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“There is a special trick that we use when we count!”

“Place the stick upright on the table”

Funneling Through Rerouting a Strategy

“Count the spaces in between the numbers”

“No matter where you start”

“If you chose to start the measurement of your stick at 5 cm…”

“You don’t count the first line”

“Can you draw me a 1 cm long line”

What Do We Learn About Prospective Teachers’ Ideas of Measurement?

5 On Divisibility by 4

Retrieving the Correct Divisibility Rule

“Who can remember?”

“Look it up in my notes” or elsewhere

“We can look in the glossary of the textbook”

“I think we are confusing…”

“Some wonderful little tricks”

“You’ve won the concert tickets!”

“How about 1000456814?”

Moving Toward Student Reasoning

“See if you can find a rule that does work”

“That’s easy, they’re all [16, 20 and 24] divisible by four”

“Start looking”… “after the break”

“How does it work?”

“I imagine that each one of these is a chocolate bar”

Uses of Mathematical Language

“A number is divisible by 4 if…”

“Just look at the last two numbers”

“What about the number 6, what’s the rule for that one?”

More Troublesome Expressions

Because... Alternative Diagnoses and Remediation

“Because 354 has 4 in the one’s place”

“Let’s try doing long division”

“Because 354 is an even number”

“I worked through all the division steps”

“4 goes into 354, 88.5 times”

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