Abstract  Our material world is reasonably well described by the set of working models that have been systematically derived in classical physics to map our perception of reality into compressed mathematical descriptions. These models are valid for most medium-intensity external conditions. For the purpose of our discussion, we accept them with their limitations and approximations as if they were the complete truth. That is, in this book we are not concerned with explaining the exact nature of all things. Instead, we merrily move forward and learn how to use the various phenomena and design new effects. In this section, we present the formulations of basic variables that are needed for RF circuit design and define the basic terminology.

2.1 Matter and Electricity

Conclusions that all matter consists mostly of electricity and that electrons are responsible for all chemical reactions in the known universe have been among the top intellectual achievements in human history. A positively charged nucleus contains almost all the mass of the atom and determines which element the atom is, e.g. silicon, oxygen, or any other element. The nucleus is generally very stable and it takes high amounts of energy to take it apart, i.e., to convert an atom into another element. In contrast, the fast-moving, negatively charged electrons contribute only a small percentage to the total atomic mass, however they determine the types of chemical reaction into which the atom enters. We can visualize fast-moving electrons as creating “shells” around the nucleus that are not easily penetrable, similar to the barrier created by a fast-spinning airplane propeller. However, a behaviour that is of much more relevance to our subject is that, in some cases, an electron may easily leave the atom and, in other cases, an external electron may join an atom by finding a place in the outermost shell. These “free-moving electrons” are responsible for a myriad of phenomena, only a small part of which are studied in this book (Fig. 2.1).

2.2 Electromotive Force

Once an electron leaves its native atom or joins some other hosting atom, a number of interesting things start happening. To start with, an atom that has lost one or more of its electrons is not electrically neutral—it now has an overall surplus of positive charges and is referred to as a “positive ion”. Similarly, an atom that receives one or more external electrons in its outermost shell gains a surplus of negative charges and is referred to as a “negative ion”. It is important to notice that ions do not
change the material itself, i.e., silicon is still silicon, oxygen is still oxygen, etc. However, charged particles interact by means of the electric field between them (see Fig. 1.19), which is a source of stress in the material that, theoretically, extends into the space infinitely far away. This stress due to unequal distribution of charges is manifested by an attractive or repelling force between the charged particles. Furthermore, the electric force is directly related to “potential energy”, which defines the “potential” at a point inside the electric field, so that the potential energy of the charged particle at that point is measured relative to the reference point in infinity. A relative, and more practical, measure of potential is the “potential difference” (also known as the voltage) between two particles (or charged objects), where one of the objects serves as the reference point. In other words, the voltage $V$ between objects $A$ and $B$, is measured as the difference between their potentials, i.e., $V = V_A - V_B$. Most solid materials have their ions fixed (liquids and gases do not), while free electrons are pushed by electric field forces. Therefore, inside an electric field, the negatively charged electrons keep moving until, eventually, the overall electrical balance is restored. It is important to note that, by being negatively charged, the natural direction of the electron flow is towards the positive ions.

An interesting situation arises when, for example, a metallic wire (that happens to have a large number of free electrons) is connected to a device called a “battery”. Then, the imbalance of the charges is maintained because the battery serves as an infinite source of free electrons because it provides “electromotive force” that moves the electrons from higher to lower potentials (which is opposite to their natural flow). The assembly of the metallic wire and the battery is referred to as a “closed circuit”, where the battery enables the constant flow of electrons within the loop, as long as the path stays closed. This flow of free charged particles from higher potential to lower potential\(^1\) caused by an electromotive force is referred to as “electric current”. A battery serves a similar role to a water pump, which constantly pumps water to the top of a hill (i.e., increases its potential energy) and the water is pulled back to the bottom of the hill by the gravitational force (i.e., the potential energy is converted into kinetic energy). On its way down, the water flow may be used to do some extra work, for example, to spin a watermill.

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\(^1\)Keep in mind that, for historical reasons, the definition of the positive electric current direction is opposite to the direction of the moving electrons (a surplus of electrons means more negative charge).
2.3 Electric Current Effects

Now that we have established why and how the electric current came to be, it is natural to ask what it can do. At the very fundamental level, an electric current:

- **Generates heat**: The interaction of flowing electrons with the atomic lattice of a metallic material causes the atoms to increase the amplitude of their vibrations, which manifests as an increase in the material’s temperature. Sometimes this heat generation is desirable, e.g. in an electric heater, and sometimes it is not, e.g. in a light bulb. Regardless of its desirability, it is very important to quantify the rate of heat generation (more details are presented in Sect. 4.1.4).
- **Generates a magnetic field around itself**: This property of an electric current is fundamental for wireless communication systems and is studied in great detail through the rest of the book.
- **Causes a chemical change in some materials**: This property of an electric current is exploited in chemistry, especially when a current passes through a liquid and enables the process of charging chemical batteries.

It should be noted that all of the above phenomena are bidirectional. Although of minor importance, the production of electricity in thermocouples is widely used for manufacturing thermal sensors. By far, the most exploited mechanism of electricity production is based on moving magnetic fields. Chemical batteries are still the most commonly used source of electric current for our mobile electronic devices.

2.4 Conductors, Semiconductors, and Insulators

In general, electrons do not have enough energy to leave a material. Instead, they keep exchanging their position by jumping from one atom to another. Every electron jump leaves a vacant spot, referred to as a positively charged “hole”, behind in the positively charged ion, which in turn attracts some other electron and becomes neutral again. Due to large number of joggling electrons at any given time, a useful model is to treat them as an “electron cloud”. Electron movements are induced in many ways, e.g. heat, and they happen randomly in time, which means that the average direction of the moving electron cloud is zero, similar to a swarm of bees that stays at one spot even though all the bees are very busy buzzing around; i.e., there is no spontaneous current flow in any particular direction. In order to force the electron cloud inside the material to have a non-zero average movement, an external electric field must be applied, for example by means of a battery. The external battery that is connected to a conductor serves as a “pump” that forces flow of the electrons from the battery’s negative terminal through the conductor to its positive terminal. Materials that easily allow this directional drift of their electron cloud are called *conductors*. Most metallic materials are good conductors of electric current. A common model of a conductor assumes an ideal metallic wire, which allows an infinite electric current flow, even if an infinitely small voltage is applied at its ends. In other words, the ideal conductor is capable of dissipating an infinite amount of heat, which is to say that it can handle infinite power. Although real conductors do not have these properties, this idealization is very useful and commonly used every time you draw an electric schematic diagram. The connecting lines between the circuit components are assumed to be ideal wire conductors. This approximation is mostly valid, especially for circuits using low levels of current and operating at low frequencies.

