Preface

In recent years, derivative trading and research has refocused on liquid instruments, and in particular on European options. Increased market turbulence, unrelenting standardisation and stronger regulatory oversight altogether call for robust and well-calibrated models of the static smile. Furthermore, many agents (e.g. banks, hedge funds) now deploy sophisticated strategies, involving both assets and options, to capture some type of *alpha* or relative value. These algorithms demand accurate, non-arbitrable modelling of the joint dynamics of the underlying and its implied volatility surface.

In principle, Stochastic Volatility (SV) model classes (such as SABR, Heston, LSV or SV term structure frameworks) offer the most potential to fulfil these objectives. Indeed they can reach the statics and represent the dynamics of the smile in a rich, realistic and flexible fashion. In practice however, their lack of tractability makes classical SV models difficult to manage. The primary cause is that the derivation of the smile's exact shape and dynamics from the model's SDE is rarely achievable in closed form, which leaves only numerical methods. This is an issue not only for calibration, but also for computing and hedging the risk of complex derivatives (especially Vega risk) and thus for model design and analysis.

The academic answer to these limitations of stochastic *instantaneous* volatility (SInsV) models has been twofold. The first tack has been to develop numerous *approximation* methods for the static smile of specific SInsV models, mostly using small-time asymptotic techniques up to some low order. These methods exploit *either* an analytic (i.e. PDE) *or* a probabilistic (i.e. SDE) approach, and include for instance heat kernel and WKB expansions, singular perturbations, Malliavin calculus or saddlepoint approximations. Yet none of these approximation methods is flexible enough to provide arbitrary precision across a wide range of SInsV models, and neither do they address the dynamics of the smile. Therefore they cannot adapt easily to rapidly changing and challenging market conditions.

The second academic direction has led to new SV *market model* classes, which take as input *some* representation of the option price surface, such as implied volatility (SImpV models), local volatility (SLocV) or variance swaps (SVarS). Although these frameworks are very informative and theoretically promising, they have not been widely adopted by practitioners, mainly for issues of validity or speed.

This book presents a *third way*: the Asymptotic Chaos Expansion approach (ACE). The ACE algorithm links the SInsV and SImpV classes by *combining* standard PDE and SDE approaches, to provide pure asymptotics of the smile's shape and dynamics. These differentials are computed in closed form, at any order, and are established for a generic SV model.

Its model versatility allows ACE to cover vanilla models such as SABR, Heston or FL-SV, but also the case of fixed—and stochastic-weights baskets, or powerful interest rates/term-structure frameworks such as SV-HJM and SV-LMM.

At low order, the ACE results can be used for rapid model design and analysis. For instance, given some model specification they explain the influence of each parameter on the smile's level, skew and curvature, as well as on their joint evolution (e.g. the backbone). As a corollary, ACE shows easily which systematic bias affects the *most probable path* heuristic for local volatility models. Conversely, the model's parameters can be replaced by these smile-related quantities, which are more meaningful for trading, leading to an *intuitive re-parametrisation*.

Also, since its algorithm is programmable, ACE gives straightforward access to higher orders, which provides fast and arbitrary-precision approximations. The latter can naturally be employed for calibration, since *shape* proxies are traditionally used to mark the model to the static smile. But approximations of the *dynamics* can also be matched to the time series for the underlying and its (implied and/or realised) volatility: this is often called *dynamic calibration*. These approximations also benefit the valuation and hedging of structured trades. For instance, in an American Monte Carlo context, such fast access to the smile associated to the model state variables allows us to significantly refine the exercise boundary.

Importantly, although ACE results can be used on their own, they can also complement other asymptotic approaches, which are usually specialised to a given model. For instance they can increase the precision of the static approximations, and/or provide the missing dynamic information.

This monograph is based on my Ph.D. manuscript [1]. Among other improvements, a large part of the calculus has been simplified and/or extracted from the main body. This has increased clarity and will hopefully help convince the reader that ACE's practical mechanics are actually quite simple. Other enhancements incorporate some updated references, as well as new comments and interpretations pertaining to both results and applications.

The intended audience of this book includes researchers and academics interested in stochastic volatility, market models and/or asymptotics. It also comprises practitioners, especially *quants* supporting vanilla/structured trading desks who wish to improve their calibration or hedging procedures.

London, December 2012 David Nicolay

Reference

1. Nicolay, D.: Volatility dynamics. Ph.D. thesis, Ecole Polytechnique (2011)

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