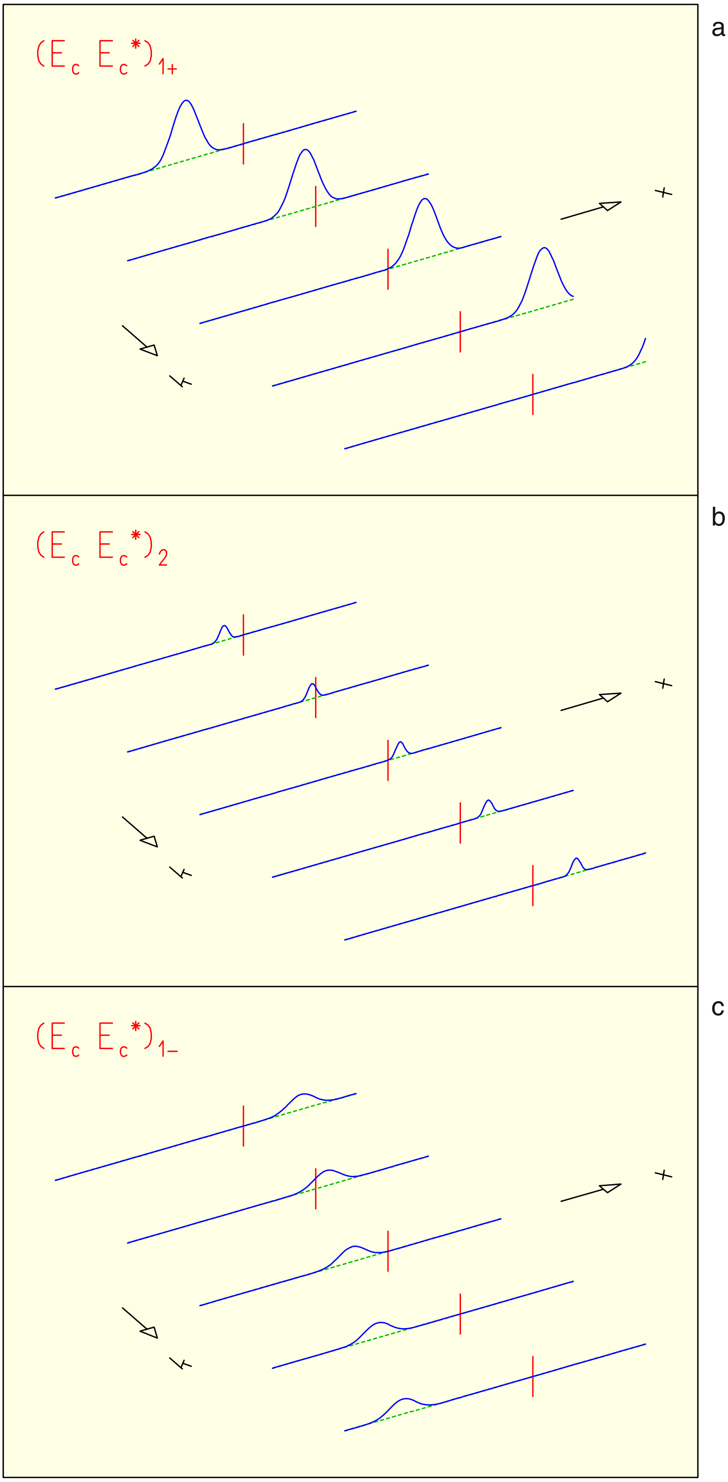

Fig. 2.10. **Time developments of the quantity** $E_c E_c^*$ **(which except for a factor** n^2 **is proportional to the average energy density) of the constituent waves in a wave packet of light incident on a glass surface: (a) incoming wave, (b) transmitted wave, and (c) reflected wave.**