Materials that do not have enough free charge–carriers to form the electron cloud are called *insulators*. Most plastic and glass-based materials are good insulators. That means that even if an internal electric field is created by an external potential difference across the isolating material and electric stress is induced in the material, (to the first approximation) there is no free current flow...
through the insulator. A common model for the ideal insulator assumes that no single electron can leave the insulating material if a constant electric field is applied. Moreover, the ideal insulator is capable of handling infinite voltage across its terminals without allowing any current to flow. That is, because no current flow is allowed, the ideal insulator does not dissipate any amount of heat. This approximation is very useful because it enables us to model ideal discrete circuit components. In reality, there is always a small “current leakage” flowing through an insulator, however in applications with moderate requirements the leakage of current is safely ignored.

A third, and equally important, category of materials is known as “semiconductors”. In general, semiconductive materials are neither good conductors nor good insulators. However, they do have a sufficient number of freely moving electric charges for a given volume of the material, which is strictly controlled so that the population of free charges is in the minority from the macro perspective. Under specially orchestrated conditions, for some types of semiconductor structures, it is possible to temporarily collect these free charges and to turn the non-conductive and localized volume of the semiconductor material into a very good conductor, i.e., to locally “invert” its conductive property. Once the controlling conditions are removed, everything reverts to the initial non-inverted state. This process is non-destructive, repeatable, and under full control of the circuit designer. Important variants of controlling current flow in semiconductor devices are outlined in Sect. 4.3.

2.5 Basic Electrical Variables

In this chapter, we have intuitively introduced the concept of “matter”, which is a form of energy, and its basic property of “electric charge”. In that model, the fine point is that electrons and protons are assumed to be merely material “carriers” of their respective charges.

2.5.1 Voltage

The concept of a particle charge, \( q \) (a scalar variable), leads to the concept of an electric field \( \mathbf{E} \) (a vector variable), and to the electrical potential \( V_X \) (a scalar variable that is relative to a point infinitely far away) of a charged particle that occupies a point in space at the coordinate \( X \). Two particle charges, occupying different points \( X \) and \( Y \) in space in the electric field, are therefore at different potentials. Thus they are said to have potential difference (p.d.) between them, which is referred to as “voltage” \( V \) and calculated as \( \Delta V = V_X - V_Y \). Note that voltage is a relative measure that can be either positive or negative, depending upon which of the two charge potentials is assumed to be the local reference.

In a broad sense, electric fields are classified as either “static” or “dynamic”. In the case of a static electric field, the electric potential created by a point charge \( q \) is described by Coulomb’s law

\[
\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \mathbf{r},
\]

where \( \mathbf{E} \) is the electric field vector, \( \varepsilon_0 \) is the vacuum permittivity (or electric constant), \( q \) is a single particle charge, \( r \) is the distance from the charge, and \( \mathbf{r} \) is the unit vector pointing from the particle charge to the evaluation point in space.

By definition, the electric potential at a point \( r \) in a static electric field \( \mathbf{E} \) is given by the line integral

\[
\Delta V_{\mathbf{E}} = -\int_0^L \mathbf{E} \cdot \mathbf{d}l,
\]
where $L$ is an arbitrary path connecting the point in infinity (i.e., with zero potential) to point $r$ and $dl$ is the unity path element. Note the dot product of the two vectors in the integral. In physical terms, (2.2) represents the electric work $W$ (scalar variable) of the electric field along the integral path

$$W = q \int_{l_0}^{l} \mathbf{E} \cdot dl = q \Delta V_E,$$

where voltage $V$ is measured in volts [V]. Note that work represents the energy that is needed to move a particle over a certain distance.\(^2\) In the case of a single particle charge $q$ inside an electric field, (2.2) yields its potential as

$$V_E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r},$$

A time-varying electric field, which is relevant to our subject, is always linked to a time-varying magnetic field (and vice versa). Consequently, it is not possible to describe the electric field in terms of a scalar potential $V$ (because the integral (2.2) is now path dependent). Instead, one must use Maxwell’s fundamental equations. For the sake of argument, one possible solution for the scalar potential is

$$-\nabla^2 V = \frac{\rho}{\varepsilon_0},$$

where $\rho$ is the charge density. For more details on Maxwell’s equations, the reader is advised to consult more advanced textbooks on electromagnetism, some of which are listed in the references section.

### 2.5.2 Current

We have established the concept of an electric charge and concluded that a charge moves in space if a force $\mathbf{F}$ is applied, in this case, in the form of an electric field.\(^3\) Observing this flow of charged particles, we define an electric current $I$ (a scalar variable) as the net transfer of particle charges across a surface per unit of time. As a simple analogy, imagine standing on a sidewalk while a parade is marching by. Each person marching in the parade represents one unit of charge and the street width determines how many persons can fit in parallel. Start a stopwatch and count the people who pass over a certain period of time, say, one second. Obviously, the wider the street, the more people pass through the street in a given time, i.e., the higher the “current” of people. Strictly, an electric current $I$ is defined either as the rate of change of charge in time or the current density within the total conducting surface

$$I = \frac{dQ}{dt} = \int_S \mathbf{J} \cdot d\mathbf{s} \quad [A],$$

\(^2\)By implication, one could sweat for whole day while trying to push a wall but no work would be done if the wall did not move, i.e., $dl = 0.$

\(^3\)The electric field $\mathbf{E}$ is defined as the force $\mathbf{F}$ per positive charge $q$ that would be experienced by a stationary point charge at a given location in the field, i.e., $\mathbf{E} = \mathbf{F}/q.$
where current \( I \) is measured in amperes \([A]\), \( Q \) is the total amount of charge through the cross-sectional area \( S \) (not to be confused with the quality factor notification \( Q \) used in this book), \( dt \) is the differential unit of time, \( \mathbf{J} \) is the current density vector, and \( \mathbf{s} \) is the vector of the conducting surface element oriented in space. Note the dot product of the two vectors in the integral. Thus, for the known current, the total amount of transferred charge is

\[
Q = \int_0^t i(t) \, dt. \tag{2.8}
\]

**Example 2.1.** A function that represents an instantaneous current amplitude is shown in Fig. 2.2 (left). The current flows through a conductor whose cross-section is shown in Fig. 2.2 (right). Determine the total amount of charge passing through: (a) from time zero to \( t = 1 \) s; (b) in the time period from \( t_1 = 1 \) s to \( t_2 = 2 \) s; and (c) from time zero to \( t = 2 \) s. In addition, find the value of the current density \( \mathbf{J} \).

