Preface


The main aim of this short book is to present recent results concerning inequalities of the Jensen, Čebyšev and Grüss type for continuous functions of bounded selfadjoint operators on complex Hilbert spaces.

The book is intended for use by both researchers in various fields of Linear Operator Theory and Mathematical Inequalities, domains which have grown exponentially in the last decade, as well as by postgraduate students and scientists applying inequalities in their specific areas.

In Chap. 1, we recall some fundamental facts concerning bounded selfadjoint operators on complex Hilbert spaces. The generalized Schwarz’s inequality for positive selfadjoint operators as well as some results for the spectrum of this class of operators are presented. Then we introduce and explore the fundamental results for polynomials in a linear operator, continuous functions of selfadjoint operators as well as the step functions of selfadjoint operators. Using these results, we then introduce the spectral decomposition of selfadjoint operators (the Spectral Representation Theorem) that will play a central role in the rest of the book. This result is used as a key tool in obtaining various new inequalities for continuous functions of selfadjoint operators, functions that are of bounded variation, Lipschitzian, monotonic or absolutely continuous. Another tool that will greatly simplify the error bounds provided in the book is the Total Variation Schwarz’s Inequality for which a simple proof is offered.

Jensen’s type inequalities in their various settings ranging from discrete to continuous case play an important role in different branches of Modern Mathematics. A simple search in the MathSciNet database of the American Mathematical Society with the key words “jensen” and “inequality” in the title reveals more than 300 items intimately devoted to this famous result. However, the number of papers where this inequality is applied is a lot larger and far more difficult to find.
In Chap. 2, we present some recent results obtained by the author that deal with different aspects of this well-researched inequality than those recently reported in the book [19]. They include but are not restricted to the operator version of the Dragomir–Ionescu inequality, Slater’s type inequalities for operators and its inverses, Jensen’s inequality for twice differentiable functions whose second derivatives satisfy some upper and lower bounds conditions, Jensen’s type inequalities for log-convex functions and for differentiable log-convex functions. Finally, some Hermite–Hadamard’s type inequalities for convex functions and Hermite–Hadamard’s type inequalities for operator convex functions are presented as well.

Chapter 3 is devoted to Čebyšev and Grüss’ type inequalities.

The Čebyšev, or in a different spelling – Chebyshev, inequality which compares the integral/discrete mean of the product with the product of the integral/discrete means is famous in the literature devoted to Mathematical Inequalities. It has been extended, generalized, refined, etc. by many authors during the last century. A simple search utilizing either spellings and the key word “inequality” in the title in the comprehensive MathSciNet database produces more than 200 research articles devoted to this result.

The sister inequality due to Grüss which provides error bounds for the magnitude of the difference between the integral mean of the product and the product of the integral means has also attracted much interest since it has been discovered in 1935 with more than 180 papers published, as a simple search in the same database reveals. Far more publications have been devoted to the applications of these inequalities and an accurate picture of the impacted results in various fields of Modern Mathematics is difficult to provide.

In this chapter, however, we present only some recent results due to the author for the corresponding operator versions of these two famous inequalities.

For the sake of completeness, all the results presented are completely proved and the original references where they have been first obtained are mentioned. The chapters are followed by the list of references used therein and therefore are relatively independent and can be read separately.

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