Preface

The well-known monographs by G.S. Ladde, V. Lakshmikantham and B.G. Zhang [248], I. Győri and G. Ladas [192], L.H. Erbe, Q. Kong and B.G. Zhang [154], R.P. Agarwal, M. Bohner and W.-T. Li [3], R.P. Agarwal, S.R. Grace and D. O'Regan [8] and D.D. Bainov and D.P. Mishev [34] are devoted to the oscillation theory of functional differential equations. Each of these monographs contains nonoscillation tests, but their main objective was to present methods and results concerning oscillation of all solutions for the functional differential equations under consideration.

The main purpose of the present monograph is to consider nonoscillation and existence of positive solutions for functional differential equations and to describe their applications to maximum principles, boundary value problems and the stability of these equations.

In view of this objective, we consider a wide class of equations:

1. scalar equations and systems of different types: linear and nonlinear first-order functional differential equations, second-order equations with or without damping terms, high-order equations, systems of functional differential equations;
2. equations with variable types of delays: delay differential equations, integrodifferential equations, equations with a distributed delay, neutral equations;
3. equations with variable deviations of the argument: advanced and mixed (including both delayed and advanced terms) differential equations;
4. both continuous and impulsive equations: first- and second-order linear and first-order nonlinear impulsive differential equations;
5. specific classes of linear and nonlinear equations, as well as linear differential equations with abstract Volterra (causal) operators;
6. both initial and boundary value problems are considered for functional differential equations.

Note that we do not use methods specific only to equations with continuous parameters since we consider models with measurable coefficients and delays.

Nonoscillation results are applied

- to nonlinear nonautonomous equations of mathematical biology with both concentrated and distributed delays;

- to stability problems; and
- to boundary value problems.

Chapter 1 is a brief survey of introductory notions and ideas in nonoscillation theory: autonomous equations, characteristic equations, solution representations, differential and integral equations, and inequalities. Though elementary in its presentation (we believe it can easily be understood by senior undergraduate students), this chapter incorporates many basic ideas that will be employed later: equivalence of nonoscillation and existence of a nonnegative solution of the generalized characteristic inequality and the application of solution representation, linearization and the approach to impulsive equations. The main population dynamics equations (Hutchinson's, Lasota-Wazewska, Mackey-Glass, Nicholson's blowflies) are also introduced in Chap. 1.

Chapter 2 presents basic results for first-order linear delay equations with positive coefficients: nonoscillation criteria, comparison theorems, explicit nonoscillation and oscillation results, sufficient conditions for positivity of solutions with given initial conditions and slowly nonoscillating solutions. In Chap. 3, some of these results are generalized to equations with positive and negative coefficients; it is also illustrated that some of the results cannot be extended. Chapter 4 is concerned with a general linear equation with a distributed delay that is nonautonomous and can include integral and concentrated delay terms. The case of positive kernels of integrals and coefficients is considered, as well as terms of different signs.

In Chap. 5, nonoscillation of linear equations of advanced and mixed types is studied. The main results of this chapter are based on various fixed-point theorems. Chapter 6 is concerned with linear neutral equations of the first order that include the derivative of the unknown function both with and without delays.

In Chaps. 7 and 8, we consider linear second-order delay equations without damping and with damping, respectively. Chapter 9 deals with linear systems of delay differential equations and also higher-order differential equations. In addition to the problems considered in the previous chapters, Chap. 9 includes an extensive section on stability of nonoscillatory systems.

Chapters 10 and 11 are devoted to nonlinear equations. In Chap. 10, the linearization method is applied to various nonautonomous models of population dynamics (in particular, logistic, Lasota-Wazewska and Nicholson's blowflies equations), and all equations are considered with a distributed delay. In Chap. 11, some equations that cannot be handled with the linearization approach are studied (mostly different variations of the logistic model).

Chapters 12–14 are concerned with impulsive equations. Chapter 12 presents nonoscillation results for first-order linear impulsive differential equations with both concentrated and distributed delays. It is also demonstrated that nonoscillation of an impulsive equation can be reduced to nonoscillation of a specially constructed equation without impulses but with discontinuous coefficients. Chapter 13 deals with second-order differential equations, and generally in the models considered any linear jumps of both the solution and the first derivative can occur. In Chap. 14, linearization methods are applied to first-order nonlinear impulsive equations.

The study of many classical questions in the qualitative theory of linear n-th-order ordinary differential equations, such as existence and uniqueness of solutions of the interpolation boundary value problems, positivity, or a corresponding regular behavior of their Green's functions, maximum principles and stability, was connected with and even based on the notion of nonoscillation intervals of corresponding linear ordinary differential equations. In Chaps. 15–17 we create a concept of nonoscillation intervals for functional differential equations that can actually be considered as an analogue of nonoscillation theory for ordinary differential equations. Various relations between the noted properties are obtained for functional differential equations on the basis of nonoscillation. Linear and nonlinear equations with Volterra (causal) operators were previously studied in the monographs [29, 98, 239, 251]. In Chaps. 15–17, we consider equations with Volterra operators. It should be noted that it is not only a generalization but also an important instrument for studying the behavior of a corresponding component x_r of a solution vector. We construct an equation for this component in Chap. 16. Even in the case of systems of ordinary differential equations, this differential equation for x_r is of quite a general form that includes Volterra operators. In these chapters, we also study such questions as maximum principles, existence and uniqueness of solutions to boundary value problems, regular behavior of their Green's functions, and applications to study stability that are not considered in previous chapters.

All chapters conclude with a discussion, some open problems, and topics for possible future research.

Finally, Appendices A and B include some reference material. Appendix A contains all auxiliary notions and functional analysis results used in the monograph: definitions of functional spaces, measures and Volterra operators, compactness conditions for sets and linear operators, and fixed-point theorems in Banach spaces with or without order. These results are applied in the study of a variety of types of equations: with several concentrated and distributed delays, with general Volterra and non-Volterra equations and systems, linear and nonlinear, and continuous and impulsive. Appendix B presents existence and uniqueness conditions for all functional differential equations considered in this monograph; in addition, solution representations are given for linear equations.

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