**Solution 2.1.** By definition, the amount of electric charge is calculated using the integral (2.8), which in this case becomes trivial, because the flow of current is constant within each of the given time frames, hence

\[
Q_1 = \int_0^{1s} i(t) \, dt = 1 \text{ A} \int_0^{1s} \, dt = 1 \text{ C},
\]

\[
Q_2 = \int_1^{2s} i(t) \, dt = -1 \text{ A} \int_1^{2s} \, dt = -1 \text{ C},
\]

\[
\therefore \quad Q = \int_0^{2s} i(t) \, dt = 0,
\tag{2.9}
\]

that is, the net charge flow is zero. The current density is calculated by definition (2.7), which is also trivial because the current is constant from zero to \( t = 1 \) s and, therefore the current density is constant, i.e.,

\[
I = J \int_S d\mathbf{s} = J \times S \quad \therefore \quad J = \frac{I}{S} = \frac{1 \text{ A}}{7 \text{ mm}^2} = \frac{1}{7} \text{ A/mm}^2 \approx 142.86 \times 10^3 \text{ A/m}^2,
\]

where the cross-sectional area \( S \) is found by inspection of the plot Fig. 2.2 (right) to be \( S = 7 \text{ mm}^2 \). Over the time period from \( t = 1 \) s to \( t = 2 \) s, the current density is \( J = -142.86 \times 10^3 \text{ A/m}^2 \) because the current vector \( \mathbf{I} \) points to the negative side.
2.5.3 Power

From the engineering perspective, it is important to establish not only the amount of energy needed to perform a work, but also the rate of energy exchange, i.e., the rate of either generation or absorption of energy. That brings us to the concept of power \( P \) (a scalar variable), which quantifies how fast, for a given amount of energy, the work is finished. Or, in a strictly mathematical sense, after substituting (2.4) and (2.7), the electrical power \( P \) is

\[
P = \frac{dW}{dt} = \frac{dQ}{dQ} \frac{dQ}{dt} = VI \quad [W],
\]

where power \( P \) is measured in watts \([W]\). To conclude, we keep in mind that all definitions introduced in this section assume either a static or a quasi-static (i.e., steady state) electric field.

Example 2.2. Find the power being delivered to or absorbed by the three elements in Fig. 2.3 at time instance \( t = 5 \text{ ms} \).

Solution 2.2. By definition, we write, \(-454.9 \text{ mW}, 132 \mu\text{W} \) and \(1.35 \text{ W} \).

2.5.4 Impedance

It is accepted convention to reserve the term resistance \( R \) for real resistive, i.e., frequency independent, components and to use the term reactance for the equivalent resistance of an inductor \( X_L \) or a capacitor \( X_C \) at a given frequency. By definition, a reactance is described only by the imaginary term \( j\mathbb{B} \) (including the \( j \) part, which takes care of the phase) of a complex number \( Z = \Re + j\Im \). By the same convention, a serial combination of a resistance and either capacitance or inductance is referred to as impedance \( Z \). For example, a serial connection of resistance \( R \) and inductance \( L \) is said to have impedance of \( Z_L = R + j\omega L \) and \( \omega \) is the radial frequency where the inductor reactance is calculated. Similarly, a serial connection of resistance \( R \) and capacitance \( C \) is said to have impedance of \( Z_C = R + 1/j\omega C = R - j/\omega C \). We note that inductive phase is positive and capacitive phase is negative.\(^4\)

Two important parameters of any impedance are its absolute value \( |Z| \) and argument \( \phi \) (also referred to as a phase). In the complex plane, the real and imaginary axes are set at \( \pi/2 \) angle relative to each other, thus the absolute value and argument of a complex number are calculated using the Pythagorean theorem and trigonometric identities. For example, absolute values of \( RL \) and \( RC \) impedances are

\[
|Z_{RL}| = \sqrt{R^2 + (\omega L)^2} \quad |Z_{RC}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2},
\]

\(^4\)See the definition of the phase of a complex number in Appendix D.
while, by applying the same right-angle triangle rules, the phase is calculated as the ratio of the reactance and resistance (i.e., the imaginary and real) parts of the impedance, while paying attention to the sign of the reactance, i.e.

\[
\tan \phi_{RL} = \frac{j}{R} = \frac{j\omega L}{R} = \frac{\omega L}{R},
\]

\[
\tan \phi_{RC} = \frac{1}{j\omega CR} = -\frac{j}{\omega CR} = -\frac{1}{\omega CR}.
\]

We use all of these relationships extensively in the rest of the book. For the time being, just note that both \(L/R\) and \(RC\) have dimensions of time.

**Example 2.3.** For an ideal capacitor \(C = 100 \text{nF}\) and an ideal inductor \(L = 100 \text{nH}\), calculate the following values at \(f = 100 \text{MHz}\):

1. Find the impedance of a serial connection of the capacitor with a resistor \(R = 6 \text{m} \Omega\).
2. Find the phase of a serial connection of the capacitor with a resistor \(R = 6 \text{m} \Omega\).
3. Find the phase of a serial connection of the capacitor with a resistor \(R = 0 \Omega\).
4. Find the impedance of a serial connection of the inductor with a resistor \(R = 4.6 \Omega\).
5. Find the phase of a serial connection of the inductor with a resistor \(R = 4.6 \Omega\).

**Solution 2.3.** It is handy to first convert the frequency into its equivalent radial frequency, i.e., \(\omega = 2\pi \times 100 \text{MHz} = 628.319 \text{Mrad/s}\), and then by direct implementation of (2.11) to (2.13), we write:

1. \(|Z_{RC}| = \sqrt{(6 \text{m} \Omega)^2 + \left(\frac{1}{628.319 \text{Mrad/s} \times 100 \text{nF}}\right)^2} \approx 17 \text{m} \Omega\).