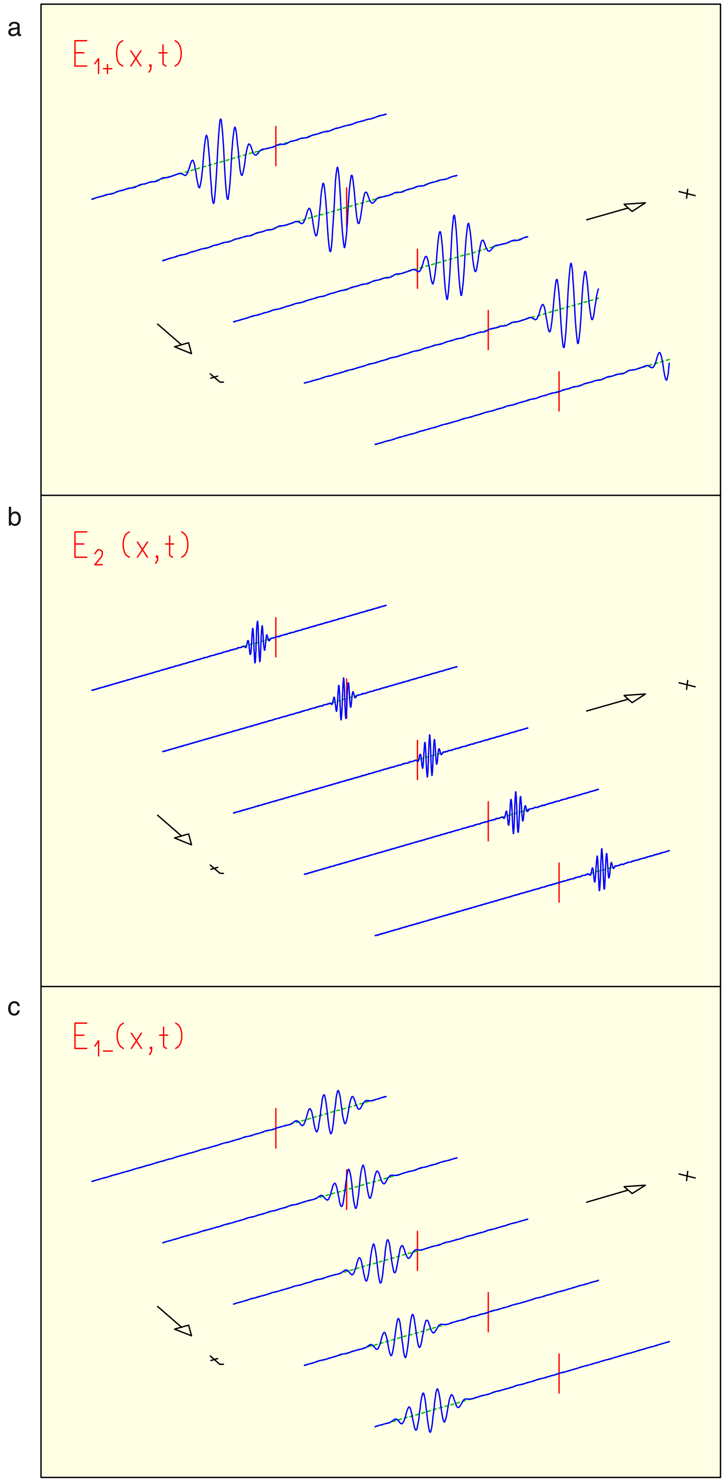

Fig. 2.11. Time developments of the electric field strengths of the constituent waves in a wave packet of light incident on a glass surface: (a) incoming wave, (b) transmitted wave, and (c) reflected wave.

