Preface

In March 2008, an *International Conference on Partitions, q-Series, and Modular Forms* was held at the University of Florida. This conference was one of the highlights of the year-long Program in Algebra, Number Theory, and Combinatorics (ANTC) held in the Mathematics Department. The University of Florida Mathematics Department has been the venue of several conferences covering the theory of partitions and q-hypergeometric series. But what made this 2008 conference so special was that its outstanding success led to the start of year-long programs in ANTC in our department, with the 2007–2008 program being the first. The 2008 conference received generous support from the National Science Foundation, the Department of Mathematics, the College of Liberal Arts and Sciences, and the Office of Research and Graduate Programs of the University of Florida and for this we are most grateful. The organizers of this conference were Krishnaswami Alladi, Alexander Berkovich, and Frank Garvan of the University of Florida, and George Andrews of The Pennsylvania State University who is Distinguished Visiting Professor in Florida each year in the Spring Term. This volume is the outgrowth of the 2008 Gainesville conference on partitions, q-series, and modular forms, and contains major surveys and research papers related to some of the talks given at the conference. The papers have been arranged in the alphabetical order of the authors names.

Major MacMahon, a towering figure in the area of Combinatory Analysis, initiated several major lines of study, one of which was the subject of plane partitions. He created a calculational and analytic method for the purpose of determining the generating function for plane partitions, but it did not turn out to be what was he had intended, so he had to develop an alternative treatment in the next two decades. George Andrews and Peter Paule provide a charming account of the resurrection of MacMahon's dream of using partition analysis to treat plane partitions and show how the computer algebra package *Omega* has now played a decisive role in a successful treatment of plane partitions via partition analysis. Andrews and Paule point out that many essential features of this approach were known to MacMahon and so this solution is very much along the lines of MacMahon's original dream.

Srinivasa Ramanujan's discovery of congruences modulo 5, 7, and 11 for the partition function stunned the mathematical world and led to a deep study of congruences not just for partition functions but for coefficients of certain types of modular forms, an area that is intensely active even today. Ramanujan published three papers on congruences for the partition function $p(n)$, but there are several fascinating identities connected to congruences for partition functions in his "Lost Notebook." In particular, page 182 of Ramanujan's Lost Notebook is devoted to partitions and here Ramanujan has results on congruences of some general partition functions. Ramanujan demonstrates how some of these congruences follow by clever use of Jacobi's triple product identity for theta functions and Euler's pentagonal number theorem. Bruce Berndt, Chadwick Gugg, and Sun Kim closely investigate Ramanujan's elementary method and provide a detailed treatment of the entries on page 182 of Ramanujan's Lost Notebook. In doing so, they deduce some new results as well, one being a new congruence result for partition functions using r colors, and for this they employ a remarkable identity due to Winquist.

One of the sensational discoveries in recent years is the connection between mock theta functions of Ramanujan and the theory of harmonic Maass forms. Kathrin Bringmann and Ken Ono, two of the primary architects of this major development, have exploited this fundamental connection to explain many intriguing links between Borcherds products, values of modular L-functions, and Dyson's generating functions for ranks of partitions, to name a few. Here Bringmann and Ono study harmonic Maass forms with a certain bound on their weights and show that such forms can be described explicitly as linear combinations of Maass-Poincare series thereby extending the fundamental results of Rademacher and Zuckerman dating back to the 1930s.

Ever since Hardy and Ramanujan produced their remarkable asymptotic series for the partition function $p(n)$ by means of the powerful circle method in 1918, there has been detailed investigation on the asymptotic sizes of several partition functions by various analytic and elementary methods. In a charming paper, Rodney Canfield and Herb Wilf show that if the set of allowable parts S is infinite, then the function $p_S(n)$, which enumerates the number of partitions of n whose parts come from S, grows faster than any polynomial, no matter how sparse S is. They show how their results are best possible by explicitly constructing sparse sets S for which $p_S(n)$ grows faster than a polynomial but smaller than a prescribed function. They conclude their paper with some interesting open problems.

Jacobi's celebrated triple product identity for theta functions may be viewed as the beginning of a chain of identities, each member of the chain being of higher complexity than its predecessor. Thus the "next level" identity is the quintuple product identity for which several proofs are known. These identities built upward from the Jacobi triple product identity are viewed as special cases of the Macdonald identities. The next paper in this volume is by Zhu Cao who shows how the quintuple and septuple product identities can be proved in a simple manner by utilizing properties of the cubic and fifth roots of unity. Cao's method is a variation of the ideas used by Shaun Cooper in his 2006 survey of the proofs of the quintuple product identity.