2. For the phase calculation, we must pay attention to the sign of reactance, i.e.

\[
\tan \phi_{RC} = -\frac{1}{\omega CR} = -\frac{1}{628.319 \text{Mrad/s} \times 100 \text{nF} \times 6 \text{m} \Omega} = -2.65258,
\]

\[
\therefore \phi_{RC} = -69.344^\circ \approx -70^\circ.
\]

3. When resistance \(R = 0\), then \(1/\omega CR \rightarrow \infty\), hence we must take a look at the limit of the tan function. However, this time we pay attention to the sign of the reactance, and we look only at the continuous range of angles within the range \(-90^\circ\) to \(90^\circ\),

\[
\tan \phi_{RC} = -\lim_{R \to 0} \frac{1}{\omega CR} = -\infty \therefore \phi_{RC} = -90^\circ.
\]

4. \(|Z_{RL}| = \sqrt{(4.6 \Omega)^2 + \left(628.319 \frac{\text{Mrad}}{\text{s}} \times 100 \text{nH}\right)^2} = 63 \Omega\).

\[5\]Remember that the tan function is periodic and its values tend to \(+\infty\) on one side and \(-\infty\) on the other.
\[ \tan \phi_{RL} = \frac{628.319 \text{ Mrad/s} \times 100 \text{nH}}{4.6 \Omega} \quad \therefore \quad \phi_{RL} = 85.813^\circ \approx 86^\circ. \]

\[ \tan \phi_{RL} = \lim_{R \to 0} \frac{\omega L}{R} = +\infty \quad \therefore \quad \phi_{RL} = 90^\circ. \]

Therefore, we note that even a small resistance in series with a reactance introduces a visible phase shift relative to the case of \( R = 0 \Omega \), i.e., when the phase equals \( \pm 90^\circ \).

### 2.6 Electronic Signals

In electronic communication systems, the useful information, i.e., the signal, is embedded and carried in the form of voltage or current, or both. Time domain variations of either of these two variables are then modelled using appropriate mathematical functions. For example, digital information is transmitted by switching between two fixed voltage levels, which is modelled by using the pulse function. In wireless radio communications, at least the ones that are the subject of this book, the transmitted signal is embedded into a sinusoidal function. Therefore, we focus on properties of sine waves.

#### 2.6.1 Properties of a Sine Wave

The basic characteristics of a travelling EM wave are based on a sinusoidal function (see Sect. 1.4). Hence, in this section, we focus on several properties of a sine functions that are relevant to RF signal analysis.

It is not difficult to prove that the average value of a sine wave over any integer number of cycles \( nT \) is zero, where \( n \) is the number of cycles and \( T \) is the sine wave period. A geometrical interpretation is that each period consists of one negative and one positive half-cycle, both having the same area. Since the cosine and sine functions are related, \( \cos \omega = \sin (\omega - \pi/2) \), for the purposes of this discussion, it does not matter whether the sine or the cosine function is used in the analysis.

A very important case in engineering is the product of two sine waves. Let us consider the following two sine wave functions, with frequencies \( \omega_1 \) and \( \omega_2 \) and an initial phase difference \( \theta \) at \( t = 0 \),

\[ A = a \sin (\omega_1 t), \quad \text{(2.14)} \]
\[ B = b \sin (\omega_2 t - \theta), \quad \text{(2.15)} \]

so that their product \( x = AB \) is simply written as\(^6\)

\[ \sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]. \]

\(^6\)Use the trigonometric identity \( \sin(\alpha) \sin(\beta) = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]. \]
\[ x = ab \sin(\omega_1 t) \sin(\omega_2 t - \theta) \]
\[ = \frac{ab}{2} \left\{ \cos[(\omega_1 - \omega_2)t + \theta] - \cos[(\omega_1 + \omega_2)t - \theta] \right\} \]
\[ = \frac{ab}{2} (x_1 - x_2) \] (2.16)

and the average value \( x_{avg} \) is then calculated as the sum of the averages of the two terms \( x_1 \) and \( x_2 \). When \( \omega_1 \neq \omega_2 \), the average of the first term \( x_{1avg} \) is

\[ x_{1avg} = \cos[(\omega_1 - \omega_2)t + \theta]_{avg} = 0 \] (2.17)

for an integer number of cycles \( nT \). Note, from (1.7), that the first term has a period of \( T = 1/(f_1 - f_2) \). Following the same argument, the same result is obtained for the second term,

\[ x_{2avg} = \cos[(\omega_1 + \omega_2)t - \theta]_{avg} = 0, \] (2.18)

which is to say that, for the case of \( \omega_1 \neq \omega_2 \), the average value over the integer number of cycles of the product of two sine waves is zero.

However, for the case of identical frequencies \( \omega_1 = \omega_2 = \omega \), (2.16) becomes

\[ x = \frac{ab}{2} \cos \theta - \frac{ab}{2} \cos(2\omega t - \theta), \] (2.19)

where the average of the second term \( \cos(2\omega t - \theta) \) is zero, which leads to

\[ x_{avg} = \frac{ab}{2} \cos \theta. \] (2.20)

In this case, the average value depends upon the phase difference (the two frequencies are identical) and can, therefore, be adjusted to zero or anywhere between \( \pm ab/2 \). As will be demonstrated many times in this book, this observation is very important for RF design because the operation of RF circuits for wireless communication is based on perfect frequency relationships among multiple sinusoidal signals.

2.6.1.1 Root Mean Square

One possible view of a resistor is that it is a device that converts electrical energy into heat energy, which is then dissipated either intentionally (as in a stove heater, for example) or as wasted energy (as in a bulb, for example). Hence, it is important to know how much power is dissipated by the resistor in case of both DC and AC over an integer number of cycles. To do so, let us first consider the simple problem of calculating electrical power \( P \) dissipated by an ideal resistor \( R \) while conducting direct (i.e., constant in time) current \( I \). Electric power was defined in (2.10) and additional forms are

\[ P = VI = I^2 R = \frac{V^2}{R}, \] (2.21)

which, for a given resistance \( R \), is dependent upon the current’s (or the voltage’s) squared value.

To find the answer for the case of periodic alternating current (e.g., \( i = I_m \sin \omega t \)), the calculation of the constant current term \( I^2 \) has to be replaced with the average value of time-varying quadratic
current, i.e. $i_{\text{avg}}^2$, which, by definition, represents a “quadratic mean” or root mean square (RMS) of the current. Hence, calculation of the equivalent dissipated power is as follows:

$$P_{\text{avg}} = i v_{\text{avg}} = i_{\text{avg}}^2 R = i_{\text{rms}} R$$

$$= \sqrt{\frac{1}{T} \int_0^T |i(t)|^2 \, dt} R = \sqrt{\frac{1}{T} \int_0^T (I_m \sin \omega t)^2 \, dt} R$$

$$= \frac{I_m R}{T} \sqrt{\int_0^T \sin^2 \omega t \, dt} = \frac{I_m R}{T} \sqrt{\frac{t}{2} - \frac{\sin 2 \omega t}{4 \omega}}$$

$$= \frac{I_m}{\sqrt{2}} R.$$  \hspace{1cm} (2.22)

The equivalent effective direct current (DC) of a sinusoidal alternating current is the AC peak divided by the square root of two \((2.22)\). In the case of a square wave, $i_{\text{rms}} = I_m$, while for a sawtooth wave, $i_{\text{rms}} = I_m / \sqrt{3}$.