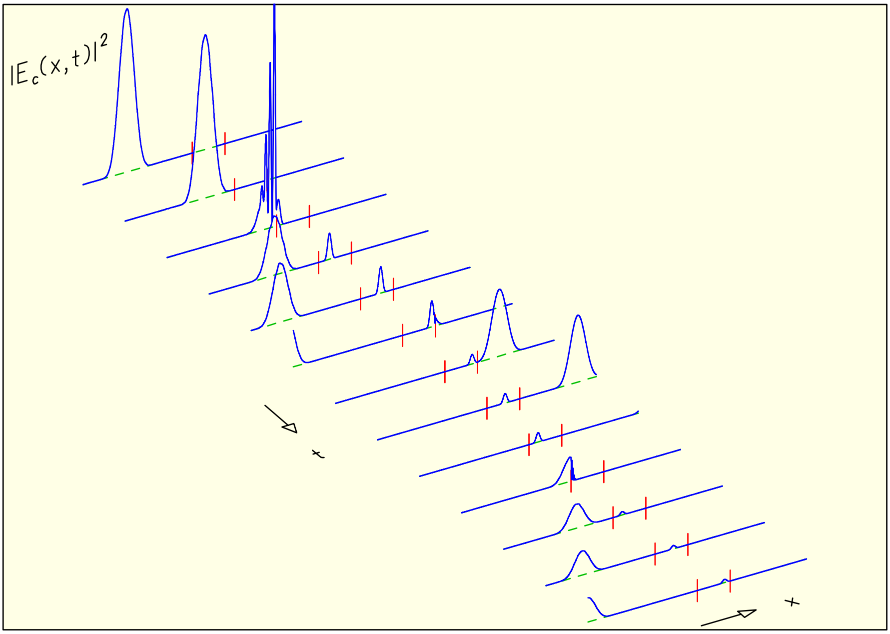

Fig. 2.12. Time development of the quantity $E_c E_c^*$ (which except for a factor n^2 is proportional to the average energy density) in a wave packet of light incident on a glass plate.

2.6 Wave Packet Traveling through a Glass Plate

Let us study a wave packet that is relatively narrow in space, that is, one containing a wide range of frequencies. The time development of its average energy density (Figure 2.12) shows that, as expected, at the front surface of the glass plate part of the packet is reflected. Another part enters the plate, where it is compressed and travels with reduced speed. At the rear surface this packet is again partly reflected while another part leaves the plate, traveling to the right with the original width and speed. The small packet traveling back and forth in the glass suffers multiple reflections on the glass surfaces, each time losing part of its energy to packets leaving the glass.

Problems

2.1. Estimate the refractive index n_2 of the glass plate in Figure 2.4b.

2.2. Calculate the energy density for the plane electromagnetic wave described by the complex electric field strength

$$E_{\rm c} = E_0\,{\rm e}^{-{\rm i}(\omega t - kx)}$$

and show that its average over a temporal period T is $\omega = (\varepsilon_0/2)E_{\rm c}E_{\rm c}^*$.

2.3. Give the qualitative reason why the resonance phenomena in Figure 2.5 (top) occurs for the wavelengths

$$\lambda = \ell\frac{nd}{2} \quad , \qquad \ell = 1, 2, 3\ldots \quad .$$

Use the continuity condition of the electric field strength and its derivative. Here n is the refractive index of the glass plate of thickness d.

2.4. Calculate the ratio of the frequencies of the two electric field strengths, as they are plotted in Figure 2.6, from the beat in their superposition.

2.5. The one-dimensional wave packet of light does not show any dispersion, that is, spreading with time. What causes the dispersion of a wave packet of light confined in all three spatial dimensions?

2.6. Estimate the refractive index of the glass, using the change in width or velocity of the light pulse in Figure 2.9 (top).

2.7. Verify in Figure 2.12 that the stepwise reduction of the amplitude of the pulse within the glass plate proceeds with approximately the same reduction factor, thus following on the average an exponential decay law.

2.8. Calculate energy E and momentum p of a photon of blue ($\lambda = 450 \times 10^{-9}$ m), green ($\lambda = 530 \times 10^{-9}$ m), yellow ($\lambda = 580 \times 10^{-9}$ m), and red ($\lambda = 700 \times 10^{-9}$ m) light. Use Einstein's formula $E = Mc^2$ to calculate the relativistic mass of the photon. Give the results in SI units.

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