In 1998, Bousquet-Melou and Kimmo Eriksson produced a startling refinement of Euler's rudimentary result connecting partitions into odd parts and distinct parts by means of the idea of Lecture Hall Partitions. This led to a flurry of activity on Lecture Hall-type identities where there is a constraint on the ratio between consecutive parts. Carla Savage, Sylvie Corteel, and Andrew Sills conduct a novel study here of Lecture Hall-type identities. They first extend the approach of Bousquet-Melou and Eriksson to encompass sequences of ratios in which the denominators are not monotone, and by doing so they derive new partition identities which are reminiscent of the classical partition theorems of Göllnitz.

The classical theta functions of Jacobi are considered among the most significant discoveries of the nineteenth century in the field of analytic functions. The concept of a theta function has been vastly generalized to include multivariable versions and to extend the domain of definition to Riemann surfaces. The famous Thomae problem deals with proportionalities between theta constants associated with certain singular curves called Hutchinson's curves, which define compact Riemann surfaces of genus 2. Hershel Farkas, a leading authority on Riemann surfaces and the study of theta functions and theta constants, discusses generalizations of Hutchinson's curves to higher genus values and the theta relations that can be deduced from such generalizations.

In the entire theory of partitions and q-series, the celebrated Rogers-Ramanujan identities are unmatched in simplicity of form, elegance, and depth. These identities and their generalizations arise in a variety of settings ranging from the study of vertex operators in the theory Lie algebras to conformal field theory in physics. Basil Gordon, one of the foremost authorities in the theory of Rogers-Ramanujan-type identities, studies the parity of the coefficients of the original two Rogers-Ramanujan identities. He shows, how in contrast to the partition function, the parity of these coefficients can be determined much more precisely.

Ramanujan's mock theta functions are considered among his deepest contributions. These intriguing functions, the discovery of which Ramanujan communicated in his last letter to Hardy from India in 1920 shortly before he died, continue to fascinate mathematicians to this day. We owe much to George Andrews and others for the present understanding of mock theta functions in the context of the theory of partitions and q-hypergeometric series. The recent work of Ono, Brigmann, Zwegers, and others connecting mock theta functions to Maass forms has led to a clearer understanding of how mock theta functions are connected to the theory of modular forms. This is a modern point of view, yet there is much significant work that has been done in recent years in the classical theory of mock theta functions. Basil Gordon has discovered new mock theta functions of order 8 and Richard McIntosh has conducted a systematic investigation on the asymptotics of the coefficients of mock theta functions, to give examples of significant work on the classical aspects of the subject. In this volume Gordon and McIntosh provide a fine comprehensive survey of the classical theory of the mock theta functions from Ramanujan's time to the present.

The Cauchy-Sylvester theorem on compound determinants is at the interface of q-series, algebraic combinatorics, special functions, and combinatorial representation theory. Masahiko Ito and Soichi Okada discuss the application of the Cauchy-Sylvester theorem to the evaluation of a certain multivariable integral of Jackson and discuss implications of the approach to determinant formulae for certain classical group characters.

Ramanujan's original notebooks contain hundreds of formulas that have inspired major lines of research in the twentieth century. The paper by Yasushi Kajihara in this volume contains a vast number of multiple series identities associated with root systems; most of these identities concern multiple series generalizations of q-series identities found in Ramanujan's notebooks.

In the last decade, one of the most important developments in the theory of theta function identities of Jacobi is the work of Steve Milne who provided multivariable versions and obtained in that process exact formulas for various sums of squares representations. In this volume Milne provides a comprehensive treatment of nonterminating Whipple transformations for basic hypergeometric series in $U(n)$. Among other things, classical work on very well-poised series on unitary groups is extended. It is expected that this approach will extend to a similar treatment of multiple basic hypergeometric series associated with the root system D_n.

In summary, the Conference on Partitions, q-Series, and Modular Forms held in Gainesville in 2008 was a meeting ground for the world's experts in these areas to interact and discuss the latest advances. This book contains survey and research papers by leading experts as outgrowths of that conference and covers a broad area of mathematics covering significant parts of number theory, combinatorics, and analysis. We are most thankful to Elizabeth Loew of Springer for including this volume in the Developments in Mathematics series.

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