It should be noted that most handheld multimeters assume a sine waveform. They filter the measured signal into an average value and then apply the $1 / \sqrt{2}$ correction RMS factor. Therefore, the measured RMS voltage or current value is correct only if the input signal is sinusoidal. This is because the true RMS value is proportional to the area under the waveform, not to the average value of the waveform itself. For a sinusoidal waveform, the ratio of the average value to the area under the curve is constant, so most of the time the measured result is correct but any distortion or offset leads to errors.

If several sinusoidal functions with various frequencies are added together, for example

$$i = a \sin (\omega_1 t + \alpha) + b \sin (\omega_2 t + \beta) + c \sin (\omega_3 t + \gamma) + \cdots$$  \hspace{1cm} (2.23)

then the RMS value of the sum \((2.23)\) must be squared, however, in this case all inter–products between terms at different frequencies may be ignored because the average values of those products are zero, which leads to

$$i_{\text{rms}} = \sqrt{\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2} + \cdots}.$$  \hspace{1cm} (2.24)

This result shows that, when calculating the power of a multi-tone signal, each tone’s power can be calculated separately, which is the property exploited in Fourier’s analysis.

Finding the RMS value of a random signal is a bit more complicated and is left for a more advanced course.

2.6.1.2 Common Mode of a Signal

A periodic function that fluctuates around an average value other than zero may be thought of as being composed of a DC component $I_{\text{CM}}$ and an AC component added together (see Fig. 2.4), i.e.

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7Use the trigonometric identity $\sin^2 \alpha = \frac{1}{2}(1 - \cos(2\alpha))$. 

A sinusoidal current signal whose common mode, i.e., average, level is $I_{CM}$

$$A = I_{CM} + I_m \sin \omega t, \quad (2.25)$$

where $I_{CM}$ is the constant value and $I_m$ is the maximum sine amplitude. Usually, $I_{CM} > I_m$ (often $I_{CM} \gg I_m$).

**Example 2.4.** Calculate the common mode level $I_{CM}$, AC amplitude $I$, RMS value of the AC component, and RMS value of the square signal in Fig. 2.5 (left). The value of current $I$ is measured in [A] and the time is measured in [ms].

**Solution 2.4.** A square function consists only of linear sections, hence the integration is simplified to a simple addition over the period $T$. By inspection, the function period is $T = 30$ ms; write,

- The common mode of Fig. 2.5 (left), i.e., the DC level, is

$$I_{avg} = \frac{4A \times 10\text{ms} + 1A \times 20\text{ms}}{30\text{ms}} = 2A. \quad (2.26)$$

- The AC component is found by realizing that the square waveform is the sum of its DC and AC components. By inspection, it is straightforward to recognize that the AC waveform must have $I_{AC} = 2A$ during the first 10 ms and $I_{AC} = -1A$ from 10 to 30 ms.

- The RMS value can be calculated, by definition, first for the AC component as

$$I_{rms}(AC) = \sqrt{\frac{(2A)^2 \times 10\text{ms} + (-1A)^2 + 20\text{ms}}{30\text{ms}}} = 1.414A \quad (2.27)$$

then, for the complete square waveform as

$$I_{rms} = \sqrt{\frac{(4A)^2 \times 10\text{ms} + (1A)^2 + 20\text{ms}}{30\text{ms}}} = 2.45A \quad (2.28)$$
or, alternatively, the total RMS value could be calculated as the sum of the RMS squares of the DC and AC components, as

\[ I_{\text{rms}} = \sqrt{I_{\text{DC}}^2 + I_{\text{rms}}^2 (AC)} = \sqrt{(2 \text{A})^2 + (1.414)^2} = 2.45 \text{A}, \]

which gives the same result because the RMS value of a DC level does not change.

### 2.6.2 DC and AC Signals

A *signal* is loosely defined as any time–varying event being observed. In electronic communications, signals are processed in form of either *current* or *voltage*; signal transmission can be either wired or wireless.

Two general categories of electronic signals are DC signals that have constant amplitude in time (for example, a battery voltage) and AC signals that have varying amplitude in time (for example, voltage amplitude measured at the wall power outlet). Further, an AC signal can be either periodic or aperiodic. Examples of periodic AC signal shapes are sinusoidal, square and saw waveforms, i.e., signals consisting of fixed, time-repetitive patterns. An example of an aperiodic electronic AC signal waveform is thermal noise. Naturally, DC signals have a simpler mathematical representation and treatment than periodic AC signals. On the other hand, aperiodic, or random, signals are more complicated than periodic signals and they are treated using mathematical tools from statistical analysis.

In this section, we review terminology related only to the most important form of AC signals, sinusoidal signals. Without being concerned about the nature of the signal, how it was generated, or what physical quantity it represents, a general sine-wave function is represented by

\[ a = A_p \sin(\omega t + \phi) \quad (2.30) \]

where:

- \(a\) is the instantaneous value of time-varying quantity (voltage, current, power, …).
- \(A_p\) is the maximum or peak amplitude.
- \(\omega\) is the angular frequency (related to frequency as \(\omega = 2\pi f\)).
- \(\phi\) is the initial phase (often assumed to be zero).
- \(t\) is the time variable.

Figure 2.6 shows two common representations of AC signal (2.30), namely a phasor (or rotating vector) and its equivalent time-domain graph, where:

- \(T\) is the period, i.e., the time interval required by the rotating vector to finish one full \(2\pi\) cycle \((T = 1/f)\).
- \(\theta\) is the instantaneous angle (not to be confused with the initial phase \(\phi\)).

And, as we explained in Sect. 1.4.4, if two sinusoidal waveforms have the same frequency, then they are also related by phase difference \(\Delta = \phi_1 - \phi_2\) (see Fig. 1.10). Further, depending upon the relative values of the instantaneous phases of the two, it is said that one waveform is either “lagging” or “leading” the other. For example, waveform \(A_1\) in Fig. 1.10 is leading waveform \(A_2\) by \(\Delta\). Of course, we keep in mind that, because the two waveforms have the same frequency, the phase difference is constant, otherwise it would not have been defined at all.
2.6.3 Single-Ended and Differential Signals

Typical signals, such as the sinusoids in Figs. 2.4 and 2.6, are also known as “single-ended” signals because they consist of only one waveform that is referenced to the local ground. In this section, we introduce a signal form that is very important to engineering, known as a differential signal, which is created by using two single-ended sinusoidal waveforms in the following relationship. They have:

- Equal amplitudes
- Equal frequencies
- Equal common mode
- Opposite phases, i.e., the phase difference is $\pi$

Let us consider two sinusoidal signals $v_1$ and $v_2$ as

$$v_1 = V_{CM} + V_m \sin \omega t,$$  \hspace{1cm} (2.31)

$$v_2 = V_{CM} - V_m \sin \omega t,$$  \hspace{1cm} (2.32)

where (2.31) and (2.32) formalize the required relationship between the two waveforms, shown in Fig. 2.7.
If these two signals are added, then the sum is, obviously, \( v_1 + v_2 = 2V_{CM} \), which is a DC signal and the \( v_1 \) and \( v_2 \) waveforms are lost. However, if they are subtracted, then we write

\[
v_{\text{diff}}(t) = v_1 - v_2 = 2V_m \sin \omega t, \tag{2.33}
\]

which is a very interesting result because the original waveform\(^8\) is still preserved, amplified by a factor of two and shifted down by the common mode value. The interesting part is that the amplification was achieved by the addition of two signals (one of which was negative) instead of multiplication. We note that the gain of factor two is significant, especially when we have weak signals to start with.

Let us explore this idea a bit further and assume that two conductive wires carrying the \( v_1 \) and \( v_2 \) signals are located physically close to each other. With that assumption, any interference signal \( n(t) \) is added equally to both \( v_1 \) and \( v_2 \), i.e.

\[
v_1 = V_{CM} + n(t) + V_m \sin \omega t, \tag{2.34}
\]

\[
v_2 = V_{CM} + n(t) - V_m \sin \omega t, \tag{2.35}
\]

which, after subtraction again, results in (2.33). In other words, the common interfering signal is removed from the differential signal. These two properties of differential signals, namely the gain and the immunity to common noise, are beneficial and important enough that most modern, high-performance, signal-processing circuits are designed to process differential signals. However, for the sake of simplicity and accepted educational methodology, all circuits in this textbook are assumed to be single-ended, leaving differential architectures for more advanced courses.

### 2.6.4 Constructive and Destructive Signal Interactions

The relative phase between two periodic signals is very important from the perspective of their sum. In a circuit network, two currents entering the same node add up in accordance with KCL, while two voltages within the same branch add in accordance with KVL. In a realistic circuit implementation, it is almost inevitable to have two or more conductive wires in close proximity to each other. Unless they have exactly the same potential along their full respective lengths at all times, there is always capacitive cross-coupling between the two. Consequently, the two signals do interact, i.e., add, with each other.

In Sect. 2.6.3, we encountered the intentional subtraction of two signals with opposite phases for the purpose of creating a differential signal and exploiting its benefits, which is an example of constructive signal addition. However, in general, the amplitudes, phases, and frequencies of two adjacent signals are not equal (see Fig. 2.8). A special case of interest is when the two interacting signals are opposite in phase and have equal frequency and equal (or almost equal) amplitude (see Fig. 2.9). Under these special conditions, the two signals cancel, i.e., their sum is zero, and we refer to this interaction as destructive addition. We keep in mind that the concept of signal addition applies to all signals, not only to single tones. It is not difficult to see, for example, how one harmonic within the complicated signal spectrum is easily removed from the spectrum with destructive addition of the appropriate single tone, i.e., the one with the same frequency and amplitude and opposite phase.

\(^8\)Remember, except for the phase difference, the two initial waveforms are identical.
It is important to realize that both constructive and destructive signal additions are used intentionally in signal processing, as we will see later in this book.

2.7 Signal Quantification

Periodic signals are arguably the most important category of signals in the design of RF communication systems. Therefore, it is important that we become familiar with the metrics used to quantify periodic RF signals. Specifically, we are more interested in RF signal power levels than in the instantaneous values of individual voltages and currents. The level of an RF signal’s RMS power is traditionally expressed in $dB$.

2.7.1 AC Signal Power

So far, we have introduced AC through a pure resistive network. In general, we need to expand our analysis to include inductive and capacitive elements as well. Being energy storage components, these reactive elements may cause reversal of energy flow (i.e., power flow) within the network. Consequently, it is common in the engineering community to define three “types” of power: real power $P$ (i.e., power delivered to a pure resistive network); reactive power $Q$ (i.e., power delivered to reactive components $L$ and $C$); and complex power $S$ (i.e., power delivered to a general RLC network); where the modulus of complex power $|S|$ is referred to as apparent power. At any given moment, the
Fig. 2.10 The instantaneous voltage (thin solid line), current (dashed line), and power (thick solid line) in an AC circuit branch showing phase difference $\phi$

Instantaneous power delivered to any circuit element or network is given by product $p = vi$, where $p$ is the instantaneous power, $v$ is the instantaneous voltage and $i$ is the instantaneous current. However, in the case of alternating currents and voltages, there is a very important consequence to notice, which we show here.

Let us assume that the instantaneous values of current and voltage in one branch of a circuit are given as follows:

\[
i = I_p \sin \omega t, \tag{2.36}
\]

\[
v = V_p \sin(\omega t + \phi). \tag{2.37}
\]

In other words, there is a phase difference of $\phi$ between the current and the voltage of that particular branch. Then, the instantaneous power is calculated as

\[
p = vi = V_p I_p \sin \omega t \sin(\omega t + \phi). \tag{2.38}
\]

Surprisingly, (2.38) suggests that at some instances in time the power is positive and at other instances the power is negative (see Fig. 2.10). In order to correctly interpret the above statement, keep in mind that the sign of power indicates only the direction of energy flow. Simply put, “positive power” indicates that external world is supplying power to the circuit, while “negative power” indicates that the circuit is delivering power to the world. This is possible only if some devices capable of storing energy are present in the circuit, i.e., inductors or capacitors.

Using the same method to calculate the average power of this circuit branch as for obtaining (2.22), we write

\[
P = \frac{1}{T} \int_0^T vi \, dt = \frac{V_p I_p}{T} \int_0^T \sin \omega t \sin(\omega t + \phi) \, dt
\]

\[
= \frac{V_p I_p}{T} \left[ \cos \phi \int_0^T \sin^2 \omega t \, dt + \sin \phi \int_0^T \cos \omega t \sin \omega t \, dt \right],
\]

\[
\therefore P = \frac{V_p I_p}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi = \frac{V_{\text{rms}}^2}{R} \cos \phi. \tag{2.39}
\]

An important observation regarding this result is that AC power depends upon the cosine of the phase difference between the corresponding current and voltage. A direct consequence of this relationship
is that in special cases when the phase difference $\phi = \pm 90^\circ$ (i.e., in a purely reactive circuit), the AC power factor $\cos \phi$ is zero. When the power factor $\cos \phi = 1$ (i.e., in a purely resistive circuit) the power is at maximum. Therefore, a power factor less than one always indicates the presence of reactive (i.e., L and C) components in the circuit. Keep this important observation in mind until we reach the discussion on capacitors and inductors in Chap. 4.

2.7.2 The Decibel Scale

In wireless communication systems, it is common to have an RF transmitter delivering signals at power levels of the order of watts, kilowatts or even megawatts. As a comparison, the signal power level at the receiving antenna can be only a few picowatts. That is, the power ratio of the transmitted and received signals may be as large as $1,000,000,000,000,000 : 1$. Clearly, using absolute numbers is not the most convenient way of presenting RF signal relations.

By definition, the dB is a logarithmic unit of measurement that expresses the magnitude of a physical quantity (usually power) relative to a specified or implied reference level. Its logarithmic nature allows very large and very small ratios to be represented by a convenient number. Being a simple ratio of two quantities, the dB is a dimensionless unit.

The Bel scale is defined as the logarithm of the base 10 of the power ratio. One Bel is a factor of 10, two Bels is a factor of 100, and so on. It is common, however, to use a more practical dB unit, so that 10 dB is a power ratio of 10, 20 dB is a ratio of 100, and so on. It is useful to remember that 3 dB is a power ratio of $\approx 2$, 6 dB is a power ratio of $\approx 4$, and so on.

Thus, power ratio (i.e., power gain $G$) is expressed in dB as

$$G_{\text{dB}} = 10 \log \frac{P_2}{P_1},$$  \hspace{1cm} (2.40)

where $P_1$ and $P_2$ are the two signal powers being compared, for example, the input and output powers of an amplifier. Keep in mind that when $G_{\text{dB}}$ is a positive number, it indicates that $P_2 > P_1$ (often referred to as “gain”), while a negative $G_{\text{dB}}$ number indicates that $P_1 > P_2$ (often referred to as “loss”).

If we want to express a voltage (or current) ratio (i.e., a voltage or current gain $A$) of two signals $v_2$ and $v_1$ in the dB scale, and assuming that both signals are measured at the same impedance $Z$, then the gain is expressed in dB as:

$$A_{\text{dB}} = 10 \log \frac{v_2}{v_1} = 10 \log \frac{v_2^2}{v_1^2/Z} = 20 \log \frac{v_2}{v_1},$$  \hspace{1cm} (2.41)

which is to say that a voltage (or current) ratio of 10 equals 20 dB gain, a ratio of 0.1 equals $-20$ dB gain, a ratio of 100 is equal to 40 dB gain, etc. It is handy to practice mental conversion between ratios and dB units by taking the number of zeros in the ratio exponent and multiplying it by 10 for power or by 20 for voltage or current; the final number is in dB units.

Because dB numbers are dimensionless they do not say anything about the absolute power levels being compared. Hence, from a specified gain, we can only conclude whether there was power amplification or power loss. From such a statement, however, we cannot conclude either what kind of gain it is (i.e., power, voltage, or current) or which two absolute signal values are being compared.

Therefore, for low-power applications, the standard reference value for power specification is defined in the form of the dBm scale, which is set to compare a given power level relative to the absolute power level of $P_1 = 1$ mW. After substituting the 1 mW level in (2.40), we write

$$G_{\text{dBm}} = 10 \log \frac{P_2}{1 \text{ mW}},$$  \hspace{1cm} (2.42)
indicating that 1 mW of power is equivalent to 0 dBm. Similarly, if an amplifier delivers 10 mW of power, it is usually expressed as 10 dBm gain, 100 mW as 20 dBm, etc. Note that, due to the same scale, in power calculations the units of dB and dBm are added to or subtracted from each other, i.e., they are interchangeable as long as we keep the 1 mW absolute reference in mind.

**Example 2.5.** A cell phone transmits $P_1 = +30$ dBm of signal power from its antenna. At the receiving side, the signal power is $P_2 = 5$ pW. Calculate the propagation loss of the transmitting medium.

**Solution 2.5.** We convert the received power into dBm units as

$$P_2 = 10\log\left(\frac{P_2}{1\text{mW}}\right) = 10\log\left(\frac{5\text{pW}}{1\text{mW}}\right) = -83\text{dBm}.\quad (2.43)$$

Therefore the signal experienced attenuation $A$ of

$$A = P_2 - P_1 = 30\text{dBm} - (-83\text{dBm}) = -113\text{dBm}.\quad (2.44)$$

### 2.7.3 The Meaning of “Ground”

In our discussions, we routinely assume that the concept of “ground” is clear to everyone and we simply assume that the ground is at zero potential. Often, we forget that the zero level was set as a relative point, not the absolute. Let us be reminded that any measured voltage value is implicitly assumed to be the potential difference between two points, one of which is arbitrarily declared the “ground”, i.e., the zero reference. The absolute potentials are, by definition, measured relative to some point at infinity. Because of that, it is more practical to arbitrarily pick one of the two points and declare it to be the “local ground”. When it is necessary to emphasize that a voltage is measured between two specific points in a circuit, the notation $V_{AB}$ is used, where $A$ is the node with higher potential and $B$ is the node with lower potential (keep in mind that $V_{AB} = -V_{BA}$). It is especially important to have a clear understanding of the concept of “ground” when dealing with differential signal circuits because a differential signal is always measured as a difference between the two signals and its value is independent of the ground level.

**Example 2.6.** What is the value of resistance ($R_{a,b}$) between points $a$ and $b$, in Fig. 2.11 if: (a) $V = 1\text{V}$; (b) $V = 0\text{V}$; (c) $V = -1\text{V}$; (d) $V = -1\text{MV}$?

**Solution 2.6.** If, for the moment, we completely ignore the existence of the voltage source $V$, then between points $a$ and $b$ there is a serial connection of two resistors, so we normally write $R_{a,b} = R_1 + R_2$. Did you notice that in order to calculate the equivalent serial resistance, we did not
need to find the potential at the joining node (1) between the two resistors? That is, the potential at node (1) is not part of the equation. Hence, the serial resistance stays the same whatever the potential at node (1). The voltage $V$ is referenced to an arbitrary point in space that we temporarily declared the ground; it could have been node (1) with no difference whatsoever.

### 2.8 Summary

Each profession has its own technical language and fluency in the language is critical for one’s professional career. Similar to native speakers who immediately pick up even the smallest mistake by a non-native speaker, experienced professionals in a field are able to estimate the competence of the other person simply by picking up on incorrect use of terminology. In this chapter, we reviewed some of the very basic definitions that are considered fundamental knowledge in the field and are found in the vocabulary of all engineers and scientists.

### Problems

#### 2.1

Using a graphing tool of your choice, create overlapping plots of the following single-tone signals at $f = 10$ MHz:

$$
S_1 = 2.0 \sin (\omega t), \quad S_2 = 2.0 \sin (\omega t + \pi/3), \quad S_3 = 2.0 \sin (\omega t + \pi/2),
$$

$$
S_4 = 2.0 \sin (\omega t + 3\pi/4), \quad S_5 = 2.0 \sin (\omega t + 2\pi), \quad S_6 = 2.0 \sin (\omega t + 4\pi/3). \quad (2.45)
$$

Observe the relationships between various signals in terms of their phase differences (hint: to start, begin at the zero time) and how the amplitudes are related to each other at any given point in time. Practice calculating the signal amplitudes at various time points by knowing their phase. For a given frequency, practice expressing various phase differences in the units of time.

#### 2.2

Using a graphing tool of your choice, create plots of the following signals at $f = 10$ MHz:

$$
S_1 = 2.0 \sin (\omega t), \quad S_2 = 2.0 \sin (\omega t + \theta). \quad (2.46)
$$

Plot $S_3 = S_1 + S_2$ for the following phase differences: $\theta = 0, \pi/3, \pi/2, \pi, 3\pi/2, 2\pi, 3\pi, 4\pi, \ldots$ Observe how the amplitude of $S_3$ changes relative to the phase differences between $S_1$ and $S_2$. In particular, pay attention to what happens to the amplitude of $S_3$ when $\theta = k\pi$ and $k = 0, 1, 2, 3, \ldots$.

#### 2.3

Overlap plots of the following single-tone signals (assume $f = 10$ MHz):

$$
S_1 = 2 \sin (\omega t), \quad S_2 = -\sin (2\omega t), \quad S_3 = \frac{2}{3} \sin (3\omega t),
$$

$$
S_4 = -\frac{1}{2} \sin (4\omega t), \quad S_5 = \frac{2}{5} \sin (5\omega t), \quad S = \sum_{k=1}^{5} S_k. \quad (2.47)
$$

To what waveform shape is $S$ converging, starting with $S = S_1 + S_2$, then $S = S_1 + S_2 + S_3$, etc., assuming that more $S_k$ terms are added to the sum? Now, plot the sum without, for example, the $S_2$ term and observe what the $S$ waveform looks like. What about without the $S_3$ term? Try dropping other terms or combinations of terms from the sum and observe the outcome.
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Fig. 2.12 Schematic diagram for Problem 2.7 (left) and Problem 2.9 (right)

This exercise demonstrates the frequency spectrum components of a complicated signal and how the signal becomes unrecognizable (i.e., distorted) to various extents if some of its terms are filtered out.

Note that this particular waveform consists (aside from the fundamental tone $\omega$) of both even and odd harmonics, i.e., $2\omega, 3\omega, 4\omega, \ldots$.

2.4. Overlap plots of the following single-tone signals (assume $f = 10$ MHz):

$$S_1 = \frac{4}{\pi} \sin(\omega t), \quad S_2 = \frac{4}{3\pi} \sin(3\omega t), \quad S_3 = \frac{4}{5\pi} \sin(5\omega t),$$

$$S_4 = \frac{4}{7\pi} \sin(7\omega t), \quad S_5 = \frac{4}{9\pi} \sin(9\omega t), \quad S = 5 \sum_{k=1} \frac{S_k}{k}. \quad (2.48)$$

Note that the frequency spectrum of this particular $S$ signal comprises (aside from the fundamental tone $\omega$) only odd harmonics, i.e., $3\omega, 5\omega, 7\omega, \ldots$

2.5. Calculate the average energy in a rectangular pulse whose amplitude is $v = 2V$ and width is $t = 1$ ms. The energy is dissipated in a resistor $R = 100\Omega$.

2.6. A current flowing in a positive direction through a wire is defined as:

$$i(t) = \begin{cases} 
-2t, & \text{if } t < 0 \\
+3t, & \text{if } t \geq 0 
\end{cases} \quad (2.49)$$

Find the following values:

(a) $i(-2.2s)$.
(b) $i(+2.2s)$.
(c) The total charge $q$ that has flowed through the wire within the time interval $-2s \leq t \leq 3s$.
(d) The average value of $i(t)$ within the same time interval.

2.7. Find the power absorbed by each element in the circuit shown in Fig. 2.12 (left).

2.8. For a resistor $R$ with a current $i$ entering its more positive terminal and voltage $v$ across its terminals, find:

(a) The resistance $R$ if $i = -1.6$ mA and $v = -6.3$ V.
(b) The absorbed power $P$ if $v = -6.3$ V and $R = 21\Omega$.
(c) The current $i$ if the voltage is $v = 8\text{ V}$ and $R$ absorbs power $P = 0.24\text{ W}$.
(d) The conductance $G$ if the voltage is $v = -8\text{ V}$ and $R$ absorbs power $P = 3\text{ mW}$.
2.9. Find $v_{R_2}$ and $v_x$, shown in Fig. 2.12 (right).

2.10. Find the power absorbed by each component in Fig. 2.13 (right).

2.11. Find the equivalent Thévenin (see Sect. 4.2.3) circuit in Fig. 2.13 (left).

2.12. Find the RMS for the following waveforms where $t$ is time, $f$ is frequency, $a$ is the peak amplitude, and $T$ is the function period:

(a) Square wave

$$y = \begin{cases} 
a & t < 0.5T \\
-a & t \geq 0.5T
\end{cases}$$  \hspace{1cm} (2.50)

(b) Modified square wave

$$y = \begin{cases} 
0 & t < 0.25T \\
a & 0.25 \leq t < 0.5T \\
0 & 0.5 \leq t < 0.75T \\
-a & t \geq 0.75T
\end{cases}$$  \hspace{1cm} (2.51)

(c) Sawtooth wave

$$y = 2at - a$$  \hspace{1cm} (2.52)

2.13. Calculate the average power delivered to an $R = 5\Omega$ resistor and $i_{\text{rms}}$ for the current waveform in Fig. 2.14.

2.14. Calculate the average power delivered to an $R = 4\Omega$ resistor if the instantaneous current is: (a) $i = (2\cos 10t - 3\cos 20t)$ A; and (b) $i = (2\cos 10t - 3\cos 10t)$ A.
Wireless Communication Electronics
Introduction to RF Circuits and Design Techniques
Sobot, R.
2012, XVIII, 386 p., Hardcover
ISBN: 978-1-4614-1116-1